New Framework for Optimal Scheduling of Combined Heat and Power with Electric and Thermal Storage Systems considering Industrial Customers Inter-Zonal Power Exchanges

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Abstract

Introducing Combined Heat and Power (CHP) units into Active Distribution Network (ADN) can significantly affect the problem of optimal generation scheduling. A new method for solving the problem of Optimal Scheduling of Combined Heat and Power (OSCHP) units of an ADN with Electric Storage Systems (ESSs) and Thermal Storage Systems (TSSs) considering Industrial Customers (ICs) Inter-Zonal Power Exchanges (IZPEs) is presented. The ADN operator may use CHP units to supply its ICs and based on smart grid conceptual model, it can transact electricity with upstream network. However, the electricity transactions between the ADN and its ICs in normal and contingency scenarios may highly complicate this problem. In this paper, linearization techniques are adopted to linearize equations and a two-stage stochastic mixed integer linear programming (SMILP) model is utilized to solve the problem to determine the optimal generation scheduling units. The first stage models the behaviour of operation parameters, minimizes the operation costs, and checks the feasibility of the ICs' requested firm and non-firm IZPEs, while the second stage considers system's stochastic contingency scenarios. The competitiveness of ADN in the deregulated market can be improved by adjusting the proposed decision variables in the two-stage optimization procedure. The proposed method is applied to 18- and 123-bus IEEE test systems to thoroughly demonstrate the benefits of implementing inter-zonal power exchanges.

Keywords: Combined heat and power; electric and thermal storage systems; optimization; security constrained unit commitment; inter-zonal power exchange.

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Nomenclature

Index sets

t	Hour index.
m,n	Bus indices.
i	Index of CHP units.
l	Index of transmission line.
j	Index of boiler units.
k	Index of storage units.
NCS	Contingency Scenario index.
NOS	Operation Scenario index.

Parameters

$CD_{storage}$	Cost of ESS discharge.
$CC_{storage}$	Cost of ESS charge.
$CHD_{storage}$	Cost of TSS discharge.
$CHC_{storage}$	Cost of TSS charge.
$\eta_{CHP(i)}$	Electric efficiency of CHP system connected to bus i at hour t.
gp	Gas price.
$OM_{CHP(i)}$	Operation and maintenance variable cost of CHP system connected to bus i.
$HR_{CHP(i)}$	Heat rate of CHP system connected to bus i.
$P^{min}_{CHP(i)}$	Minimum active power of CHP system connected to bus i at hour t
$P^{max}_{CHP(i)}$	Maximum active power of CHP system connected to bus i at hour t
$\alpha^{th}_{CHP},\beta^{th}_{CHP},\gamma^{th}_{CHP}$	Coefficient of heat-power feasible region for CHP units.
$H^{min}_{Boiler(i)}$	Minimum output of the boiler connected to bus i at hour t.
$H^{max}_{Boiler(i)}$	Maximum output of the boiler connected to bus i at hour t.
$\eta_{Boiler(i)}$	Efficiency of the boiler connected to bus i at hour t.
$ILT_{\max(i,t)}$	Maximum ILC of bus i at hour t.

$ILT^{bus}_{\max(i)}$	Maximum ILC of bus *i*.
$CIC_{(i,t,NCS)}$	Customer interruption cost in bus *i* at hour *t* in contingency scenario *NCS*
$PMax^{firm}_{IZPE(i,t)}$	Maximum firm IZPE of bus *i* at hour *t*.
$PMax^{Non-firm}_{IZPE(i,t)}$	Maximum non-firm IZPE of bus *i* at hour *t*.
$IZPE^{firm}_{\text{Trans\_price}}$	Transmission price of firm IZPE in operation scenario *NOS*.
$IZPE^{Non-firm}_{\text{Trans\_price}}$	Transmission price of non-firm IZPE in operation scenario *NOS*.
$ICPE^{firm}_{\text{Trans\_price}}$	Interruption cost of firm IZPE in contingency scenario *NCS*.
$ICPE^{Non-firm}_{\text{Trans\_price}}$	Interruption cost of non-firm IZPE in contingency scenario *NCS*.
$\zeta_{Gen}, \zeta_{Line}$	Outage of generating units and transmission lines matrix.
$PE_{Demand(i,t)}$	Total load demand of bus *i* at hour *t*.
$HE_{Demand(i,t)}$	Thermal power required by bus *i* at hour *t*.
$Prob_{(NCS)}$	Probability of the contingency scenario *NCS*.
$Prob_{(NOS)}$	Probability of the operation scenario *NOS*.
WF	Weight factor of cost.
W	Weighing factor of objective function.
$C_{(d)}$	Interruption cost from the composite customer damage function (monetary unit/kW).
$X_{mn(l)}$	Reactance of line *l*.
$Pflow^{max}_{(l)}$	Maximum capacity of line *l*.
$ep_{(t)}$	Wholesale market electricity price at hour *t*.
L_b	Number of transmission lines connected to bus *b*.
T	Number of scheduling hours.
N	Number of buses.
$NCHP$	Number of CHP units.
$NBoiler$	Number of boiler units.
$NCscenario$	Number of contingency scenarios.
$NOscenario$	Number of operation scenarios.

Variables

Obj	Objective function.
$Benefit$	Benefit of ESU.
$Revenue$	Revenue of ESU.
$Cost$	Overall cost during the schedule period.
$Cost_{CHP(i,t)}$	Cost of CHP system connected to bus i at hour t.
$Cost_{Boiler(i,t)}$	Cost of generating heat via boiler connected to bus i at hour t.
$Cost_{Buy(t)}$	Cost of purchased electricity from upstream network at hour t.
$Cost_{Storage(t)}$	Cost of generating electricity and heat via ESS and TSS connected at hour t.
$Cost_{ENSC(t,NCS)}$	Energy Not Supplied Cost at hour t in contingency scenario NCS.
$Cost_{firm}$	Cost of firm IZPE in operation scenario NOS at hour t.
$Cost_{Non-firm}$	Cost of non-firm IZPE in operation scenario NOS at hour t.
$PD_{storage(t)}$	Power discharge of ESS at hour t.
$PC_{storage(t)}$	Power charge of ESS at hour t.
$Pg_{(t)}$	Total active power generation at bus i at hour t
$HD_{storage(t)}$	Power discharge of TSS at hour t
$HC_{storage(t)}$	Power charge of TSS at hour t.
$P_{CHP(i,t)}$	Active power generation via CHP system connected to bus i at hour t.
$H_{CHP(i,t)}$	Heat generation via CHP system connected to bus i at hour t.
$H_{Boiler(i,t)}$	Heat generation via boiler connected to bus i at hour t.
$Grid_{sell(t)}$	Purchased electricity by the network from the upstream network at hour t
$Grid_{buy(t)}$	electricity sold to the upstream network by network at hour t.
$P^{firm}_{IZPE(i,t)}$	Firm IZPE of bus i at hour t.
$P^{Non-firm}_{IZPE(i,t)}$	Non-firm IZPE of bus i at hour t.
$ILT_{(i,t)}$	ILC in bus i at hour t.
$ILT_{NS(i,t,NCS)}$	ILC in bus i at hour t in contingency scenario NCS.

