New probabilistic method for solving economic dispatch and unit commitment problems incorporating uncertainty due to renewable energy integration

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Abstract

In this paper, a methodology to solve Unit Commitment (UC) problem from a probabilistic perspective is developed and illustrated. The method presented is based on solving the Economic Dispatch (ED) problem describing the Probability Distribution Function (PDF) of the output power of thermal generators, energy not supplied, excess of electricity, Generation Cost (GC), and Spinning Reserve (SR). The obtained ED solution is combined with Priority List (PL) method in order to solve UC problem probabilistically, giving especial attention to the probability of providing a determined amount of SR at each time step. Three case studies are analysed; the first case study explains how PDF of SR can be used as a metric to decide the amount of power that should be committed; while in the second and third case studies, two systems of 10-units and 110-units are analysed in order to evaluate the quality of the obtained solution from the proposed approach. Results are thoroughly compared to those offered by a stochastic programming approach based on mixed-integer linear programming formulation, observing a difference on GCs between 1.41% and 1.43% for the 10-units system, and between 3.75% and 4.5% for the 110-units system, depending on the chosen significance level of the probabilistic analysis.

Keywords: Forecasting error, probabilistic economic dispatch, priority list, probabilistic unit commitment, wind power.

Nomenclature

Sets

j	Index for conventional generators ($j = 1, \dots, J$)
t	Index for time step ($t = 1, \dots, T$)
i	Index for the discrete states of discrete distribution G_j^t ($i = 1, \dots, I$)
q	Index for discrete states of forecasted wind power distribution ($q = 1, \dots, Q$)
r	Index for sampling point of output power at $t-1$ ($r = 1, \dots, R$)
l	Discretization state (bin) of forecasted wind generation ($l = 1, \dots, L$)

Parameters

A_j, B_j, C_j	Parameters of fuel consumption cost of unit j
UR_j	Ramp up rate limit of unit j
DR_j	Ramp down rate limit of unit j
SUR_j	Start-up ramp rate limit of unit j
SDR_j	Shut-down ramp rate limit of unit j

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HSU_j	Hot start-up cost of unit j
CSU_j	Cold start-up cost of unit j
CST_j	Cold start-up time of unit j
MDT_j	Minimum down time of unit j
MUT_j	Minimum up time of unit j
AWG^t	Discretized distribution of forecasted wind generation at time t
AWG^t_{max}	Maximum forecasted wind generation at time t
AWG^t_{min}	Minimum forecasted wind generation at time t
α^t, β^t	Parameters of beta distribution at time t
BWC	Battery wear cost
$VOLL$	Value of lost load
γ	Significance level of the probabilistic analysis
SR^t_{req}	Spinning reserve requirements at time t
δ	Discretization parameter of beta distribution

Variables

G^t_j	Output power of unit j at time t
G^{min}_j	Minimum power generation of unit j
G^{max}_j	Maximum power generation of unit j
$g^{t-1}_{j,r}$	Power generation for unit j and sampling point r at time $t-1$
$g^{t,max}_{j,r}$	Maximum power of unit j at time t (limited by ramp constraint and rated capacity)
G^{max}	Maximum value of power to be represented on discrete distribution G^t_j
ΔG	Discretization step of discrete distribution G^t_j
$G_{pdf}(i,j)$	Tabular representation of G^t_j for discrete state i and unit j
SUC^t_j	Start-up cost of unit j at time t
CWG^t	Discretized distribution of consumed wind generation at time t
awg^t_l	Forecasted wind generation for the state (bin) l at time t
cwg^t_l	Consumed wind generation for the state (bin) l at time t
G_i	Power value of the bin i
μ_r	Sampling point of the interval [0,1]
μ^{max}	Maximum value of μ_r
μ^{min}	Minimum value of μ_r
$\Delta\mu$	Step used for sampling interval $[\mu^{min}, \mu^{max}]$
$SP(j,r)$	Tabular representation of sampled points of distribution G^{t-1}_j
n_q	Values of the support over the interval [0,1] of discretized beta distribution
Ω	Discretized beta distribution (interval [0,1])
φ	Intermediate variable for discretization of beta discretization
$\overline{P}_r\{\cdot\}$	Calculation of a normalized probability value
$P_r\{\cdot\}$	Calculation of a probability value
$E\{\cdot\}$	Calculation of an expected value
HL^t	Hourly load at time t
HNL^t	Hourly net load at time t
EE^t	Discretized probability distribution of excess of electricity at time t
ENS^t	Discretized probability distribution of energy not supplied at time t
K	Discretized probability distribution of total generation cost
ΔK	Difference between generation cost obtained from proposed approach and reference method
SR^t	Discretized probability distribution of spinning reserve at time t
$k_{r,l}$	Total generation cost for sampling point r and state (bin) l

ee_r^t	Excess of electricity for sampling point r at time t
ens_r^t	Energy not supplied for sampling point r at time t
sr_r^t	Measurement of spinning reserve for sampling point r at time t
u_j^t	Binary variable to represent offline ($u_j^t = 0$) or online ($u_j^t = 1$) conditions
$T_{o,j}^t$	Amount of time that unit j has been online
$T_{f,j}^t$	Amount of time that unit j has been offline
FCC_j^{avg}	Average fuel consumption cost of unit j
G_j^{avg}	Average power production of unit j
CP^t	Cumulative committed capacity at time t
PUS_j^t	Element that corresponds to unit j at time t of primary unit scheduling
ΔAWG^t	Increment of committed capacity due to forecasting error
AWG_f^t	Mode of forecasted wind-generation probability distribution at time t
S	Intermediate variable of addition of power generation process

