



Risk Aversion in a Mixed-Integer Nonlinear Approach to Support Decision-Making for a Hydro Power Producer

J. P. S. Catalão¹, H. M. I. Pousinho¹, V. M. F. Mendes²

Abstract – In this paper, a mixed-integer nonlinear approach is proposed to support decision-making for a hydro power producer, considering a head-dependent hydro chain. The aim is to maximize the profit of the hydro power producer from selling energy into the electric market. As a new contribution to earlier studies, a risk aversion criterion is taken into account, as well as head-dependency. The volatility of the expected profit is limited through the conditional value-at-risk (CVaR). The proposed approach has been applied successfully to solve a case study based on one of the main Portuguese cascaded hydro systems. **Copyright © 2010 Praise Worthy Prize S.r.l. - All rights reserved.**

Keywords: Decision Support, Hydro Power Producer, Mixed-Integer Nonlinear Programming, Risk Aversion

Nomenclature

| | |
|------------------------|--|
| I, i | Set and index of reservoirs |
| K, k | Set and index of hours in the time horizon |
| N, n | Set and index of scenarios |
| B_n | Benefit in scenario n |
| ζ | Value-at-risk |
| δ | Per unit confidence level |
| ρ_n | Probability of occurrence of scenario n |
| η_n | Auxiliary variable used to compute the conditional value-at-risk |
| α | Weighting positive factor to achieve appropriate tradeoff profit versus risk |
| λ_{kn} | Electricity price for scenario n in hour k |
| p_{ik} | Power generation of plant i in hour k |
| v_{ik} | Water storage of reservoir i at end of hour k |
| a_{ik} | Inflow to reservoir i in hour k |
| M_i | Set of upstream reservoirs of plant i |
| q_{ik} | Water discharge of plant i in hour k |
| s_{ik} | Water spillage by reservoir i in hour k |
| η_{ik} | Power efficiency of plant i in hour k |
| h_{ik} | Head of plant i in hour k |
| l_{ik} | Water level in reservoir i in hour k |
| v_i^{min}, v_i^{max} | Water storage limits of reservoir i |
| q_i^{min}, q_i^{max} | Water discharge limits of plant i |
| u_{ik} | Commitment decision of plant i in hour k |
| H | Hessian matrix |

| | |
|------------------------------|--|
| f | Vector of coefficients for the linear term |
| x | Vector of decision variables |
| A | Constraint matrix |
| b^{min}, b^{max} | Lower and upper bound vectors on constraints |
| x^{min}, x^{max} | Lower and upper bound vectors on variables |
| $\eta_i^{min}, \eta_i^{max}$ | Power efficiency limits of plant i |
| h_i^{min}, h_i^{max} | Head limits of plant i |
| l_i^{min}, l_i^{max} | Water level limits of reservoir i |

I. Introduction

Electricity restructuring has offered us additional flexibility at both levels of generation and consumption [1]. Generation scheduling is a crucial challenge in power systems especially under new environment of liberalization of electricity industry [2].

In this paper, the short-term hydro scheduling (STHS) problem of a head-dependent hydro chain is considered. Hydro plants with only a small storage capacity available are known as run-of-the-river. Due to the reservoirs small storage capacity, the operating efficiency becomes sensitive to the head—head change effect. Thus, hydroelectric power generation has to be considered as a function of water discharge and also of the head, in order to adequately address this effect. Run-of-the-river hydro plants are considered to provide an environmentally friendly energy option, while fossil-fuelled plants are considered to provide an environmentally aggressive energy option, but still nowadays a necessary option [3].

The cascaded hydraulic configuration, coupled with the nonlinear head change effect, augments the problem

dimension and the complexity.

In a competitive environment, the most advantageous management of the conversion of the potential energy available in the reservoirs into electric energy, without affecting future operation use, represents a major advantage for the hydroelectric utilities to face competition.

The main goal in the STHS problem is to maximize the value of total hydroelectric power generation throughout the time horizon, satisfying all physical and operational constraints, and consequently to maximize the profit of the hydro power producer from selling energy into the electric market [4]-[5].

Dynamic programming (DP) is among the earliest methods applied to the STHS problem [6]. However, for hydro systems with cascaded reservoirs, CPU-time and memory requirements expand exponentially with problem size making DP unsuitable.

Artificial intelligence techniques have also been applied to the STHS problem [7]. However, a significant computational effort is necessary to solve the problem for cascaded hydro systems.