$PS^{firm}_{IZPE(i,t,NCS)}$	Interruptible firm IZPE in bus i at hour t in contingency scenario *NCS*.
$PS^{Non-firm}_{IZPE(i,t,NCS)}$	Interruptible non-firm IZPE in bus i at hour t in contingency scenario *NCS*.
$ISPE^{firm}_{IZPE}$	Binary variable associated with commitment state of interruptible firm IZPE in bus i at hour t in contingency scenario *NCS*.
$ISPE^{Non-firm}_{IZPE}$	Binary variable associated with commitment state of interruptible non-firm IZPE in bus i at hour t in contingency scenario *NCS*.
$IPE^{firm}_{IZPE(i,t)}$	Binary variable associated with commitment state of firm IZPE; 1 if the firm IZPE is committed at hour t and 0 otherwise.
$IPE^{Non-firm}_{IZPE(i,t)}$	Binary variable associated with commitment state of non-firm IZPE; 1 if the Non-firm IZPE is committed at hour t and 0 otherwise.
$Pflow_{(l,t)}$	Real power flow of line l at t hour t.
$Pflow_{NS(l,t,NCS)}$	Real power flow of line l at t hour t in contingency scenario *NCS*.

1. Introduction

Nowadays, many of Energy Supplying Utilities (ESUs) are utilizing CHP systems to supply Industrial Customers (ICs) with electricity and heat [1]. The ICs' electric loads are usually supplied through a common electric distribution network and each IC is coupled with the main utility grid (denoted as 'main grid') through the point of common coupling. However, many of the ICs may have CHP facilities that supply energy to their energy-intensive industries and they may behave as dispatchable loads by reducing their electricity withdrawal from the ESU network and increasing the utilization of their electricity generation systems. In addition, the main grid may transact electricity with upward wholesale electricity market. Hence, the main grid behaves as an Active Distribution Network (ADN) that transacts electricity with upward wholesale electricity market and its downward ICs' systems. Based on the ICs' electrical and thermal load group characteristics, land ownership and operational constraints, the main grid can be segmented into different operational zones or areas. In addition, for an open access ADN's main grid, the ICs of different zones can transact energy with each other and they may form various Inter-zonal Power Exchange (IZPE) patterns [2]. However, any inter-zonal electricity transactions between the ICs must be analysed and approved by the ESU's day-ahead optimization procedures in advance and then, the transactions can be performed.

The Optimal day ahead Scheduling of Combined Heat and Power (OSCHP) units problem consists of determining the optimal day ahead unit commitment (UC) of generation resources and depend on the system loads, reliability criteria, dynamic and static characteristics of devices and cost-benefit analysis [3]. The OSCHP must be logical in light of demands and heat-electric energy systems optimal operation. However, the main operation decisions are critical due to the two-way ESU and ICs interactions in terms of what will happen now to what will happen later based on system dynamic constraints. In addition, many dynamic interdependencies of heat and electric systems should be adequately modelled to capture the real nature of the problem. Dynamic constraints may consist of additional operational constraints (power ramp-rate constraints, heat storage constraints, and min. up/down times constraints) [4]. The ESU must encounter different parameters uncertainties that can be classified to: upward wholesale market price uncertainties, IZPE uncertainties and main grid contingency uncertainties. In addition, the OSCHP problem has a slave problem that optimizes the hourly electric and heat dispatch that is constrained as the system's static and dynamic constraints. This problem is generally known as the optimal power flow or economic dispatch problem based on its objective function formulation [5].

The current research focuses on the OSCHP problem for an ADN considering energy storage devices commitment and ICs' IZPEs.

In the following, efforts have been done to answer the main questions related to the OSCHP problem:

- Is it economically and technically to operate an ADN that transacts energy with ICs and upward network while the ICs are transacting energy with each other?
- How can the system components be modelled such that the dynamic interdependencies of heat and electric systems are captured appropriately?
- And finally, how can the OSCHP problem be formulated to meet the feasibility and optimality studies of IZPEs?

When an ADN transacts energy with upward wholesale market, the OSCHP problem is asymmetrical with wholesale market based on the fact that the produced heat must meet heat load and the produced electricity must respond to the upward market prices [4]. In addition, the ICs IZPE will significantly complicate this problem. The OSCHP optimization procedure that is performed for a day-ahead horizon must be faster than ordinary OSCHP problems for three reasons. First, the rescheduling of OSCHP problem must be rapidly performed when the parameters of upward market or ICs transactions are changed. Second, the stochastic behaviour of system's contingencies must be considered in the optimization procedure based on the fact that critical contingencies may highly change the scheduled operating points. And third, considering the system's contingencies may highly increase the state space of the problem and the curse of dimensionality of the defined problem may lead to unacceptable solution times. Over the years, extensive works in the operational optimization of CHP have been performed and categorized into the three groups. The first group developed accurate model of CHP-based system components to capture physical reality of the modelled system. The second group of researches proposed efficient solution algorithms that try to determine the global optimum of the CHP-based operational planning problem [6]. The third group encounters new conceptual ideas in the operational planning paradigms of the CHP-based systems.

Based on the above-mentioned categorization and for the 2nd group of works, the different solution techniques have been introduced to solve the OSCHP problem that some of these techniques used Mathematical Programming (MP) algorithms such as linear programming, nonlinear programming, Mix Integer Linear Programming (MILP), and Mix Integer Non Linear Programming (MINLP), whereas others are based on heuristic and meta-heuristic methods [3]. Ref. [7] formulates the decision problem of a ESU as a MILP model. The hourly CHP operation planning problem as an LP model presented in [8] that focused to develop the extended simplex technique for solving the problem. Ref. [9] proposes two versions of an algorithm for CHP production planning: the online and offline envelope construction algorithms where the online algorithm are constructed based on the power price and the offline algorithm are pre-estimated for all different power prices. The optimal UC of CHP units is solved by dynamic programming algorithm in [10] based on linear relaxation of the states of the units. The output results show that the dynamic programming based algorithm presents more accurate results while is faster than the tabular Simplex unit decommitment algorithm. Ref. [11] proposed a sequential dynamic programming algorithm for UC problem that the relaxed states for reduce the dimension of algorithm.