1. Introduction

During many years, wind energy has experienced a relevant development from a technological and economic point of views, incrementing its participation and importance to supply energetic requirements in many countries around the world in order to reduce oil consumption and consequently the emission of Green-House Gases (GHG) [1]. However, the variability of wind resources is an aspect that limits the integration of wind power at high penetration due that the variability of wind power generation introduces uncertainty into the scheduling problem, which makes difficult determining the optimal amount of power that should be committed in order to compensate the variability with the lowest Generation Cost (GC). In fact, this problem has inter-temporal characteristics that depends on the integration level; according to the analysis of Electric Reliability Council of Texas (ERCOT) data [2], GC related to the variability of wind generation in the interval from 15 minutes to 1 hour decreases as capacity factor increases; or in other words, those wind farms installed in places with high wind resources has a low integration cost; however, the benefit obtained from the integration of an additional wind farm reduces suddenly. Regarding the emissions of GHG, wind power variability can impact their emissions in a negative way due that cycling units are partially loaded so that their efficiency is reduced while GHG emissions are increased; besides of this, a recent analysis of Spanish power system [3] suggests that reduction of CO_2 emissions and their corresponding benefits are still important.

Nowadays, solving Economic Dispatch (ED) and Unit Commitment (UC) problems considering uncertainty of wind power generation have been extensively analysed by many authors. This problem could be solved by applying scenario generation/reduction methods as well as probabilistic methods. Scenario generation/reduction methods have been widely suggested in the technical literature due to extreme operating conditions can be easily represented in order to obtain a robust and cost-effective schedule; for this reason, it is likely that this methodology being adopted and implemented by the power industry. Other approaches, still under development, are those based on probabilistic analyses, which studies the probabilistic optimization problem since an analytical point of view; these methodologies have not been totally accepted because the reliability of the obtained results from their implementation has not been proved yet [4].

A representative methodology to solve stochastic UC problem by scenario generation/reduction method was proposed by Tuohy *et al.* [5] at which, correlated scenarios of wind generation and hourly load are generated by means of Monte Carlo Simulation (MCS) approach; more specifically, by evaluating an Autoregressive Moving Average (ARMA) model in order to describe the inter-temporal characteristics of wind power time series. The optimization model used to determine UC solution is based on a mixed-integer, stochastic optimization formulation. Additionally, an operation policy based on rolling planning is implemented in order to take advantage of wind generation and hourly load predictions with lower forecasting error; in consequence, a more robust solution could be obtained. However, this approach can be carried out only analysing a scenario set with a reduced number of trials, which could be a source of error. To solve this problem, Ruiz *et al.* [6] proposed the incorporation of Spinning Reserve (SR) requirements for each scenario to improve the robustness of the solution; this strategy compensates the problems related to consider a limited number of scenarios. Other important conclusion of this study is related to the computational time, which notably increases with the number of scenarios due that the solution of the corresponding stochastic optimization problem requires the solution of the equivalent deterministic problem; in this sense, the implementation of decomposition techniques was suggested by the authors. Besides of this, other way to reduce computational efforts consists on relaxing the integrality constraint of fast-start units as well as representing generation unit outages as a load increment [7].

Scenario generation/reduction method depends on the type and quality of trials used; in this sense, Constantinescu *et al.* [8] developed a tool able to integrate Numerical Weather Prediction (NWP) models on stochastic UC formulation, this tool allows us evaluating the capabilities of forecasting methods based on NWP from an operational point of view. According to the reported results, it was not found a considerable benefit from the intra-day operation, which contradicts those results reported by other authors whose used scenarios synthetically generated; this fact shows how the quality of the scenario set used can influence the day-ahead scheduling.

Other approach previously proposed to solve stochastic UC is Chance-Constrained Programming (CCP); Ozturk *et al.* [9] developed an optimization model able to consider uncertainty of hourly load and its correlation so that unit scheduling is estimated in order to meet demand with a high probability over the scheduling horizon; the methodology is based on the solution of several deterministic UC problems which gradually converges to the final solution. In a similar way, Ding *et al.* [10] developed a scheduling model based on CCP at which, several stochastic variables such as hourly demand, generation unit forced outages, energy prices, and wind power generation are taken into account; this method changes probabilistic constraints by their corresponding deterministic ones; then, they are incorporated in a optimization problem solved by branch and bound algorithm.

Modelling wind power uncertainty through a Markov process has been proposed in the literature, as well. In this regard, Hargreaves *et al.* [11] proposed the application of stochastic dynamic programming approach, where the optimization is represented by a two-stage problem using the recursive Bellman equation and wind power generation is modelled as a first-order Markov process through the corresponding transition matrix; to overcome the problems related to the analysis of large-scale systems, a unit aggregation algorithm based on Mixed-Integer Linear Programming (MILP) is developed.

Another work in this field is proposed by Luh *et al.* [12] where was used a Markov process to model wind generation; then, stochastic UC problem is modelled in terms of states instead of scenarios, which allows us reducing the mathematical complexity due that the information related to the previous time steps is concentrated in the transition matrix; to consider extreme operating conditions, rare events are efficiently taken into account by means of importance sampling.

