# SCHEDULING OF HEAD-SENSITIVE CASCADED HYDRO SYSTEMS: A COMPARISON BASED ON NUMERICAL SIMULATION RESULTS

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## Abstract

In the present day, with the deregulation of the electric power sector, business is recognized as a bid to win the best profit. In this new and competitive environment, a hydroelectric power utility has to decide the optimal management of the inflows and the water stored in its reservoirs, maximizing profit from selling energy without compromising future potential profit. This article is on the problem of short-term hydro scheduling, concerning head-sensitive cascaded reservoirs, and the algorithmic aspects of its solution. We propose and compare optimization methods based on dynamic programming, linear and non-linear network programming. Finally, based on numerical simulation results, we report and illustrate our experience.

## Key Words

Hydro scheduling, optimization methods, numerical simulation, cascaded reservoirs, variable head

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## 1. Introduction

The electric power industry has undergone significant transformations in the past decades. Prior to 1973, successive advances in the domains of generation, transmission and distribution of electrical power lead to relatively low production costs. The redundancy of installed and available equipments kept the reliability levels high. The energy crisis of 1973 and its repercussion over the cost of the equipments introduced new economic concerns to the issue. Since then, the electricity price increased, becoming increasingly difficult to maintain the reliability levels relying on the former strategy, within a healthy economic prospective.

Nowadays, the deregulation of the electric power sector and, particularly concerning Portugal and Spain, the creation of the Iberian Electricity Market (MIBEL), poses new challenges to the electric power utilities. The possibility of choosing the electricity supplier by the consumers, presents the electric power utilities a new concern: competitiveness.

The development of new methodologies carrying away an improved planning is absolutely crucial in a competitive environment. In the electric liberalized energy framework, a utility with hydroelectric facilities faces the optimal trade-off problem of how to make the present profit by the management of the water resources without compromising future potential profit [1]. This problem is known as the hydro scheduling problem. Hydro scheduling is a very important activity for hydroelectric power utilities because of its significant economic impact [2, 3].

Short-term hydro scheduling is concerned with the operation during a time horizon of one to seven days, usually discretized in hourly intervals. In this case, the problem is treated as a deterministic one. Where the problem includes stochastic quantities, such as inflows to reservoirs or energy prices, the corresponding forecasts are used [4]. Hydro scheduling is guided by prespecified hourly weighting factors, which quantify the energy price in the corresponding hours [4]. The goal is to maximize the value of total hydroelectric generation throughout the time horizon considered satisfying all hydraulic constraints, and consequently to maximize the profit of the electric utility from selling energy. A reservoir has a dynamic behaviour and constraints coupling the hourly generation across time. Furthermore, since the reservoirs in a river catchment are hydraulically coupled, the generation of an upstream reservoir affects the water volumes of the downstream reservoirs [5].

To solve this complex large-scale problem many algorithms have been developed, including dynamic programming, linear network programming, mixed-integer linear programming and non-linear network programming.

Dynamic programming (DP) is flexible and can handle the previous mentioned constraints in a straightforward way [6-8]. However, direct application of DP methods, for systems with cascaded reservoirs and discrete operating states, is impractical due to the curse of dimensionality, since the computational burden increases exponentially with problem size.

Modern codes and computers make linear network programming (LNP) a widely used method for hydro scheduling [8-11]. Indeed, these algorithms accommodate easily constraints such as the water balance equation for cascaded systems, reservoirs limits of operation, water flow through canals and spillways, and water draft through powerhouses, transit times in between reservoirs, and other constraints. In addition, LNP algorithms lead to extremely efficient codes, implementations of which can be found commercially. However, its major limitation is the inability to deal with head-sensitive reservoirs, since in this case power generation is a non-linear function of water discharge and head.

Also, mixed-integer linear programming (MILP) is becoming frequently used for hydro scheduling [12-16], where binary variables allow modelling of start-up costs to avoid unnecessary start-ups and of discrete hydro unit-commitment constraints. Even though the objective function of MILP is linear, it can easily accommodate a convex, piece-wise linear representation of the power generation versus water discharge curve. However, the piecewise-linear approximations augment the computational burden required to solve this problem.

This article proposes a non-linear network programming (NLNP) method to handle such non-linear functions as they relate to head changes. A NLNP approach [1-3, 17] is more realistic and justified for improving the results, particularly in reservoirs where the head greatly depends on the water volume stored.

## 2. Notation

The notation used throughout the article is described as follows.