A MINLP formulation is presented in [12] for operational planning in CHP systems that considers nonlinear limits in unit performance characteristics with changes in temperature conditions and system load. Ref. [13] proposes a nonlinear formulation for economic optimization of a CHP-based district heating system and it uses genetic algorithm for solving the problem. A deterministic MILP model is proposed in [14] that considers the transition of operating modes with logic constraints. Ref. [15] extends the previous work of [14] and considered transitional behaviour consists of startup and shutdown limits with different operating modes. Ref. [16] proposes a MILP formulation of OSCHP problem to commitment of boilers and turbines. The computational time in most cases is less than 5 minutes for an optimality gap equal or less than 1%. However, in [6-16] contingency constraints and ICs' IZPE scenarios have not been considered. Ref. [17], proposed a stochastic programming framework for OSCHP that the uncertainty of market prices is modelled as the stochastic parameters. However, in [17] the ICs' IZPE scenarios have not been considered. In [18], a model has been presented for involuntary load curtailment (ILC) implementation in the UC problem. The OSCHP problem in MG is proposed in [1], which simulates the impact of energy storages.

To illustrate the benefits of the proposed mixed-integer linear programming (MILP) formulation in this paper, a comparison has been done with the other methods and the results are presented in Table I. The heuristic algorithm in most of the cases cannot provide the accurate results due to exploring a limited region of the search space while there is the possibility of getting stuck into a local optimum solution. However, parameter tuning and lack of information in terms of the quality of solution are two drawbacks of the heuristic methods, especially if the aim is to provide a useful technique for a company. To find the exact solution to the problem, mathematically optimizing methods such as mixed integer linear programming, have proven to attain the global optimal solution in a bounded number of steps, besides providing an accurate and flexible model.

The described OSCHP problem is affected by different source of uncertainties such as contingencies, upward wholesale market prices and ICs' IZPE scenarios where these uncertainties highly increase the state space of the problem. Thus, the authors try to find the reasonable trade-off between solution quality and calculation time. To the authors' knowledge, no research work has considered customers' power exchange scenarios in OSCHP. In addition, no research work has considered firm and non-firm power exchange of ICs based on the fact that the OSCHP decisions are critical due to the two-way ESU and ICs interactions what will happen now to what will happen later based on system dynamic constraints.

The present research proposes a new OSCHP formulation that uses a two-stage SMILP model and it considers IZPEs. The contribution of the proposed method can be summarized as follows:

- It represents the impact of IZPE on the operation scheduling scenarios,
- It proposes firm and non-firm model for ICs' IZPEs,
- The proposed algorithm uses a two-stage SMILP framework for optimal operation of CHPs, electric and thermal storage systems based on different wholesale market prices, power exchanges and contingency scenarios.

The rest sections of this research is organized as follows: A brief model of the problem is introduced. in Section II. In Section III, details of the proposed formulation is described. The numerical results of the case studies and discussion are presented in Section IV. Finally, the relevant conclusions of the research are included in Section V.

2. System Modelling

The ESU utilizes CHP systems to supply its electric loads through the main grid with a restricted energy-handling capability. Any inter-zonal electricity transactions between the ICs must be analysed and approved by the ESU's day-ahead optimization procedures in advance and then, the transactions can be performed [2]. Two types of the ICs' IZPEs can be performed: firm and non-firm power exchanges. The firm (or non-recallable) power exchanges are usually high priority in using the ESU main grid than non-firm (or recallable) power exchanges and their transmission service costs are higher than non-firm power exchanges. The ICs may behave as the dispatchable loads by reducing their electricity withdrawal from the ESU main and increasing the utilization of their electricity generation systems. The ESU must estimate the ICs' electricity and heat consumptions for a day-ahead horizon and it receives the firm and non-firm power exchanges requests from the ICs for the next 24 hours. The main grid is introduced by the balanced power flow method for all hours [20], considering grid operation limits. The electric power flow model should be considered the main grid load and power generation at the network nodes. Some of the ICs' heat loads may be supplied through the local ESU's boilers and there is no heat transmission system.

A. Energy storage systems modelling

By definition, a source of energy storage system is "a physical system with the ability to obtain energy for replacement and dispatch of energy at later times" [21]. The constraints include the limits of energy storage, charge and discharge, impact on the power balance constraint, and impact on the objective function. The ESS and TSS constraints can be formulated as [19].

B. Inter-zonal power exchange modelling

The Firm and non-firm ICs' IZPEs between different nodes must be approved by the ESU in advance based on the OSCHP results. It is very important that the IZPEs must be economically and technically feasible.

- Power flow equations before IZPE:

$$P_{CHP(i,t)} + Grid_{sell(t)} - Grid_{buy(t)} - PE_{Demand(i,t)} + PD_{storage(t)} - PC_{storage(t)} = \sum_{l=1}^{L_b} Pflow_{(l,t)} \tag{1}$$

$$\left| Pflow_{(l,t)} \right| \le Pflow_{(t)}^{max} \tag{2}$$

- Equation (1) after IZPE:

$$\begin{aligned} & P_{CHP(i,t)} + Grid_{sell(t)} - Grid_{buy(t)} - PE_{Demand(i,t)} + PD_{storage(t)} - PC_{storage(t)} \\ & = \sum_{l=1}^{L_b} Pflow_{(l,t)} - P_{IZPE(i,t)} \quad \forall \;\; i = \text{injection point of power to network} \;, \; t \end{aligned} \tag{3}$$

$$\begin{aligned} & P_{CHP(i,t)} + Grid_{sell(t)} - Grid_{buy(t)} - PE_{Demand(i,t)} + PD_{storage(t)} - PC_{storage(t)} \\ & = \sum_{l=1}^{L_b} Pflow_{(l,t)} + P_{IZPE(i,t)} \quad \forall \;\; i = \text{delivery point of power to the network} \;, \; t \end{aligned} \tag{4}$$

3. Problem Formulation

The OSCHP problem is subject to four sources of uncertainty: wholesale electricity market prices, firm and non-firm ICs' IZPEs, and system contingencies. The uncertainty can be modelled as a two-stage decision making approach based on a stochastic programming [22]. The first stage illustrates a point in time where the decision variables of ESU's CHP generation schedules, electricity transactions with wholesale market, check the feasibility of the ICs' requested firm and non-firm IZPEs, charging and discharging schedules of electric and thermal storage systems for the feasible and initial point of operation are made. The uncertainty of the first stage variables are consisting of upward wholesale electricity market prices for next 24 hours. Finally, at the second stage, the values of ILC and interruption of firm and non-firm IZPEs must be determined for the system contingencies.