More recently, other methodologies based on the combination of some of the approaches previously explained have been presented. Wang *et al.* [13] proposed the combination of CCP and Two-Stage Stochastic Programming (CCTS). CCTS is mathematically formulated in order to ensure the integration of the available wind generation with a high probability for each time step over the scheduling horizon; this model consists of a combined sample average approximation algorithm composed of scenario generation, convergence analysis, and solution validation, while the optimization model is formulated as a MILP problem. In other approach, Zhao *et al.* [14] proposed a model that combines two-stage stochastic UC and robust optimization; hence, a solution to stochastic UC problem with low generation cost and high robustness is reached. This approach incorporates weights at the objective function that can be adjusted by system operator according to its preferences, while optimization model is based on MILP formulation combined to Benders decomposition algorithm.

Regarding probabilistic approaches, Hetzer *et al.* [15] proposed a model to solve ED problem at which wind power generation is modelled through a Weibull distribution evaluated on a linearized power curve of the wind farm, while the objective function is formulated in terms of fuel consumption cost, wind generation cost, cost related to the consumption of Excess of Electricity (EE), and SR provision; optimal power dispatch is analysed as a function of scale factor, SR cost, among others factors. Similarly, the model proposed by Liu *et al.* [16] includes wind generation as a constraint in the optimization problem and is numerically analysed through Lagrange multiplier approach, considering several values of shape, scale, and penetration factors. In other work, Liu [17] analyses the expected value of GC by means of a linear representation of the obtained solutions. Liu *et al.* [18] analytically study the Oxides of Nitrogen (NO_x) emissions through a model expressed in terms of incomplete gamma function to describe the effects of wind generation on ED problem. Roy [19] presented an optimization model which analyses ED problem during a short duration interval (validity interval); this approach avoids stochastic relations on the optimization problem by taking advantage of the characteristics of aggregated wind generation; under this context, turbulence intensity is used to represent short variations. Osório *et al.* [20] proposed a model at which forecasting error is represented as a discretized Beta distribution and incorporated in ED problem under uncertainty. Besides of this, proposed model is able to probabilistically represent ramping capabilities of thermal generators and their GHG emissions.

In this paper, a UC model that combines probabilistic ED model with Priority List (PL) method is described to represent all the variables of interest (power production of thermal units, Energy Not Supplied (ENS), EE, GC, and SR) by means of their discretized Probability Distribution Function (PDF), giving a completely analytical treatment to the UC problem under uncertainty. The rest of the paper is organized as follows: Section 2 briefly describes the probabilistic ED model; Section 3 describes the probabilistic UC model; Section 4 describes the analysis of three case studies to illustrate the capabilities of proposed analytical model. Finally, conclusions and remarks are presented in Section 5.

2. Probabilistic economic dispatch

Solving probabilistic ED problem consists on finding PDF of output power of conventional generators, ENS, GC, SR, and EE considering the uncertainty of wind generation. Fig. 1 presents a simplified representation of the system under study at which, wind generation is modelled by the aggregated wind generation; as can be observed wind generation is described by two PDF, one to represent the available wind generation (AWG^t), and another one to represent the consumed or dispatched wind generation (CWG^t). Available wind generation is obtained from forecasting tools (ARMA models, NWP, etc.), while consumed wind generation is obtained from the solution of probabilistic ED problem; by this way it is possible representing wind power curtailment from a probabilistic point of view.

"See Fig. 1"

The analytical approach used to solve probabilistic ED problem is composed by three processes [20]: discretization of the PDF of available wind generation, simplification of the PDF of conventional power generation at $t-1$, and inclusion of discretized PDF of forecasting error on probabilistic ED problem. These processes are briefly describes in the next sub-sections.

2.1 Discretization of forecasting error probability distribution

From the analysis of a one-year time series obtained from a real installation and simulations based on the persistence forecasting method; in [21], the capabilities of Beta distribution to model wind power forecasting error were extensively evaluated, concluding that this distribution can reasonably representing its fat-tailed characteristic. Nowadays, other probability distributions have been suggested in the literature (Versatile distribution [22], Lévy α-stable distribution [23], Cauchy distribution [24]); however, to illustrate our proposed methodology, in this paper Beta distribution is going to be used. The parameters used to model Beta distribution are those required to represent the shape of the PDF (α^t and β^t) and those required to represent the minimum (AWG^t_{min}) and maximum (AWG^t_{max}) available wind power generation.

Discretization process used in this paper was initially proposed by Punzo and Zini [25]; according to this method, discretized Beta distribution can be described by the set Ω, as is presented in (1):

$$\Omega = \{n_q, P_r\{n_q\}; q = 0,1,\dots,Q\} \tag{1}$$

where the total amount of discrete states (Q) or bins are selected by system operator taking into account the precision required and the available computational resources, this representation is shown in Fig. 2, where the values of ($n_q \; \epsilon \; [0,1]\,; \; q = 1,\dots,Q$) are estimated according to (2):

$$n_q = \begin{cases} max\left(\left\{\frac{q}{Q} - \frac{\delta}{Q}, 0\right\}, \frac{q}{Q} - \frac{\delta}{Q} + \frac{1}{Q}\right); & q = 0,1,\dots,Q-1 \\ \left[\frac{q}{Q} - \frac{\delta}{Q}, 1\right]; & q = Q \end{cases} \tag{2}$$