A natural approach to STHS is to model the system as a network flow model, because of the underlying network structure subjacent in hydro chains [8]. This network flow model is often simplified to a linear or piecewise linear one. Mixed-integer linear programming (MILP) is becoming frequently used for STHS [9]-[11], where integer variables allow modeling of discrete hydro unit-commitment constraints.

A nonlinear model has advantages compared with a linear one. The use of nonlinear programming (NLP) in some case studies leads to a result that exceeds by four percent what is obtained by linear programming (LP), requiring a negligible extra computation time [12]-[13].

Since the nonlinear model cannot avoid water discharges at forbidden areas, a mixed-integer nonlinear approach is proposed to solve the STHS problem [14].

As a new contribution to earlier studies, risk aversion is incorporated into the model by limiting the volatility of the expected profit through the conditional value-at-risk.

The mixed-integer nonlinear approach proposed to solve the STHS problem considers not only head-dependency, but also risk aversion. The proposed approach has been applied successfully to solve a case study based on one of the main Portuguese cascaded hydro systems.

II. Problem Formulation

II.1. Objective Function

In this paper, the objective function takes into account all the price scenarios at once, weighed by their occurrence probability:

$$\max \sum_{n=1}^N \rho_n B_n + \alpha \left(\zeta - \frac{1}{1-\delta} \sum_{n=1}^N \rho_n \eta_n \right) \quad (1)$$

The benefit for each price scenario is calculated as:

$$B_n = \sum_{k=1}^K \lambda_{kn} \sum_{i=1}^I p_{ik} \quad (2)$$

II.2. Conditional Value at Risk

Conditional value-at-risk (CVaR) is an alternative risk assessment tool that does quantify the losses associated with the tail of the profit distribution. CVaR computation does not require the use of binary variables and it can be modeled by the simple use of linear constraints. The concept of CVaR is illustrated in Fig. 1.

The CVaR is the expected profit not exceeding a measure called Value at Risk:

$$CVaR = E(B | B < \zeta) \quad (3)$$

Value at Risk, ζ , is a measure defined as the maximum profit value such that the probability of the profit being lower than or equal to this value than or equal to $1-\delta$:

$$VaR = \max(x | p\{B \leq x\} \leq 1-\delta) \quad (4)$$

The literature refers that δ assumes values usually between 0.9 and 0.99 [15].

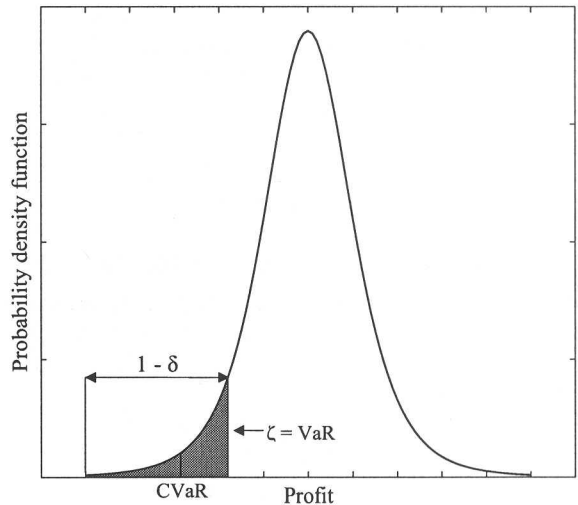


Fig. 1. VaR and CVaR illustration

In this paper, δ is considered equal to 0.95. Mathematically, CVaR can be defined as:

$$\max \zeta - \frac{1}{1-\delta} \sum_n \rho_n \eta_n \quad (5)$$

subject to:

$$-B_n + \zeta - \eta_n \leq 0 \quad (6)$$

$$\eta_n \geq 0 \quad (7)$$

In constraint (6), η_n is a variable whose value is equal to zero if the scenario n has a profit greater than ζ . For the rest of scenarios, η_n is equal to the difference of ζ and the corresponding profit.

II.3. Hydro Constrains

1) Water Balance:

The water balance equation for each reservoir is formulated as:

$$v_{ik} = v_{i,k-1} + a_{ik} + \sum_{m \in M_i} (q_{mk} + s_{mk}) - q_{ik} - s_{ik} \quad (8)$$

In (8) it is assumed that the time required for water to travel from a reservoir to a reservoir directly downstream is less than the one hour period, independently of water discharge, due to the small distance between consecutive reservoirs.