A. Objective Function

An optimal OSCHP must locate the minimize ESU's cost solution while the system loads are supplied, and other operational constraints are satisfied. According to the discussed model, the objective function of OSCHP problem is proposed as (5), in which the first part is the cost of energy production, and the second part is the revenue of energy selling.

$$Obj = Cost - Revenue \tag{5}$$

$$Revenue = \sum_{NOS=1}^{NOscenario} Prob_{(NOS)} \times \sum_{t=1}^{T} \begin{bmatrix} ep_{(t)} \times Grid_{sell(t,NOS)} \\ +Cost_{firm(t,NOS)} + Cost_{Non-firm(t,NOS)} \end{bmatrix} \tag{6}$$

The revenue consists of energy selling revenue and IZPE transmission service revenue. The cost function is decomposed into seven groups: CHP electricity production costs, boilers heat production costs, costs of electricity purchased from the upward network, costs of ESS and TSS, costs of firm and non-firm power exchanges, and energy not supplied cost:

$$Cost = \sum_{NOS=1}^{NOscenario} Prob_{(NOS)} \times \sum_{t=1}^{T} \begin{pmatrix} \sum_{i=1}^{NCHP} W_1.Cost_{CHP(i,t,NOS)} + \sum_{j=1}^{NBoiler} W_2.Cost_{Boiler(j,t,NOS)} \\ +W_3.Cost_{Buy(t,NOS)} + \sum_{k=1}^{Nstorage} W_4.Cost_{storage(k,t,NOS)} \\ +Cost_{firm(t,NOS)} + Cost_{Non-firm(t,NOS)} \\ + \sum_{NCS=1}^{NCscenario} W_5.Cost_{ENSC(t,NOS,NCS)} \end{pmatrix} \tag{7}$$

$$Cost_{CHP(i,t)} = \left(\frac{P_{CHP(i,t)}}{\eta_{CHP(i)}} \times gp \right) + P_{CHP(i,t)} \times OM_{CHP(i)} + \left(\frac{H_{CHP(i,t)}}{\eta_{CHP(i)}} \times gp \right), \quad \forall i,t \tag{8}$$

$$\eta_{CHP(i)} = \frac{1}{HR_{CHP(i)} / 3600}, \quad \forall i \tag{9}$$

$$Cost_{Boiler(j,t)} = \left(\frac{H_{Boiler(j,t)}}{\eta_{Boiler(j)}} \times gp \right), \quad \forall t, j \tag{10}$$

$$Cost_{Buy(t)} = ep_{(t)} \times Grid_{buy(t)}, \quad \forall t \tag{11}$$

$$Cost_{storgae(k,t)} = \begin{pmatrix} CD_{storage} \times PD_{storage(k,t)} \\ -CC_{storage} \times PC_{storage_{(k,t)}} \end{pmatrix} + \begin{pmatrix} CHD_{storage} \times HD_{storage(k,t)} \\ -CHC_{storage} \times HC_{storage(k,t)} \end{pmatrix} \quad \forall t, k \tag{12}$$

$$Cost_{firm(t,NOS)} = P_{IZPE(t,NOS)}^{firm} \times IZPE_{\text{Trans\_price}(t,NOS)}^{firm} \tag{13}$$

$$Cost_{Non-firm(t,NOS)} = P_{IZPE(t,NOS)}^{Non-firm} \times IZPE_{\text{Trans\_price}(t,NOS)}^{Non-firm} \tag{14}$$

$$Cost_{ENSC(t,NCS)} = \left[\sum_{i=1}^{N} \begin{Bmatrix} CIC_{(i,t,NCS)} + PS_{IZPE(i,t,NCS)}^{firm} \times ICPE_{\text{Trans\_price}(t,NCS)}^{firm} \\ +PS_{IZPE(i,t,NCS)}^{Non-firm} \times ICPE_{\text{Trans\_price}(t,NCS)}^{Non-firm} \end{Bmatrix} \right] \tag{15}$$

$$CIC_{(i,t,NCS)} = \begin{bmatrix} WF_1 \times C_{(d_1)} \times ILT_{1(i,t,NCS)} + WF_2 \times \left\{ C_{(d_2)} - C_{(d_1)} \right\} \times ILT_{2(i,t,NCS)} + WF_3 \times \\ \left\{ C_{(d_3)} - C_{(d_2)} \right\} \times ILT_{3(i,t,NCS)} + .. + WF_\zeta \times \left\{ C_{(d_\zeta)} - C_{(d_{(\zeta-1)})} \right\} \times ILT_{\zeta(i,t,NCS)} \end{bmatrix} \tag{16}$$

B. Constraints

The first stage decision variables are the active power generation (via ESS and CHP systems), heat generation (via TSS and boilers), purchased and sold electricity from/to the upward network, firm and non-firm power exchanges, respectively. The second stage decision variables are the ILC variables and interruption of the firm and non-firm power exchanges. The equality and inequality constraints can be written as:

- Feasible operating region for CHP units [3]:

$$\alpha_{CHP(i)}^{th} \times P_{CHP(i,t)} + \beta_{CHP(i)}^{th} \times H_{CHP(i,t)} \geq \gamma_{CHP(i)}^{th} \tag{17}$$

$$P_{CHP(i)}^{min} \leq P_{CHP(i,t)} \leq P_{CHP(i)}^{max}, \quad \forall\ t, i \tag{18}$$

- Power flow limits:

Constraints (1)-(4) (19)

- Heat output limit for boilers:

$$H_{Boiler(j)}^{min} \leq H_{Boiler(j,t)} \leq H_{Boiler(j)}^{max}, \quad \forall\ t, j \tag{20}$$

- Electric balance constraint equation:

$$PE_{Demand(t)} = \sum_{i=1}^{N} \left(P_{CHP(i,t)} + Grid_{sell(t)} - Grid_{buy(t)} + PD_{storage(t)} - PC_{storage(t)} \right) \tag{21}$$

- Thermal power balance constraint equation:

$$PH_{Demand(t)} = \sum_{i=1}^{N} \left(H_{CHP(i,t)} + \sum_{j=1}^{NBoiler} H_{Boiler(j,t)} + HD_{storage(t)} - HC_{storage(t)} \right) \tag{22}$$

- ESS and TSS constraints [19]. (23)

- Firm and non-firm power exchanges limits:

$$IPE_{IZPE(i,t)}^{Non-firm} \leq IPE_{IZPE(i,t)}^{firm}\ , \quad \forall\ i\ ,\ t \tag{24}$$

$$P_{IZPE(i,t)}^{firm} \leq PMax_{IZPE(i,t)}^{firm} \times IPE_{IZPE(i,t)}^{firm}\ , \quad \forall\ i\ ,\ t \tag{25}$$

$$P_{IZPE(i,t)}^{Non-firm} \leq PMax_{IZPE(i,t)}^{Non-firm} \times ISPE_{IZPE(i,t)}^{Non-firm}\ , \quad \forall\ i, t \tag{26}$$

- ILC limits:

$$ILT_{NS(i,t,NCS)} \leq ILT_{\max(i,t)}\ , \quad \forall\, i, t, NCS \tag{27}$$

$$\sum_{t=1}^{T} ILT_{NS(i,t,NCS)} \leq ILT_{\max(i)}^{bus}\ , \quad \forall\ t, NCS \tag{28}$$

- Power balance constraint for contingencies:

$$PE_{Demand(t)} = \begin{pmatrix} \sum_{i=1}^{N} \zeta_{(i,NCS)} \times P_{CHP(i,t)} + Grid_{sell(t)} - Grid_{buy(t)} + \sum_{i=1}^{N} ILT_{NS(i,t,NCS)} \\ + PD_{storage(t)} - PC_{storage(t)} + \sum_{i=1}^{N} PS_{IZPE(i,t,NCS)}^{firm} + \sum_{i=1}^{N} PS_{IZPE\ \ (i,t,NCS)}^{Non-firm} \end{pmatrix} \tag{29}$$