### *A. Indices and sets*

| | |
|---|---|
| $i$ | Index of hydro resource |
| $k$ | Index of hour in scheduling period |
| $I$ | Total number of hydro resources |
| $K$ | Total number of hours in scheduling period |
| $M_i$ | Set of reservoirs upstream to reservoir $i$ |

### *B. Variables and constants*

| | |
|---|---|
| $l_{ik}$ | Water level in reservoir $i$ in hour $k$ |
| $h_{ik}$ | Head of plant $i$ in hour $k$ |
| $\overline{h}_i$ | Maximum head of plant $i$ |
| $\underline{h}_i$ | Minimum head of plant $i$ |
| $v_{ik}$ | Water volume of reservoir $i$ at end of hour $k$ |
| $\overline{v}_i$ | Maximum water volume of reservoir $i$ |
| $\underline{v}_i$ | Minimum water volume of reservoir $i$ |
| $q_{ik}$ | Water discharge of plant $i$ in hour $k$ |
| $\overline{q}_i$ | Maximum water discharge of plant $i$ |
| $\underline{q}_i$ | Minimum water discharge of plant $i$ |
| $s_{ik}$ | Water spillage by reservoir $i$ in hour $k$ |
| $a_{ik}$ | Natural inflow to reservoir $i$ in hour $k$ |
| $p_{ik}$ | Power generation of plant $i$ in hour $k$ |
| $\overline{p}_i$ | Maximum power generation of plant $i$ |
| $\underline{p}_i$ | Minimum power generation of plant $i$ |

| | |
|---|---|
| $\lambda_k$ | Forecasted energy price in hour *k* |
| $\Psi_i$ | Future value of water stored in reservoir *i* |
| $\tau^{q}_{mi}$ | Time required for the water discharged from reservoir *m* to reach reservoir *i*, in hours |
| $\tau^{s}_{mi}$ | Time required for the water spilled by reservoir *m* to reach reservoir *i*, in hours |

### C. *Vectors and matrixes*

| | |
|---|---|
| $A$ | Node-arc incident matrix |
| $x$ | Vector of the flux variables corresponding to the arcs of the network |
| $b$ | Right hand side vector |
| $\overline{x}$ | Upper bound vector |
| $\underline{x}$ | Lower bound vector |
| $f$ | Vector of coefficients for the linear term |
| $H$ | Hessian matrix |

## 3. Formulation

The water balance equation of reservoir *i* in hour *k* can be stated as:

$$v_{i,k-1} + a_{ik} + \sum_{m \in M_i} (q_{m,k} - \tau^{q}_{mi} + s_{m,k} - \tau^{s}_{mi}) = v_{ik} + q_{ik} + s_{ik} \tag{1}$$

In this article, we assume that the discharge from any upstream reservoir flows directly into the succeeding downstream reservoir with no time lag, thus (1) becomes:

$$v_{i,k-1} + a_{ik} + q_{i-1,k} + s_{i-1,k} = v_{ik} + q_{ik} + s_{ik} \tag{2}$$

This assumption is only for theorical simplification and poses no difficulties in the problem formulation.

The objective function considered for the hydro scheduling problem is composed of two terms: the first term represents the profit with the watershed during the short-term time horizon and the last term expresses the economic value of the future use of the final stored water in the reservoirs.

Thus, the hydro scheduling problem can be formulated as:

$$Max \sum_{i=1}^{I} \left\{ \sum_{k=1}^{K} \left[ \lambda_k \, p_{ik} \left( q_{ik}, h_{ik} \right) \right] + \Psi_i \left( v_{iK} \right) \right\} \tag{3}$$

*subject to*:

- Water balance equation:

$$v_{i,k-1} + a_{ik} + q_{i-1,k} + s_{i-1,k} = v_{ik} + q_{ik} + s_{ik}$$

- Power generation equation:

$$p_{ik} = q_{ik} \, (\alpha_i \, h_{ik} + \eta_{i0}), \quad \text{where } \alpha_i, \eta_{i0} \in \Re \tag{4}$$

- Head equation:

$$h_{ik} = l_{ik} - l_{i+1,k} \tag{5}$$

- Water level equation:

$$l_{ik} = \beta_i \, v_{ik} + l_{i0}, \quad \text{where } \beta_i, l_{i0} \in \Re \tag{6}$$

- Reservoir water volume constraints:

$$\underline{v}_i \leq v_{ik} \leq \overline{v}_i \tag{7}$$

- Plant water discharge constraints:

$$\underline{q}_i \leq q_{ik} \leq \overline{q}_i \tag{8}$$

- Plant power generation constraints:

$$\underline{p}_i \leq p_{ik} \leq \overline{p}_i \tag{9}$$

- Reservoir water spillage constraints:

$$s_{ik} \geq 0 \tag{10}$$

The optimal value of the objective function (3) is determined subject to constraints.