2) Head:

The head h_{ik} is considered a function of the water levels in the upstream $l_{f(i)k}$ and downstream $l_{t(i)k}$ reservoirs:

$$h_{ik} = l_{f(i)k}(v_{f(i)k}) - l_{t(i)k}(v_{t(i)k}) \quad (9)$$

depending on the water storages in the respectively reservoirs. Typically for a powerhouse with a reaction turbine, where the tail water elevation is not constant, the head is modeled as in Eq. (9), and for a powerhouse with an impulse turbine, where the tail water elevation remains constant, the head depends only on the water level in the upstream reservoir as in [10].

3) Power Generation:

Power generation p_{ik} is considered a function of water discharge q_{ik} and hydro power efficiency η_{ik} :

$$p_{ik} = q_{ik} \eta_{ik}(h_{ik}) \quad (10)$$

depending on the head h_{ik} . Hence, the electrical output of a hydro plant depends on the water discharge, the head, and the efficiency.

4) Water Storage:

Water storage has lower and upper bounds:

$$v_i^{min} \leq v_{ik} \leq v_i^{max} \quad (11)$$

5) Water Discharge:

Water discharge has lower and upper bounds:

$$u_{ik} q_i^{min} \leq q_{ik} \leq u_{ik} q_i^{max} \quad (12)$$

The binary variable, u_{ik} , is equal to 1 if plant i is on-line in hour k , otherwise is equal to 0.

6) Water Spillage:

We consider a null lower bound for water spillage:

$$s_{ik} \geq 0 \quad (13)$$

Water spillage can occur when without it the water storage exceeds its upper bound, so spilling is necessary due to safety considerations.

The initial water storages and inflows to reservoirs are assumed known.

III. Mixed-Integer Nonlinear Approach

Mixed-integer nonlinear programming can be stated as to maximize:

$$Max \ F(x) = f^T x + 1/2 x^T H x \quad (14)$$

subject to:

$$x^{min} \leq x \leq x^{max} \quad (15)$$

$$b^{min} \leq A x \leq b^{max} \quad (16)$$

$$x_j \text{ integer}, \quad j \in J \quad (17)$$

The lower and upper bounds for water discharge imply new inequality constraints that will be rewritten into (16). As expressed in (9) and (10), water level and hydro power efficiency depend respectively on water storage and head.

We consider a linearization of hydro power efficiency of plants, expressed as the output-input ratio, given by:

$$\eta_{ik} = \alpha_i h_{ik} + \eta_i^0 \quad (18)$$

where the parameters α_i and η_i^0 are given by:

$$\alpha_i = (\eta_i^{max} - \eta_i^{min}) / (h_i^{max} - h_i^{min}) \quad (19)$$

$$\eta_i^0 = \eta_i^{max} - \alpha_i h_i^{max} \quad (20)$$

Also, we consider a linearization of the water level function given by:

$$l_{ik} = \beta_i v_{ik} + l_i^0 \quad (21)$$

where the parameters β_i and l_i^0 are given by:

$$\beta_i = (l_i^{max} - l_i^{min}) / (v_i^{max} - v_i^{min}) \quad (22)$$

$$l_i^0 = l_i^{max} - \beta_i v_i^{max} \quad (23)$$

Substituting (18) into (10) we have:

$$p_{ik} = q_{ik} (\alpha_i h_{ik} + \eta_i^0) \quad (24)$$

Therefore, power generation is considered a nonlinear function of water discharge and water storage, given by:

$$p_{ik} = \alpha_i \beta_{f(i)} q_{ik} v_{f(i)k} + \alpha_i \beta_{t(i)} q_{ik} v_{t(i)k} + \chi_i q_{ik} \quad (25)$$

with:

$$\chi_i = \alpha_i (l_{f(i)}^0 - l_{t(i)}^0) + \eta_i^0 \quad (26)$$

A major advantage of our approach is to consider the head change effect in a single function (25) of water discharge and water storage that can be used in a straightforward way, instead of deriving several curves for different heads.

IV. Case Study

The proposed mixed-integer nonlinear approach has been applied on one of the main Portuguese cascaded hydro systems. The realistic hydro chain has three cascaded reservoirs and is shown in Fig. 2.

Only the first reservoir has inflow. This inflow is due to an upstream watershed belonging to a different company. The inflow is shown in Fig. 3.

Our model has been developed and implemented in MATLAB and solved using the optimization solver package Xpress-MP. Hence, the proposed mixed-integer nonlinear approach uses general software and no algorithmic work is performed.