- Power flow equation and line flow limits for contingencies:

$$\begin{aligned} &\zeta_{Gen(i,NCS)} \times P_{CHP(i,t)} + Grid_{sell(t)} - Grid_{buy(t)} - PE_{Demand(i,t)} + PD_{storage(t)} - PC_{storage(t)} \\ &+ ILT_{NS(i,t,NCS)} + PS_{IZPE(i,t,NCS)}^{Non-firm} + PS_{IZPE(i,t,NCS)}^{firm} = \sum_{l=1}^{L_b} Pflow_{NS(l,t,NCS)} \end{aligned} \tag{30}$$

$$\left| Pflow_{(l,t)} \right| \leq \zeta_{Line(i,NCS)} \times Pflow_{(l)}^{max}, \ \forall \ \ l, t, NCS \tag{31}$$

- Firm and non-firm power exchanges limits for contingencies:

$$PS_{IZPE(i,t,NCS)}^{firm} \leq PMax_{IZPE(i,t)}^{firm} \times ISPE_{IZPE(i,t,NCS)}^{firm} \tag{32}$$

$$PS_{IZPE(i,t,NCS)}^{Non-firm} \leq PMax_{IZPE(i,t)}^{Non-firm} \times ISPE_{IZPE(i,t,NCS)}^{Non-firm} \tag{33}$$

C. Solution Algorithm

The proposed OSCHP has real and binary variables and it is a MILP problem with linear and convex parameters. Based on the problem formulation, a two-stage SMILP framework is proposed as shown in Fig. 1. At the first stage, the ESU determines the optimal hourly schedule of CHPs, electricity transactions with upward wholesale market, charging and discharging schedules of electric and thermal storage systems, and checks the feasibility of requested ICs' firm and non-firm IZPEs, along with optimal schedule of its boilers for each operation scenario.

Finally, at the second stage, the contingency scenarios are also considered and system parameters, ILC variables and interruption of the feasible firm and non-firm IZPEs are determined for each contingency scenario. The proposed optimization algorithm has advantages compare to the single stage stochastic optimization for three reasons. First, a two stage optimization procedure enables the ESU to exploit the substitutability among different system operational conditions and IZPE scenarios. Second, for a specified volatility of wholesale energy market prices and contingency selection analysis, the competitiveness of the ADN can be improved by a two-stage optimization procedure. And finally, the probability of scenarios and variables are not precisely known and may vary greatly from estimated values that can be reconsidered in the optimization procedure.

4. Simulation Results and Discussion

Two test systems are used to assess the proposed algorithm for different zone segmentation methods. The proposed method for the OSCHP is demonstrated on the 18-bus, and 123-bus IEEE test networks. Additional data associated with system are extracted from [19].

A. 18-bus system

The proposed method is implemented on the 18-bus IEEE distribution system (Fig. 2) [19]. The proposed method is modelled using MILP and it is solved using CPLEX solver [23] under GAMS optimization software [24]. The daily electric and heat peak load is assumed to be 1.160 kW and 870 kW, respectively. The cost of ILC is assumed to be 10 $/kWh.

Simulations were studied in the following cases:

Case 1: OSCHP units assuming *NOscenario=1*, *Prob(NOS=1)=1* and considering contingency scenarios and ESS and TSS systems, without firm and non-firm i IZPEs.

Case 2: OSCHP units assuming *NOscenario=1*, *Prob(NOS=1)=1* and considering contingency scenarios, ESS and TSS, and non-firm IZPE.

Case 3: OSCHP units assuming *NOscenario=1*, *Prob(NOS=1)=1* and considering contingency scenarios, ESS and TSS, and firm IZPE.

Case 4: OSCHP units assuming *NOscenario=1*, *Prob(NOS=1)=1* and considering contingency scenarios, ESS and TSS, and firm and non-firm IZPEs.

For the 4th to 8th Cases, it was assumed that the NOS=1 was the base case condition with the defined parameters. However, NOS=2 assumed that the wholesale market electricity price and gas price were increased by +50%. At first, for all cases, a 10 kW IZPEs was considered from bus 7 to bus 15. Then, the whole state space of power exchanges scenarios is considered. The transmission service price is assumed 1.5 ¢/kWh.

Table II, displays the cost of the 18-bus test system for different cases. As shown in this table, the operation cost highly increased when the system's contingency scenarios were considered in the optimization procedure. The 4th case considered the ILC and interruption costs of feasible firm and non-firm IZPEs for contingency scenarios. The elimination of the firm IZPEs had led to increase the 2nd objective function case about 15.06% with respect to the 4th objective function. In addition, the elimination of the non-firm IZPEs increased the 3rd objective function case about 10.53% with respect to the 2nd objective function. Using of the ILC had increased the 1st case objective function about 0.23%, 4.33% and 15.33%, with respect to the 2nd, 3rd and 4th cases objective functions based on the fact that the ILC costs were about 5 and 25 times of the firm and non-firm IZPEs interruption costs, respectively. The same results can be concluded for the 5th-8th cases that the objective function of the 8th case decreased about 10.77%, 10.41% and 8.61% with respect to the 5th, 6th and 7th cases, respectively.

Fig. 3 show the electricity and heat generation of CHP units for different cases. For the 4th case, the specified IZPE changed the heat generation of CHP units with respect to the all cases. Maximum deviation of electricity and heat generation of CHP units in 1-4 cases were about 25.78% and 145.9% with respect to the corresponding values of the 1st case. Thus, it can be concluded that the firm and non-firm IZPEs can decrease operating costs. Table III shows the number of continuous and discrete variables and number of equations in optimization procedure for 1-8 cases.

Maximum deviation of electricity generation of CHP units in 5-8 cases was about 3.26% with respect to the corresponding values of 1-4 cases. However, maximum deviation of heat generation of CHP units in 5-8 cases was about 89.86% with respect to the corresponding values of 1-4 cases. Thus, it can be concluded that the increasing of the operation cases had led to more changes in heat generation of CHP units. By comparison of the cases, it is obvious that the energy production of ESU's generation systems were increased based on the fact that the optimization procedure tried to implement a preventive action for critical contingencies. However, the procedure involved post-contingency corrective actions for other contingencies. In addition, the system cost was reduced when the energy storage systems were used since the storage systems were shifting the peak load and the storages were committed when the upward electricity market prices were high. The optimal commitment of energy storage systems improved the competitiveness of ADN in a deregulated market. In addition, for the 4th case, the IZPEs reduced the system's costs. Moreover, the revenue from selling power in the upward market for the 5th to 8th cases was increased.