The constraints are of two kinds: equality constraints and inequality constraints or simple bounds on the variables.

In (4) power generation is considered a non-linear function of water discharge and head. In (5) the head is considered a function of the water levels in the reservoirs upstream and downstream to the plant. In (6) the water level is considered a linear function of the reservoir water volume.

Using (5) and (6) in (4) power generation becomes a non-linear function of water discharge and reservoir volume, given by:

$$p_{ik}=\alpha_i\,\beta_i\,q_{ik}\,v_{ik}-\alpha_i\,\beta_{i+1}\,q_{ik}\,v_{i+1,k}+\delta_i\,q_{ik},\quad \text{where } \alpha_i,\beta_i,\delta_i\in\Re \tag{11}$$

In (7), (8), (9), reservoirs water volume, plant water discharge and power generation have lower and upper bounds. In (10) is considered that the lower bound of water spillage is zero. Water spillage by the reservoirs exists when maximum volume of the water stored exceeds its upper bound or it is profitable to discharge to the downstream reservoir. The initial water volumes and inflows to reservoirs are known, for this problem.

## 4. Solution Methodologies

DP methods were among the earliest methods applied to the hydro scheduling problem. DP has the advantage that it can directly handle non-convex, non-linear, and even discrete characteristics present in the hydro model [9]. These methods, however, are not ideally suited for hydro systems with two or more cascaded reservoirs, due to excessive computation times and the well-known curse of dimensionality.

For these kinds of hydro systems, LNP and NLNP methods are, therefore, superior to DP methods.

LNP can be stated as:

$$Max \quad f^{\mathrm{T}}\,x \tag{12}$$

$$subject\ to\!: \quad A\ x\ =\ b \tag{13}$$

$$\underline{x}\ \leq\ x\ \leq\ \overline{x} \tag{14}$$

In this approach, we ignore the head change effect and non-linearities thus power generation is linearly dependent on water discharge, given by:

$$p_{ik} = \sigma_i \, q_{ik}, \quad \text{where } \sigma_i \in \Re \tag{15}$$

The previous simplification may lead to inaccuracies. A more accurate model should include the hydro generation characteristic describing the relationship between the head, the water discharge and the power generation. This relationship can be represented as a family of non-linear, non-convex curves (fig. 1), which are also known as plant performance curves, each for a specified value of the head.

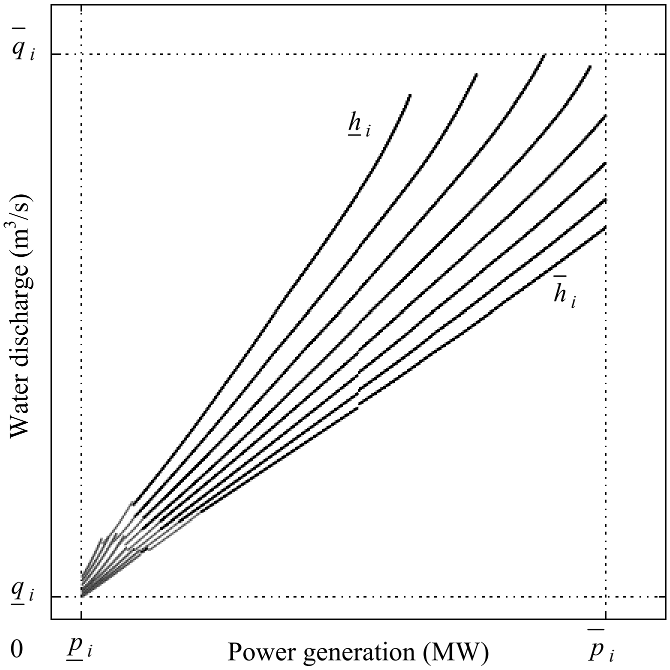

figure 1. Plant performance curves.