The numerical testing has been performed on a 600-MHz-based processor with 256 MB of RAM. The scheduling time horizon chosen is one day divided into 24 hourly periods.

The prices scenarios over the time horizon are shown in Fig. 4 (where \$ is a symbolic economic quantity).

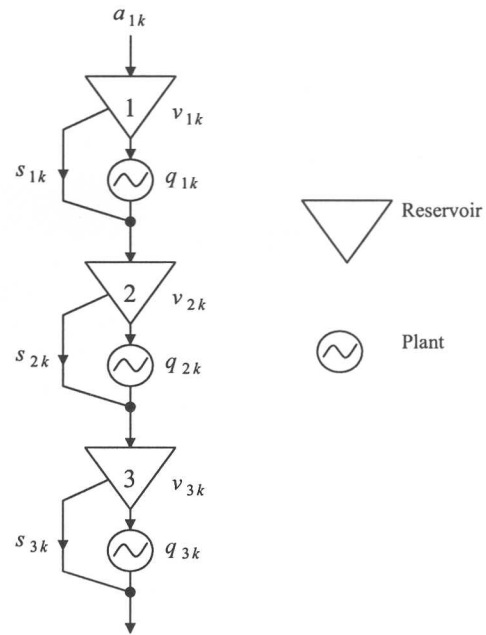


Fig. 2. Cascaded hydro system

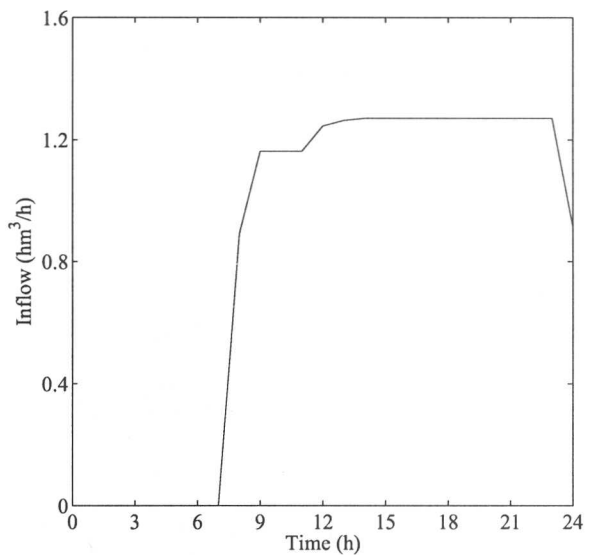


Fig. 3. Inflow on the first reservoir

The number of prices scenarios generated in the optimization problem is $N = 5$. This number has been selected arbitrarily, and the probability of each generated scenario will be $1/N$.

Several techniques have been tried out for energy prices forecasting [16], mainly based on time series models [17], [18], or on artificial intelligence techniques [19], [20]. In this case study, the prices scenarios are obtained using the approach in [20].

The expected profit and the profit standard deviation obtained for four values of α are presented in Fig. 5.