"See Fig. 2"

where discretization parameter (δ) could be selected by the system operator considering the precision required and the available computational resources, its typical value is ($\delta = 3$). The corresponding probability value of each state or bin (q) is calculated by using (3):

$$P_r\{n_q\} = \frac{(1+q)^{\alpha^{t}-1}(Q+1-q)^{\beta^{t}-1}}{\sum_{\varphi=0}^{Q}(1+\varphi)^{\alpha^{t}-1}(Q+1-\varphi)^{\beta^{t}-1}}; \quad q = 0, 1, \dots, Q \tag{3}$$

Once the set Ω has been described, the values (n_q) are displaced in order to obtain the set of available power (AWG^t) to be injected into the system, this is carried out by including an additional index (l); hence, $l = q + 1$ and using (4):

$$AWG^t = \{awg_l^t = (AWG_{max}^t - AWG_{min}^t)n_{l-1} + n_{l-1}; \quad l = 1,2,\dots,L\} \tag{4}$$

2.2 Solution of probabilistic economic dispatch

For each time step $t = 1, 2, \dots, T$, the power production at previous time instant $t-1$ should be taken into account to solve probabilistic ED problem considering the ramp rate capabilities of conventional generators. In the probabilistic ED model used in this paper, power generation at any time interval is represented by a discretized PDF able to represent power values (G_i) between 0 and G^{max}, using I discrete states or bins; then, considering this information discretization step (ΔG) is calculated according to (5):

$$\Delta G = \frac{G^{max}}{I-1} \tag{5}$$

Output power of conventional units are probabilistically described by using these discrete states ($i = 1, \dots, I$) to represent values as high as G^{max}, with precision ΔG; including those generated at time $t-1$ (G_j^{t-1}), which are assumed to be known in our analysis.

Cumulative Distribution Function (CDF) of G_j^{t-1} is presented in Fig. 3, this function is used to sample some power values determined by means of the quantile concept; in more detail, the interval [0, 1] is sampled by the variable $\mu_r \in [\mu^{min}, \mu^{max}]$ with a determined step ($\Delta\mu$) defined by the system operator; then, for each of the values of μ_r, the inverse CDF is evaluated by using a linear interpolation algorithm in order to obtain the power value $g_{j,r}^{t-1}$, which is saved in the table of sampled points ($SP(j,r)$ with J rows and R columns). The quantity R is selected by the system operator according to the required precision, while μ^{min} and μ^{max} are selected by considering a determined significance level (γ); so that, $\mu^{min} = \gamma$, and $\mu^{max} = 1 - \gamma$. For example, if a significance level of 1% is selected, μ^{min} = 1% and μ^{max} = 99%. It is possible to note that as lower is the significance level selected, more extreme are the conditions of power generation included in the probabilistic ED analysis.

Once the table $SP(j,r)$ is completed by evaluating the inverse CDF for $j = 1, \dots, J$ and $r = 1, \dots, R$; the corresponding weighting factors or normalized probabilities ($\overline{P_r}(g_{j,r}^{t-1}) \forall j \in [1, J]$) should be calculated; it is required because not all the possible combinations of power generation between the units of the system are taken into account at time $t-1$; then, the probability of the selected trials by means of quantile evaluation does not sum up one; then, normalized probabilities are calculated by evaluating (6) [20]:

"See Fig. 3"

$$\bar{P}_r\{G_j^{t-1} = g_{j,r}^{t-1}\} = \frac{\prod_{j=1}^{J}\left(P_r\{G_j^{t-1} = g_{j,r}^{t-1}\}\right)}{\sum_{r=1}^{R}\prod_{j=1}^{J}\left(P_r\{G_j^{t-1} = g_{j,r}^{t-1}\}\right)} \tag{6}$$

Modelling of output power at time $t-1$ allows us estimating the consumed or dispatched wind generation by means of the solution of probabilistic ED problem with ramp constraints; in a similar way as the available wind generation, dispatched wind power is analytically represented through the set (CWG^t) according to (7):

$$CWG^t = \{cwg_l^t,\ \ P_r\{cwg_l^t\};\ \ l = 1, 2, \ldots, L\} \tag{7}$$

Once the available wind generation has been discretized and PDF of power generation of each unit at time $t-1$ has been simplified (by sampling some points according to quantile evaluation), the next step in our analysis is the solution of probabilistic ED problem; this is carried out by following the algorithm presented as follow [20]:

- Step 1: Select the first column of the table $SP(j,r)$; or in other words, set $r \leftarrow 1\ \forall\, j \in [1,\ J]$, (choose the first column).
- Step 2: Select the first state (bin) of discretized available wind generation distribution by setting $l \leftarrow 1$.
- Step 3: Solve ED problem for the pair (r, l) previously selected in steps 1 and 2; the optimization problem to be solved is presented in (8)-(15) [26]:

$$k_{r,l} = \sum_{j=1}^{J}\left\{A_j u_j^t + B_j\left(g_{j,r}^t\right) + C_j\left(g_{j,r}^t\right)^2\right\} + BWC(ee_r^t) + VOLL(ens_r^t) \tag{8}$$

$$\sum_{j=1}^{J} g_{j,r}^t + cwg_l^t = HL^t \tag{9}$$

$$g_{j,r}^t - g_{j,r}^{t-1} \le UR_j;\quad u_j^t = 1,\ \ u_j^{t-1} = 1 \tag{10}$$

$$g_{j,r}^{t-1} - g_{j,r}^t \le DR_j;\quad u_j^t = 1,\ \ u_j^{t-1} = 1 \tag{11}$$