NLNP, namely quadratic programming, can be stated as:

$$Max \quad \frac{1}{2} x^{\mathrm{T}} H x + f^{\mathrm{T}} x \tag{16}$$

$$subject\ to: \quad A \ x = b$$

$$\underline{x} \le x \le \bar{x}$$

In this approach, power generation is a non-linear function of water discharge and reservoir volume:

$$p_{ik} = \alpha_i \beta_i q_{ik} v_{ik} - \alpha_i \beta_{i+1} q_{ik} v_{i+1,k} + \delta_i q_{ik}, \quad \text{where } \alpha_i, \beta_i, \delta_i \in \Re$$

The sparsity pattern of matrix $H$ is given below in fig. 2, considering a case with three cascaded reservoirs and three hours in the time horizon.

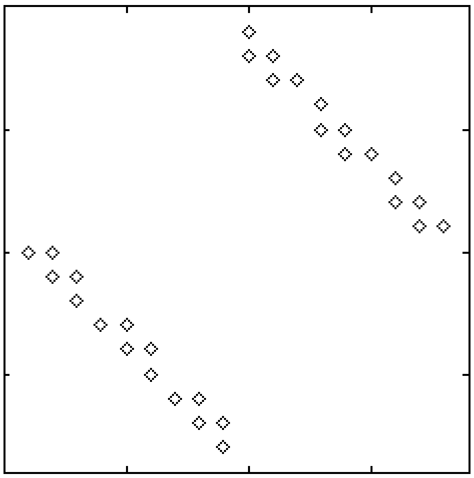

figure 2. Structure of matrix $H$: sparse and symmetric.

### 5. Numerical Simulation Results

The numerical simulation, based on two test cases, was performed on a 1.6-GHz-based processor with 512 MB of RAM. A time horizon of 168 hours was used for both test cases.

The forecasted energy price considered for the time horizon is shown in fig. 3 ($ is a symbolic quantity).

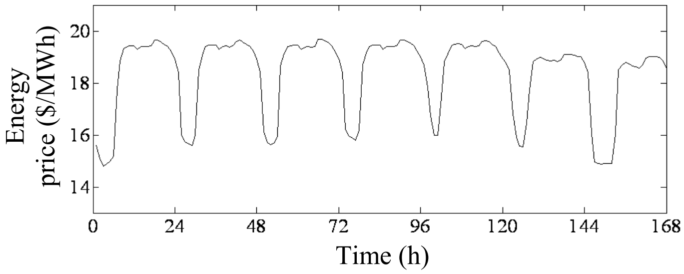

figure 3. Forecasted energy price.

The first test case, consisting of a single hydroelectric plant, is discussed for illustrating the use and the success of the proposed NLNP approach in comparison with a DP approach that provides an exact solution for the problem.

The DP approach was developed using a discretization level of 0.01 $hm^3$, thus with 2000 states on each hour given a maximum volume of 20 $hm^3$, considering the head change effect using the non-linear objective function.

In the next figure (fig. 4) the solid line denotes NLNP results while the dashed line denotes DP results.

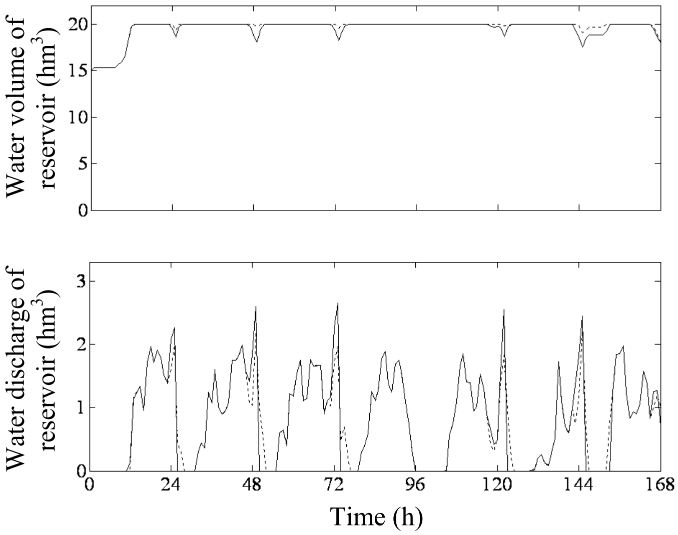

figure 4. NLNP results denoted by the solid line versus DP results denoted by the dashed line.

To compare DP with NLNP, the main results are summarized in table 1.