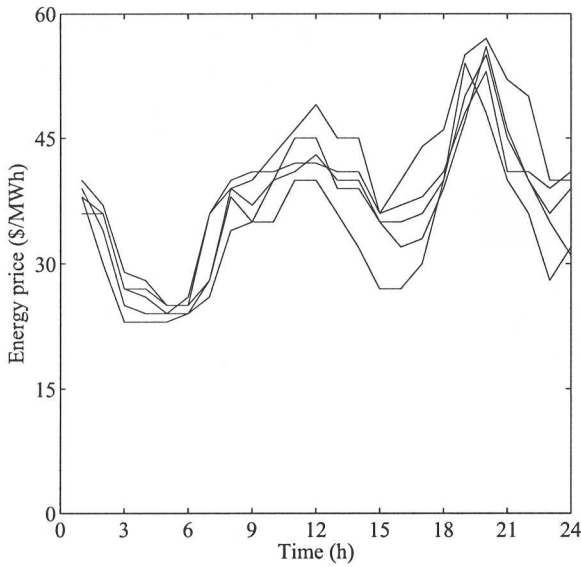


Fig. 4. Prices scenarios considered in the case study

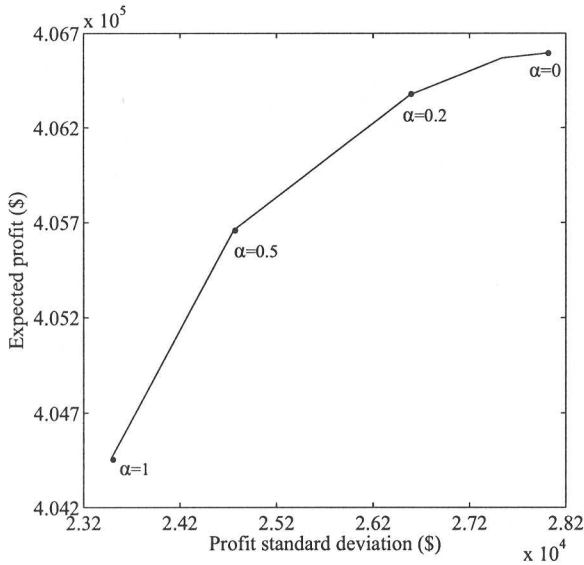


Fig. 5. Expected profit versus profit standard deviation

An analysis of Fig. 5, known as efficient frontier, reveals that for a risk-neutral producer ($\alpha = 0$), the expected profit is \$406,569 with a standard deviation of \$28,038. On the other hand, a risk averse producer ($\alpha = 1$) expects to achieve a profit of \$404,455 with a lower standard deviation of \$23,480.

Table I establishes a numerical comparison of the increase in profit for several levels of risk. The maximum profit represents an increase of 0.52% corresponding to level of risk $\alpha = 0$. Hence, different hydro power producers may choose different behaviors towards risk.

The optimal reservoir storage trajectories are shown in Fig. 6. The optimal plant discharge trajectories are shown in Fig. 7.

TABLE I
COMPARISON OF THE INCREASE IN PROFIT FOR SEVERAL LEVELS OF RISK

| Level of risk | Profit standard deviation (\$) | Expected Profit (\$) | % Increase |
|---------------|--------------------------------|----------------------|------------|
| 1.0 | 23,480 | 404,455 | - |
| 0.5 | 24,758 | 405,661 | 0.30 |
| 0.2 | 26,585 | 406,373 | 0.47 |
| 0.0 | 28,038 | 406,569 | 0.52 |

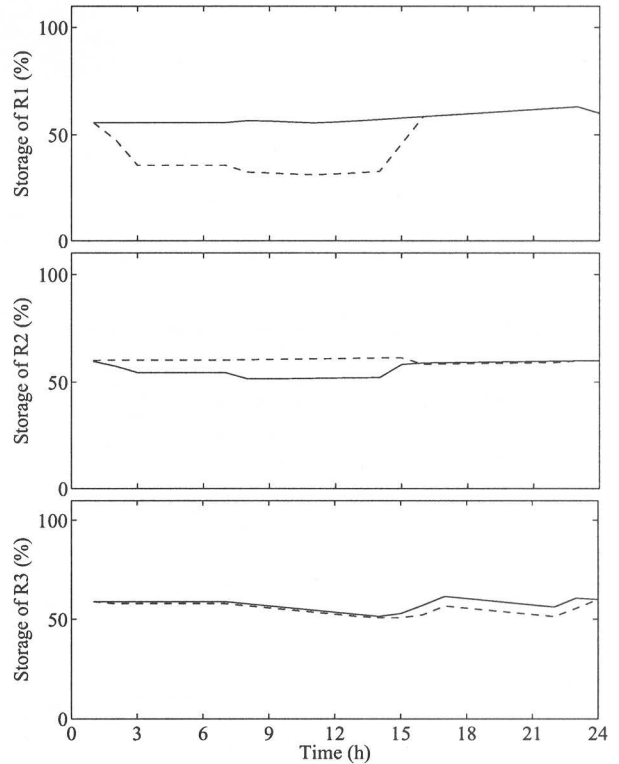


Fig. 6. Optimal reservoir storage trajectories. The solid line denotes the results obtained using a level of risk $\alpha = 0$, while the dashed line denotes the results obtained using a level of risk $\alpha = 1$

Fig. 6 shows the different storage trajectories for both levels of risk.

We verify that some different behaviors are possible to be observed for the first reservoir, because for a risk-neutral producer the influence of the head change effect is much more relevant due to increased of the water available in reservoir in order to benefit the power generation efficiency of plant.

Fig. 7 shows that when a large value is dedicated to α , the participation of the water discharge will be lower than the case when α is small. By assuming higher values for the risk penalty factors, the number of online hours tends to decrease.

Table II shows the scheduling results for two levels of risk in the third hydro plant of this case study. The values considered are maximum risk ($\alpha = 0$) and minimum risk ($\alpha = 1$).