$$G_j^{min} \le g_{j,r}^t \le G_j^{max};\quad u_j^t = 1 \tag{12}$$

$$0 \le cwg_l^t \le awg_l^t \tag{13}$$

$$g_{j,r}^t \le SUR_j;\quad u_j^t = 1,\quad u_j^{t-1} = 0 \tag{14}$$

$$g_{j,r}^t \le SDR_j;\quad u_j^t = 1,\quad u_j^{t+1} = 0 \tag{15}$$

  Using the results obtained from the solution of the optimization problem (carried out by applying quadratic programming approach), SR resources are measured by applying (16):

$$sr_r^t = \sum_{j=1}^{J} g_{j,r}^{t,max} u_j^t - g_{j,r}^t \tag{16}$$

- Step 4: Using the values obtained of power production of thermal units ($g_{j,r}^t$), energy not supplied (ens_r^t), excess of electricity (ee_r^t), generation cost ($k_{r,l}$), and spinning reserve (sr_r^t); discretized PDF of each probabilistic variable is built by finding the discrete state that corresponds to value obtained from the optimization process.

An example of this process applied to allocate $g_{j,r}^{t}$ on the G_j^t distribution is shown in Fig. 4 at which, discretized distribution $G_j^t\ \forall j\epsilon[1,J]$ is represented in tabular form through the matrix $G_{pdf}(i,j)$ with all its elements initialized to zero. It is possible to note the influences of the probability of occurrence of a determined wind power value, as well as, the normalized probability of a determined power generation of conventional units at time $t-1$. A similar algorithm can be easily adopted to build discretized PDF of other probabilistic variables of interest such as *GC, ENS, ES, SR*, among others.

"See Fig. 4"

- Step 5: If, $l < L$, set $l \leftarrow l+1$ and go back to step 3; else go to step 6.
- Step 6: If, $r < R$, set $r \leftarrow r+1$ and go back to step 2; else end.

3. Probabilistic unit commitment

Generally speaking, in the case of systems vertically integrated, day-ahead UC problem is carried out in order to minimize GC over the scheduling horizon (T); as in our model all variables are described by their corresponding PDFs, UC problem can be easily formulated in terms of these variables. Equation (17) [27] describes expected value of total GC (K) at which, fuel consumption cost, start-up cost, cost related to EE, and cost related to ENS are considered; Battery Wear Cost (BWC) is incorporated in order to evaluate the cost of storing or consuming EE:

$$E\{K\} = \sum_{t=1}^{T}\left\{\sum_{j=1}^{J}\left\{A_j u_j^t + B_j(G_j^t) + C_j(G_j^t)^2\right\} + SUC_j^t(1-u_j^{t-1})u_j^t + (BWC)E\{EE^t\} + (VOLL)E\{ENS^t\}\right\} \quad (17)$$

An additional SR requirement (SR_{req}^t) to be adjusted by system operator has been assumed; in this sense, this constraint should be fulfilled with a determined probability $(1-\gamma)$; this is mathematically expressed in (18):

$$P_r\{SR^t \geq SR_{req}^t\} \geq 1-\gamma \quad (18)$$

Power balance of the system is represented by (19), where available wind generation (AWG^t) at any time step t could be curtailed in order to preserve it; hence, consumed or dispatched wind generation (CWG^t) is incorporated to this constraint:

$$\sum_{j=1}^{J} G_j^t u_j^t + CWG^t = HL^t \quad (19)$$

A simplified definition of start-up cost is used in this paper according to (20) expressed in terms of the cumulative number of time steps that unit j has been offline ($T_{f,j}^t$) in order to properly estimate start-up cost (SUC_j^t) according to hot start-up (HSU_j) and cold start-up cost (CSU_j) [28, 29]:

$$SUC_j^t = \begin{cases} HSU_j;\ T_{f,j}^t \leq MDT_j + CST_j \\ CSU_j;\ T_{f,j}^t > MDT_j + CST_j \end{cases} \quad (20)$$

Parameters $T_{f,j}^t$ and $T_{o,j}^t$ are calculated by using (21) and (22) in a cumulative sense [30]:

$$T_{o,j}^{t} = \begin{cases} T_{o,j}^{t} + 1; & u_j^t = 1 \\ 0; & u_j^t = 0 \end{cases} \tag{21}$$

$$T_{f,j}^{t} = \begin{cases} T_{f,j}^{t} + 1; & u_j^t = 0 \\ 0; & u_j^t = 1 \end{cases} \tag{22}$$

In order to obtain a feasible solution, the amount of time that a determined unit j has to be online should be higher or equal than the corresponding minimum up time ($MUT_j,\ j \epsilon [1,\ J]$), while the amount of time that this unit should be offline has to be higher or equal than its corresponding minimum down time ($MDT_j,\ j \epsilon [1,\ J]$); this idea is mathematically expressed in (23) and (24) [30]:

$$T_{o,j}^{t} \geq MUT_j \tag{23}$$

$$T_{f,j}^{t} \geq MDT_j \tag{24}$$

Operation of each generation unit j is carried out to limit power generation to a lower (G_j^{min}) and upper (G_j^{max}) limit, this is presented in (25) [30]:

$$G_j^{min} \leq G_j^t \leq G_j^{max}; \quad u_j^t = 1 \tag{25}$$

Similarly, conventional units are operated to limit their increment or decrement on the power production according to their corresponding ramp up rate limit (UR_j and SUR_j) and ramp down rate limit (SUR_j and SDR_j) during continues, starting-up and shutting-down conditions; this is expressed in (26)-(29) [30]:

$$G_j^t - G_j^{t-1} \leq UR_j; \quad u_j^t = 1,\ u_j^{t-1} = 1 \tag{26}$$

$$G_j^{t-1} - G_j^t \leq DR_j; \quad u_j^t = 1,\ u_j^{t-1} = 1 \tag{27}$$

$$G_j^t \leq SUR_j; \quad u_j^t = 1,\ u_j^{t-1} = 0 \tag{28}$$

$$G_j^t \leq SDR_j; \quad u_j^t = 1,\ u_j^{t+1} = 0 \tag{29}$$

Finally, dispatched wind generation is limited to be lower than that value forecasted, this is presented in (30); then, power values of dispatched generation of thermal and renewable sources are completely feasible:

$$0 \leq CWG^t \leq AWG^t \tag{30}$$

Once the mathematical formulation has been presented, probabilistic UC problem should be solved; in our case by PL method. This method is composed by three main processes: Primary Unit Scheduling (PUS), minimum up down/time repairing, and addition of power generation, each of these steps is briefly described in the next subsections.

3.1 Primary unit scheduling

During PUS, an initial approximation to probabilistic UC solution is developed based on an economic criteria; this solution is built by committing conventional generators from the cheapest one to the most expensive one until a reserve margin able to face wind power variability is reached. PUS process is applied by following the algorithm presented next:

- Step 1: Create table PUS_j^t, with J rows and T columns. All elements of this table are initialled with zero.
- Step 2: Determine the order at which conventional generators are going to be committed (priority list); this order is established by means of the average fuel consumption cost (FCC_j^{avg}) presented in (31) [30]:

$$FCC_j^{avg} = \frac{A_j + B_j\left(G_j^{avg}\right) + C_j\left(G_j^{avg}\right)^2}{G_j^{avg}} \quad (31)$$

$$G_j^{avg} = \frac{G_j^{avg}}{2}\left(1 + \frac{G_j^{min}}{G_j^{max}}\right) \quad (32)$$

- Step 3: Set $t \leftarrow 1$.
- Step 4: Commit the first unit according to the priority list created in step 2 and set $j \leftarrow 1$.
- Step 5: Set $PUS_j^t \leftarrow 1$.
- Step 6: Calculate the cumulative capacity (CP^t), the increment of committed capacity due to forecasting error (ΔWF^t), and hourly net load (HNL^t) at time t according to (33), (34) and (35), respectively:

$$CP^t = \sum_{j=1}^{J} G_j^{max} PUS_j^t \quad (33)$$

$$\Delta AWG^t = max\left(AWG_{max}^t - AWG_f^t,\ AWG_f^t - AWG_{min}^t\right) \quad (34)$$

$$HNL^t = max\left(HL^t - AWG_f^t,\ 0\right) \quad (35)$$

  AWG_f^t is the wind power generation profile with the highest probability, it can be interpreted as the mode of AWG^t.
- Step 7: If $CP^t < HNL^t + \Delta AWG^t$, and $j \le J$, set $j \leftarrow j+1$ and go to step 5; else if $t \le T$, set $t \leftarrow t+1$ and go to step 4; else, stop.

3.2 Minimum up/down time repairing

As can be observed, solution obtained from PUS process (PUS_j^t) could be unfeasible due that minimum up/down time constraints (equations (23) and (24)) could not be fulfilled; in order to overcome this problem, in this paper the algorithm proposed by Dieu *et al.* [30] is used.

3.3 Addition of power generation

As was stated before, a probabilistic constraint to SR margin is considered in (18); this condition is probabilistically verified by means of discretized distribution of SR obtained from the solution of probabilistic ED problem (Sub-section 2.2); in other words, the solution obtained from the application of minimum up/down time repairing (Sub-section 3.2) is probabilistically evaluated to obtain ED solution (Section 2); then, discretized distribution of SR (SR^t) is used to evaluate $P_r\{SR^t \ge SR_{req}^t\}$; if $P_r\{SR^t \ge SR_{req}^t\}$ is lower than $(1-\gamma)$, more generation capacity is added according to the order established by PL method.

This is an iterative process that could be implemented by following the algorithm presented next [31]:

- Step 1: For $t = 1,2,\ldots,T$ verify the probability of fulfil the required SR requirements (SR_{req}^{t}) by means of $P_r\{SR^t \geq SR_{req}^{t}\}$.
- Step 2: Build a list with those hours at which SR requirements are not fulfilled; or in other words, fill out the list with those hours with probability lower than $1-\gamma$; once the list has been completed, the amount of elements of this list is saved in the factor S.
- Step 3: If ($S > 0$), create a table with S rows and two columns, this table is designed to content the units that should be committed to fulfil SR requirements at each hour. In other case; stop.
- Step 4: The information obtained in step 2 is used to fulfil the table created in step 3, the second column specifically.
- Step 5: For each element of the table created in step 3; using PL method, those units which should be committed in order to fulfil SR requirements are identified, this information is used to complete the first column of the table created in step 3.
- Step 6: The first element of column one and column two of the table completed in step 5 are selected; so that, the condition of the corresponding unit at the corresponding hour is changed from offline to online.
- Step 7: As the scheduling has been changed, minimum up/down time repairing algorithm (sub-section 3.2) is applied in order to obtain a feasible solution.
- Step 8: Go to step 1.