Table 1

Comparison of DP with NLNP Results

| Method | Average volume ($hm^3$) | Average discharge ($hm^3/h$) | Total profit ($\$\times10^3$) | CPU (s) |
|---|---|---|---|---|
| DP | 19.640 | 0.819 | 167.899 | 165 |
| NLNP | 19.520 | 0.821 | 167.840 | 0.18 |

The proposed NLNP approach achieves an optimal solution with nearly the same total profit obtained with the DP approach (less than 0.036%), but with an inferior computation time (almost 1000 times less). Thus, NLNP represents a valid and better solution for the problem in comparison with DP for a single hydroelectric plant.

The second test case, consisting of three hydroelectric plants as shown in fig. 5, is based on a realistic cascaded hydro system. Only reservoir 1 has natural inflow. This second test case is discussed for illustrating the benefits of the NLNP approach in comparison with the LNP approach that ignores the head change effect.

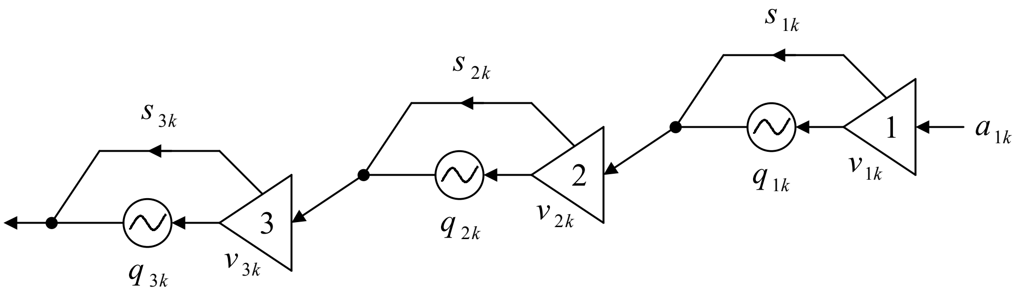

figure 5. Hydro system watershed.

The data of the hydro system is given in table 2.

Table 2

Hydro System Data

| Reservoir | Volume capacity ( $hm^3$ ) | Maximum discharge ( $hm^3/h$ ) | Power capacity ( MW ) |
|---|---|---|---|
| 1 | 9.9 | 1.404 | 174 |
| 2 | 13.5 | 1.188 | 191 |
| 3 | 26.4 | 1.512 | 240 |

This test case considers that final water volume of reservoirs is equal to its initial volume. Initial and final reservoir volumes can be obtained by a medium-term planning procedure. Consequently, the future values of water stored in reservoirs are not considered.

In the next figures, the solid line denotes NLNP results while the dashed line denotes LNP results. The results for the water volume are shown in fig. 6.

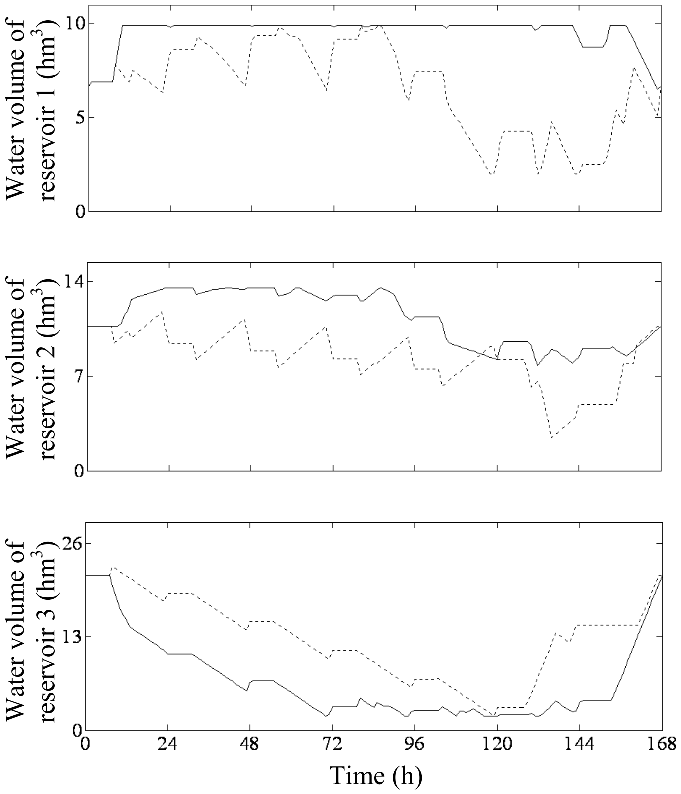

figure 6. Water volume of reservoirs. The solid line denotes NLNP results while the dashed line denotes LNP results.