However, the system costs of these cases were increased since more contingency scenarios were considered. Fig. 4 depicts the electricity transactions of the ADN with upward network. The purchased electricity is displayed by a minus sign. The ADN purchased electricity from upward network when the electricity price was high. When the ADN's contingency scenarios were considered the transacted electricity with the upward network was increased.

In addition, the IZPEs increased the transacted electricity with upward network. The maximum value of net transacted energy in the 5-8 cases with upward network was increased about 21.91% with respect to 1-4 cases. The ADN transacts more electricity with upward network meanwhile the considered IZPEs was performed. For analysing the IZPE's impacts on the objective function value and based on the 8th case assumptions, different ICs' IZPEs cases were studied. Table IV shows some of the ICs' flat profile power exchange cases for 18-bus test system. By comparison of system costs for the 8th case (Table II) and cases of IZPEs (Table IV) it can be concluded that the system costs was reduced for the cases of IZPE that had opposite direction of power flow from upward network to downstream loads. The best ICs' inter-zonal case of the 8th case was the second case. Fig. 5 shows all of the IZPE scenarios from generation units to load buses. Some of the power exchanges were economically infeasible. Total number of non-zero and feasible IZPEs was 187 (about 40.47% of total considered IZPE) that the ESU can approve some of the feasible IZPEs.

B. 123-bus system

The proposed method was also implemented on a 123-bus system. Some of the required data for the system are given in [25]. The network has 123 buses, 118 transmission lines, 25 CHPs, 24 heating boilers, one ESS and one TSS. A 5 kW IZPE was considered from bus 7 to bus 24. Then, the whole state space of IZPE scenarios is considered. The electric load of the system was more than 1,163 kW. The heat load of the system was about 872 kW. Table V presents the system costs for different cases. The overall cost in the 1st case was $13,836.40 which was reduced to $13,793.19 for the 4th case of the proposed method.

Table VI shows the number of continuous and discrete variables and number of equations for 1-8 cases. The number of continuous variables and total equations for the 4th case were 699,292 and 1,143,007, respectively. However, the maximum CPU time required solving each problem with a DELL series laptop (Vostro 5470) computer powered by 3.1 GHz Core i7 and 4 GB of RAM was less than 20 seconds.

Fig. 6 depicts the electricity transactions of ADN with upstream network. The ADN transacts more electricity with upward network meanwhile the considered inter-zonal power exchange is performed and the competitiveness of ADN is improved by adjustment of the multi stages decision variables. This Figure depicts the electricity transactions of ADN with upstream network for 5-8 cases. The maximum value of net transacted energy with upward network is increased about 7.08% with respect to 1-4 cases.

Fig. 7 shows some of the different ICs' power exchange cases for 123-bus system that were technically feasible. However, some IZPE cases were not economically feasible and these cases were highlighted by red color. Total number of non-zero and feasible IZPEs was 70 (about 51.1% of total considered IZPEs) that the ESU can approve some of the feasible IZPEs.

5. Conclusion

This paper proposed a new framework for OSCHP units of an ADN with electric and thermal storage systems considering industrial customers IZPEs. Firm and non-firm IZPEs concept was proposed for CHP-based ADN operation scheduling. A two-stage SMILP model was proposed for OSCHP problem to find the reasonable trade-off between solution quality and calculation time, considering that the trade-off depended on the linearization and solution method. The uncertainty of system contingencies and upward wholesale market were considered by a stochastic model. The two-stage SMILP framework used for optimal operation of CHPs, electric and thermal storage systems was based on different wholesale market prices, power exchanges and contingency scenarios. The first stage of the proposed model minimized the operation costs and checks the firm and non-firm IZPEs, while the second stage considered contingency scenarios. The competitiveness of ADN improved by adjusting the two stages decision variables. The proposed formulation considered dynamic interdependencies of heat and electric systems to capture the real nature of the problem. The performance of the proposed algorithm was assessed through study of different cases that were applied to 18-bus and 123-bus IEEE test systems. It was clearly shown that the proposed model and optimization algorithm explored the feasibility and optimality of the ICs IZPEs. Based on the proposed framework, the firm and non-firm IZPEs decreased the ESU's operating costs. The test systems costs were reduced when the storage systems shifted the peak load and the storage units were committed when the upward electricity market price was high.

Acknowledgment

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FIGURES

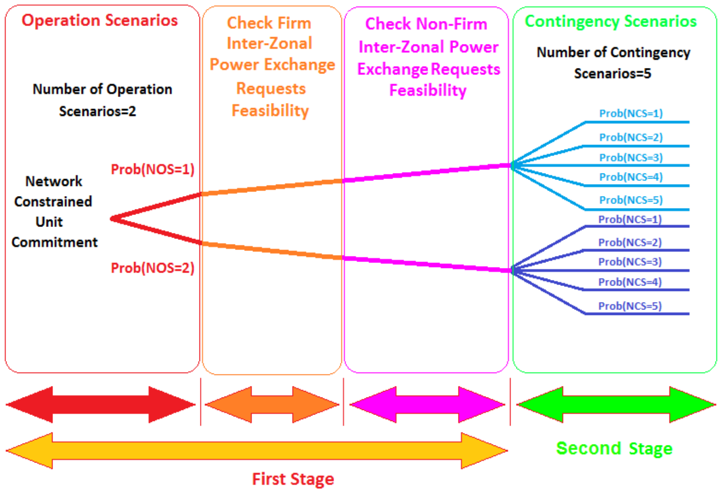

Fig. 1. Two-stage scenario-based model for day-ahead OSCHP.

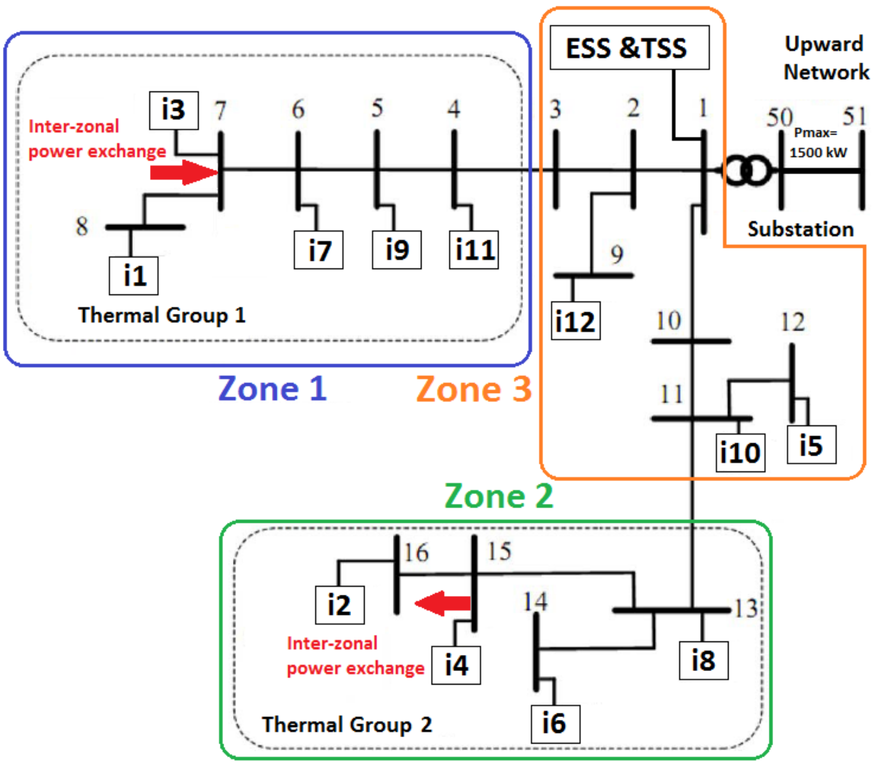

Fig. 2. 18-bus IEEE distribution system.