Methodology proposed in this paper is illustrated by analysing three case studies with 6, 10, and 110 units; in order to evaluate the performance of proposed method, where a comprehensive comparison with scenario generation/reduction approach is presented.

4. Case studies

In this section, three case studies are carefully analysed and the comparison with Stochastic Programming (SP) approach (reference method) is presented in order to evaluate the proposed probabilistic methodology. The reference method used is based on the mathematical formulation proposed in [32]. Regarding the case studies; in the first case study, a system with 6 units is analysed; in the second case study, a system with 10 units is studied; and finally in the third case study a system with 110 units is analysed. The proposed approach in this paper was implemented in MATLAB programming language, while the reference method was implemented in GAMS programming language. The computer used is provided with i7-3630QM CPU at 2.40 GHz, 8 GB of RAM and 64-bit operating system.

4.1 6-units power system

This case study is proposed and analysed in order to understand the probabilistic treatment of the SR requirements proposed in this paper, data used here was adapted from that proposed in [18] at which, ramp up/down limitations and wind generation have been added. All the relevant information related to the conventional generators is shown in Table 1, while the characteristics of wind generation are described in Table 2, the analyses is carried out for a single time step t (T = 1). Hourly load of the system is assumed to be 3.5MW (HL^t = 3.5) and reserve margin is assumed to be 0.89MW (SR_{req}^{t} = 0.89).

Discretized distribution of G_j^{t-1} has been built by representing the values shown in Table 1 in a probabilistic way, assigning probability of 1 to the corresponding state (bin) of the initial power generation of each unit. Regarding the representation of discretized PDFs, power of conventional units has been represented by using 2,500 states (I = 2,500); so that, a maximum power value of 1.3MW can be represented ($G^{max} = 1.3$); wind generation has been represented by using 100 states ($Q = 100$); hence, a maximum power value of 1.6MW (dispatched or consumed power) can be represented; ENS, EE, and SR have been modelled by using 100 states; then, a maximum power value of 1.5MW can be represented.

"See Table 1"

"See Table 2"

Fig. 5 shows discretized PDF and CDF of SR when units 1, 2 and 3 are committed, if SR requirements are evaluated in CDF, it is possible observing that there is 54% of fulfil this requirement; then, if a significance level of 1% ($\gamma = 0.01$) is adopted, more generation capacity should be committed; by this way, the required amount of capacity in order to fulfil a determined SR requirement is determined by committing each unit of the system sequentially (addition of power generation process of PL method); as can be observed, this is an iterative process.

Results obtained from this approach are shown in Fig. 6, where it is possible concludes that capacity required is 3.8MW; discretization parameters δ was adjusted to 5, while only the scenario of conventional power production at time $t - 1$ with highest probability was considered by setting R = 1 and consequently μ_1 = 0.5 (Fig. 3).

"See Fig. 5"

"See Fig. 6"

4.2 10-units power system

Technical data of conventional generators and hourly load profile used in this case study can be found in [33], while the characteristics of wind power generation over a scheduling horizon of 24 hours (T = 24) are presented in Table 3. In order to evaluate the performance of proposed approach, scenario generation/reduction method has been implemented and used as a reference of comparison; initially 3,000 scenarios of wind power production on daily basis has been generated by using a first order ARMA model; then, a scenario reduction method based on k-means++ clustering algorithm was used to reduce the scenario set to 50 trials; reference solution was estimated by using duality gap of 0%. In proposed model, only are specified the parameters of Beta distribution for each hour, while autocorrelation is not modeled; in other words, it is equivalent to assume that wind generation is not correlated in hourly basis, which general speaking is not true. In order to avoid the effects of this fact on the comparison, both SP approach and proposed method are compared by using an additional set of 1,000 scenarios.

Discretization of forecasting error was done by considering Q = 10, δ = 5, μ^{min} = 0.01, μ^{max} = 0.99, $\Delta\mu$ = 0.49 (R = 3), I = 2,500, γ = 0.01, G^{max} = 500, and SR requirements were defined as: $SR_{req}^t = 0.1(HL^t),\ t \in [1,\ T]$.

"See Table 3"

Fig. 7 shows results obtained from scenario generation/reduction algorithm used at which, extreme profiles of maximum (AWG_{max}^{t}) and minimum (AWG_{min}^{t}) wind generation has been included.

"See Fig. 7"

Fig. 8 shows a comparison between the hourly committed capacity required by our proposed method and reference approach; as can be observed, proposed approach requires the commitment of more power during almost all the hours, especially during the time interval between $t = 1$h and $t = 5$h. This difference directly impacts the expected value of GC over the entire day.

"See Fig. 8"

The proposed probabilistic approach does not take into account auto-correlated nature of wind generation; so that, expected value of GC ($E\{K\}$) has been calculated by MCS approach through the solution of ED problem using the scenario set of 1,000 trials previously mentioned. Finally, the obtained results were used to verify the probability of fulfil constraint (18) and the quality of the solution from an economical perspective.

Table 4 shows the probability of fulfil the required SR requirements measured from the evaluation of the obtained solution from the proposed method on the set of 1,000 scenarios aforementioned, including significance levels of 10% and 1% (considering wind power autocorrelation), while Fig. 9 presents a comparison between probabilities obtained from the evaluation of 1,000 scenarios (using information taken from Table 4 and known as correlated in Fig. 9) and analytically measured by using the proposed approach, at which probability values are obtained by reading discretized PDF of SR by means of a linear interpolation (known as proposed in Fig. 9).