In fig. 6 we can see the major influence of the head change effect in the optimal behaviour of the reservoirs. Considering the head change effect, upstream reservoirs should operate at the highest possible storage level in order to maximize the generation's efficiency of their associated plants. The optimization process with NLNP tries to maintain the maximum storage levels in the two upstream reservoirs, pulling up the reservoirs trajectory, opposing to the change in the third reservoir in order to maximize electric generation efficiency. This implies that reservoirs play a completely different role in the system depending on their relative position in the cascade.

The results for the water discharge are shown in fig. 7.

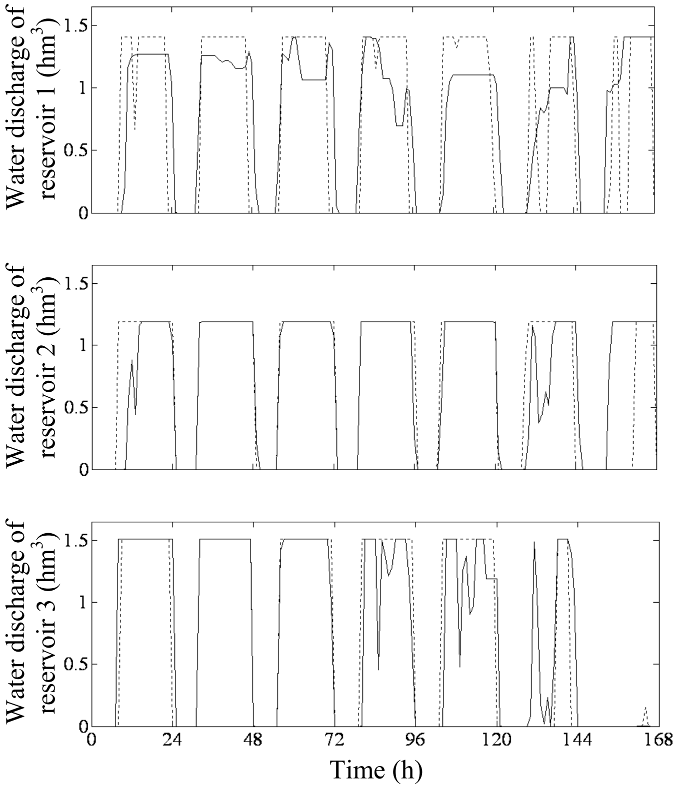

figure 7. Water discharge of plants. The solid line denotes NLNP results while the dashed line denotes LNP results.

As we can see in fig. 7, the LNP results show that the water discharge change from the minimum value, zero, quickly to the upper value thus ignoring the head change effect, while the NLNP results show that, considering the head change effect, the hydro generation in the two upstream reservoirs is postponed in order to quickly reach high reservoir storage levels and, consequently, increase hydro generation efficiency.

To compare LNP with NLNP, the main results are summarized in table 3.

Table 3

Comparison of LNP with NLNP Results

| Reservoir | Method | Average volume ( hm$^3$) | Average discharge ( hm$^3$/h ) | Total profit (\$×10$^3$) | CPU (s) |
|---|---|---|---|---|---|
| 1 | LNP | 6.584 | 0.744 | 1045.120 | 0.21 |
| 2 |  | 8.317 | 0.744 |  |  |
| 3 |  | 12.820 | 0.744 |  |  |
| 1 | NLNP | 9.552 | 0.744 | 1107.160 | 0.35 |
| 2 |  | 11.280 | 0.744 |  |  |
| 3 |  | 6.949 | 0.744 |  |  |

The average water discharge is as expected the same with both optimization methods but the average volume in the two upstream reservoirs is superior with the proposed NLNP approach. Thus, with NLNP we have a larger total profit for the hydroelectric power utility, almost 6% representing 62040 $, with negligible additional CPU time required.

## 5. Conclusion

As the traditional monopolistic scenery for the electric energy makes way to a competitive energy market, an improved planning is crucial for an electric utility to face competitiveness. We propose and compare optimization methods based on dynamic, linear and non-linear network programming, to solve the short-term hydro scheduling problem with head-sensitive cascaded reservoirs. Numerical simulation results show that non-linear network programming is justified for representing a better solution. The comparison gives a negligible extra computational effort in a realistic cascaded hydro system where the head depends on the water volume stored.

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