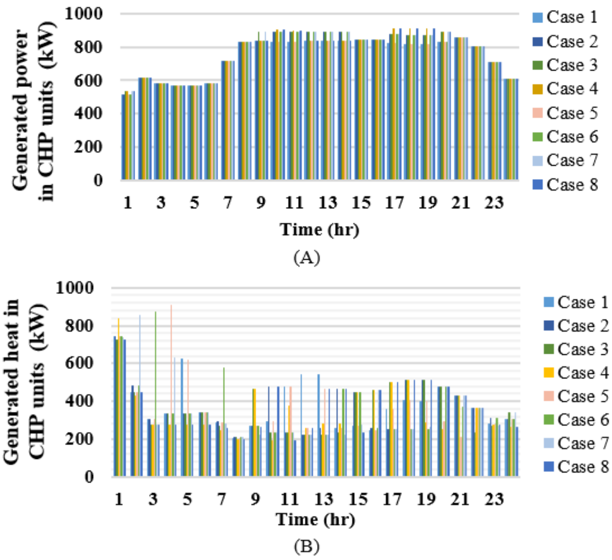

Fig. 3. (A) Electricity generation of CHP units of 18 bus network for different cases. (B) Heat generation of CHP units of 18 bus network for different cases.

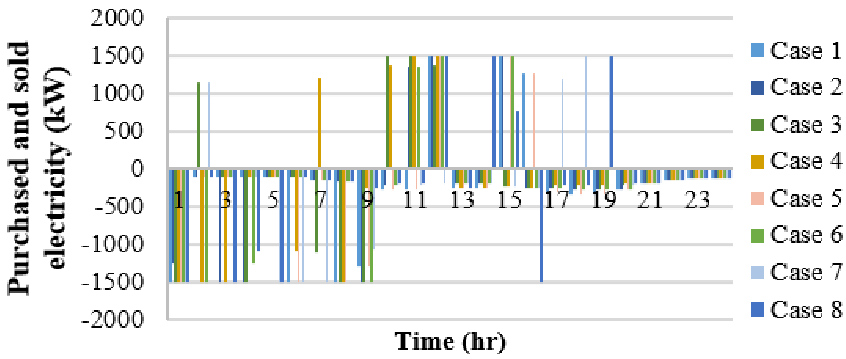

Fig. 4. Purchased and sold electricity from/to the upward network for 18-bus system for different cases.

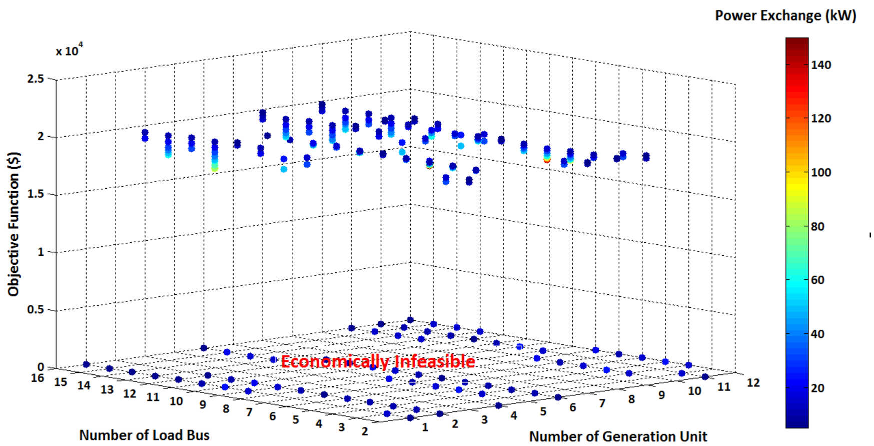

Fig. 5. Different scenarios of ICs' IZPE from system's generation units to load buses for 18 bus test system.

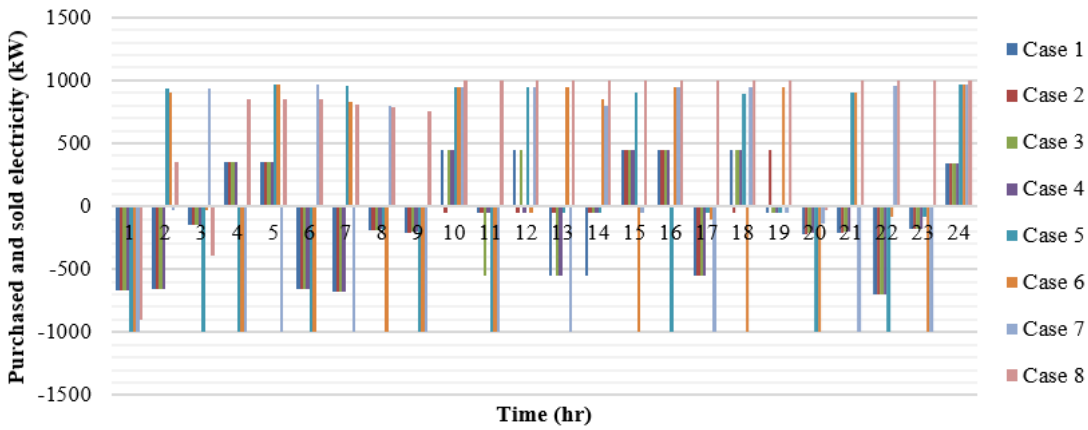

Fig. 6. Purchased and sold electricity from/to the upward network for 123-bus system for different cases.