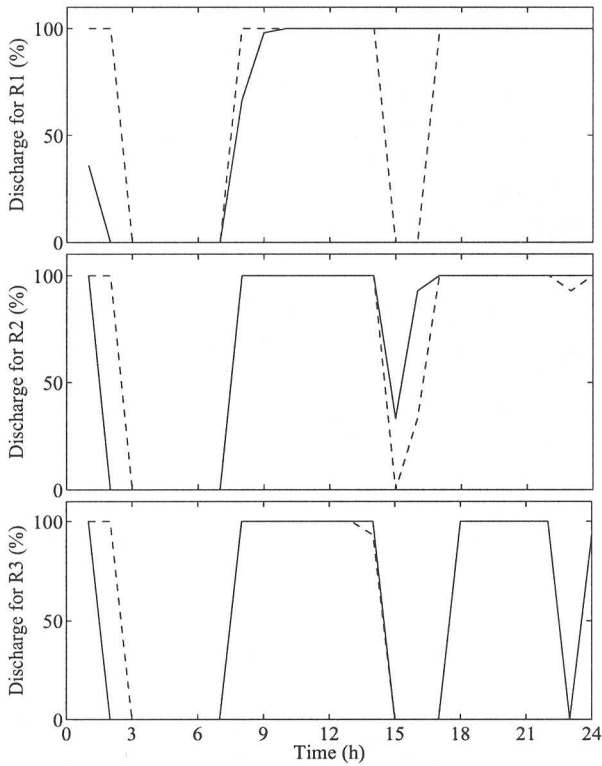


Fig. 7. Optimal plant discharge trajectories. The solid line denotes the results obtained using a level of risk $\alpha = 0$, while the dashed line denotes the results obtained using a level of risk $\alpha = 1$

TABLE II
SCHEDULING RESULTS FOR TWO LEVELS OF RISK – PLANT 3

| Hour | Power generation per water discharge [MWh/hm ³] | | Hour | Power generation per water discharge [MWh/hm ³] | |
|------|---|--------------|------|---|--------------|
| | $\alpha = 0$ | $\alpha = 1$ | | $\alpha = 0$ | $\alpha = 1$ |
| 1 | 191.74 | 191.74 | 13 | 190.58 | 190.38 |
| 2 | 0 | 191.55 | 14 | 190.38 | 190.25 |
| 3 | 0 | 0 | 15 | 0 | 0 |
| 4 | 0 | 0 | 16 | 0 | 0 |
| 5 | 0 | 0 | 17 | 0 | 0 |
| 6 | 0 | 0 | 18 | 192.03 | 191.14 |
| 7 | 0 | 0 | 19 | 191.83 | 190.95 |
| 8 | 191.55 | 191.35 | 20 | 191.64 | 190.75 |
| 9 | 191.35 | 191.16 | 21 | 191.44 | 190.56 |
| 10 | 191.16 | 190.96 | 22 | 191.25 | 190.36 |
| 11 | 190.97 | 190.77 | 23 | 0 | 0 |
| 12 | 190.77 | 190.57 | 24 | 191.94 | 0 |

The results obtained in Table II show that different risk levels provide a different scheduling, i.e., in time periods in which hydro plant is online ($\alpha = 1$) the production is lower compared to the case of maximum risk ($\alpha = 0$).

The optimal solution requires less than 2 seconds of CPU time, on a 600-MHz-based processor with 256 MB of RAM, using Xpress-MP, considering head-

dependency, discontinuous operating regions and risk constraints.

Hence, the proposed approach is both accurate and computationally acceptable, providing better results for head-sensitive cascaded hydro systems.

V. Conclusion

In this paper, the risk measure CVaR is included in the STHS problem, besides head-dependency. The objective function is the maximization of the expected total profit plus a risk measure of the profit. The problem is formulated as a mixed-integer nonlinear one, solved using the optimization solver package Xpress-MP under MATLAB. The numerical results provided by the proposed approach include the efficient frontier curve, giving the expected profit vs. profit standard deviation. Hence, the proposed approach contributes to making the STHS problem in the day-ahead electricity market better.

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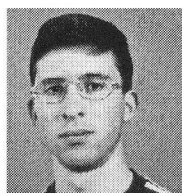
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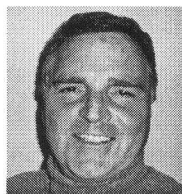
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