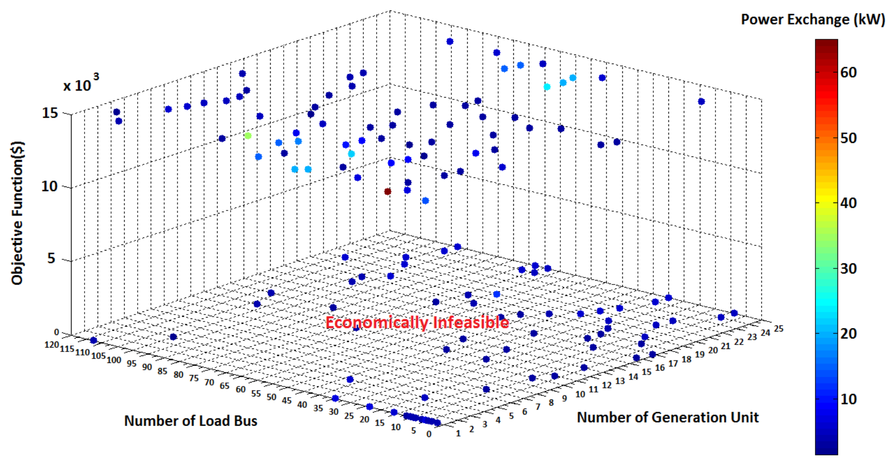

Fig. 7. Different scenarios of ICs' IZPE from system's generation units to load buses for 123 bus test system.

TABLES

Table I

Comparison of proposed Approach with other literatures

References		[12]-[13]	[14]	[16]	[17]	[8]-[10]	[7]	[4]	[18]	Proposed Approach
Method	MILP	×	✓	✓	×	✓	×	✓	✓	✓
	MINLP	×	×	×	✓	×	✓	×	×	×
	Heuristic	✓	×	×	×	×	×	×	×	×
Objective Function	Revenue	×	✓	✓	✓	×	×	×	×	✓
	Generation Cost	✓	✓	✓	✓	✓	✓	✓	✓	✓
	Storage Cost	×	✓	×	×	×	×	×	×	✓
	ENSC	×	×	×	×	×	×	×	✓	✓
	Cost of firm and non-firm power exchange	×	×	×	×	×	×	×	×	✓
	Cost of LC program	×	×	×	✓	×	×	×	✓	✓
CHP units		✓	✓	✓	✓	✓	×	✓	×	✓
Feasible operating region of a CHP units		×	✓	✓	✓	✓	×	✓	×	✓
Storage System	EES	×	✓	×	×	×	×	×	×	✓
	TES	×	✓	×	×	×	×	×	×	✓
Network Constraints		✓	✓	✓	✓	✓	✓	×	✓	✓
Grid Connected		✓	✓	✓	✓	✓	✓	✓	✓	✓
Security model	deterministic	✓	✓	✓	×	✓	×	×	×	✓
	stochastic	×	×	×	✓	×	✓	×	✓	✓
Contingency	Generation outages	×	×	×	×	×	×	×	✓	✓
	Line outages	×	×	×	×	×	×	×	✓	✓

Table II

The IEEE 18-bus system 1st-8th cases costs

Costs ($)	**Case 1**	**Case 2**	**Case 3**	**Case 4**
Obj	18,878.93	18,835.28	18,093.74	16,369.47
Cost	19,602.84	19,612.98	19,014.26	17,329.53
Revenue	723.91	777.70	920.52	960.07
$Cost_{CHP}$	4,533.80	4,313.18	4,229.82	4,046.71
$Cost_{Boiler}$	283.65	297.04	301.69	311.68
$Cost_{Buy}$	911.09	845.83	929.30	933.07
$Cost_{storage}$	5.68E-14	2.84E-14	1.42E-14	4.44E-16
$Cost_{ENSC}$	13,874.29	14,156.93	13,553.45	12,038.08
Costs ($)	**Case 5**	**Case 6**	**Case 7**	**Case 8**
Obj	20,138.19	20,056.89	19,660.63	17,967.83
Cost	21,224.05	21,223.45	20,993.86	19,479.25
Revenue	1,085.86	1,166.55	1,333.22	1,511.42
$Cost_{CHP}$	5,546.21	5,352.21	5,260.67	5,122.49
$Cost_{Boiler}$	436.92	445.56	470.96	444.01
$Cost_{Buy}$	1,366.63	1,268.74	1,351.15	1,463.11
$Cost_{storage}$	5.68E-14	2.84E-14	3.55E-15	5.68E-14
$Cost_{ENSC}$	13,874.29	14,156.93	13,553.45	12,038.08

Table III

Number of variables of the IEEE 18-bus system for different cases

Case	Continuous variables	Discrete variables	total equations
Case 1	70,420	3,168	130,102
Case 2	84,292	3,648	144,382
Case 3	84,580	4,608	143,806
Case 4	99,100	5,784	158,086
Case 5	140,840	6,336	260,204
Case 6	168,584	6,432	288,764
Case 7	169,160	8,352	287,612
Case 8	198,200	11,568	316,172

Table IV

Different ICs' IZPE cases of the 18-bus system for the 8th case

IZPE Cases	Start bus of power exchange	End bus of power exchange	Power exchange value (kW)	IZPE Revenue ($)	Obj ($)
--	--	--	--	--	20,138.2
1	7	15	10	33.3	19,448.7
2	7	15	80	266.4	17,327.3
3	6	10	25	83.25	19,111.9
4	6	10	50	166.26	18,261.9
5	13	9	15	16.70	19,842
6	13	9	30	32.24	19,637.2

Table V

The IEEE 123-bus system 1st-8th cases costs

Costs ($)	Case 1	Case 2	Case 3	Case 4
Obj	13,836.40	13,843.66	13,825.04	13,793.19
Cost	14,620.48	14,390.10	14,618.84	14,467.84
Revenue	784.08	546.44	793.80	621.01
$Cost_{CHP}$	4,524.70	4,520.25	4,494.65	4,495.65
$Cost_{Boiler}$	323.01	327.46	357.41	356.08
$Cost_{Buy}$	1,178.55	948.18	1,174.92	1,059.47
$Cost_{storage}$	0	0	0	0
$Cost_{ENSC}$	8,644.22	8,644.22	8,641.86	8,606.64
Costs ($)	**Case 5**	**Case 6**	**Case 7**	**Case 8**
Obj	14,926.54	14,899.83	14,890.67	14,901.57
Cost	17,627.80	17,954.30	17,947.40	16,637.56
Revenue	2,701.26	3,054.47	3,056.73	1,735.99
$Cost_{CHP}$	6072.65	6,065.35	6,096.26	6,045.58
$Cost_{Boiler}$	512.80	520.10	489.19	526.31
$Cost_{Buy}$	2,611.73	2,938.23	2,931.33	1,685.68
$Cost_{storage}$	1.137E-13	2.84E-14	5.68E-14	0
$Cost_{ENSC}$	8,430.62	8,430.62	8,430.62	837,9.99

Table VI

Number of variables of the IEEE 123-bus system for different cases

Case	Continuous variables	Discrete variables	total equations
Case 1	476,644	2,376	918,007
Case 2	588,854	5,369	1,033,159
Case 3	586,521	6,540	1,027,855
Case 4	699,292	10,128	1,143,007
Case 5	953,288	4,752	1,836,014
Case 6	1,177,736	4,848	2,066,318
Case 7	1,173,560	7,680	2,055,710
Case 8	1,399,592	21,264	2,286,014