From Table 4, it is possible observing how the probability of fulfil constraint (18) increases as significance level is reduced (specifically at $t = 6$h), due that more generation capacity should be committed, increasing GC. From Fig. 9, it is possible observing that this constraint is not totally fulfilled for $t = 12$h and $t = 18$h. In fact, during $t = 12$h, there is a high-load period and high wind power uncertainty; for this reason, all units in the system are committed; however, according to these results it is not enough. Regarding $t = 18$h, the proposed approach is quite sensitive to the number of intervals used for discretization process (Q), which impacts the accuracy to represent PDF of SR. The application of PL method to determine the units to be committed, combined with the solution of ED problem at each hour lead to a sub-optimal scheduling because this procedure does not have look-ahead capabilities [34]; this is the reason why to fulfill a determined SR requirements, the solution obtained by PL suggests more generation capacity (Fig. 8) than that scheduling obtained from SP approach.

This could be evaluated by observing the probability of fulfill the required reserve (Fig. 9) where at time $\mathrm{t} = 12$h, the corresponding probability value is low due to all available capacity have been committed. In SP approach, ED and UC problems are solved simultaneously for the scenario set considered (solving the equivalent deterministic problem), while in PL method power dispatch is obtained from the solution of the optimization problem at each time step; hence, SR requirements are obtained by committing additional capacity generation according to the list built by using (31) and (32). This explains the difference between generation costs and probability of provide a determined SR requirements.

"See Table 4"

"See Fig. 9"

By one hand, scheduling obtained from SP approach was obtained in 48.049 seconds with expected value of GC equals to 551,060.065$; on the other hand; the information related to the proposed approach is shown in Table 5, where according to these results, proposed method offers a unit scheduling between 1.41% and 1.43% more expensive than that obtained from SP approach. Besides of this, an additional increment on the computational time is observed (the number of iterations required by addition of power generation process of PL method is reported in the last column); differences on the computational time could be related to the languages employed to carry out the numerical analysis.

"See Table 5"

4.3 110-units power system

In this case study, data related to the conventional generators and hourly load profile was taken from reference [35], while forecasting of wind generation used is presented in Table 6; and as was stated before, corresponding comparison with SP approach was carried out; for this purpose, 3,000 scenarios of wind generation were created and then reduced to 30 scenarios; additionally, extreme conditions shown in Table 6 was included in the optimization process; all scenarios generated are shown in Fig. 10. Discretization of forecasting error was done by considering $Q = 5$, $\delta = 5$, $\mu_1 = 0.5$, $R = 1$, $G^{max} = 800$, $I = 1{,}500$ and SR requirements were defined as: $SR_{req}^{t} = 0.1\ (HL^{t}),\ t \in [1, T]$.

"See Table 6"

"See Fig. 10"

Comparison of the hourly committed capacity between the scheduling obtained from the application of our proposed methodology and SP approach is shown in Fig. 11; according to these results more capacity is committed especially during the interval between $t = 1$ h and $t = 8$ h; which relatively increases GC.

"See Fig. 11"

As in the previous case study (Sub-section 4.2), an additional set of 1,000 scenarios was created and from the evaluation of this set, SR was measured and probabilistically analysed; then, probability of fulfil constraint (18) was estimated; the results are presented in Table 7, where it is possible observing that this constraint is successfully fulfilled.

"See Table 7"

In this case, solution from SP was obtained in a limited time of 36,000 seconds, expected value of GC of the corresponding solution is 2,436,638.424$ (duality gap equals to 0.000420). Relevant information related to the solution obtained from proposed approach is shown in Table 8, where a difference between 3.8% and 4.5% with respect to SP approach was observed. Such as in previous case study analyzed, the number of iterations required by addition of power generation process of PL method is reported in the last column.

"See Table 8"

Overall, SP and the proposed method have important differences: 1) On the one hand, the scheduling obtained from SP approach strongly depends on the characteristics of the scenario set used; on the other hand, the proposed method depends on the parameters of probability distribution, significance levels, discretization intervals, and sampling points on cumulative distribution. 2) In scenario generation/reduction method if a reduced number of trials is considered, all of them are close to the point forecasting of wind generation, excluding extreme conditions of very high and very low wind generation; however, by discretization of forecasting error distribution with a few amount of intervals, extreme conditions could be considered with their corresponding probabilities. 3) Scenario reduction process used to obtain a representative set of possible wind generation profiles requires, in many of the available algorithms, the comparison of one to one scenarios in a repetitive way, which increases the computational burden. However, in our proposed methodology, discretization intervals can be easily increased.

5. Conclusions

In this paper, a probabilistic methodology to the day-ahead unit scheduling of power systems provided of wind generation was proposed and evaluated. The method was based on solving probabilistic ED problem finding the probability distribution of the variables of interest in a discrete form; then, this method was incorporated to PL method to determine a near-optimal scheduling of the system by analysing probability distribution of SP at each time step. Three case studies were analysed; the first case study explained how the probability distribution of SR can be used to determine the capacity that should be committed in order to provide a required amount of SR requirements; while in the second and third case studies, two systems of 10-units and 110-units were respectively studied. In order to evaluate the performance of the proposed approach, results were compared to those obtained from a SP approach based on MILP formulation, observing an increment on GCs between 1.41% and 1.43% for 10-units system, and between 3.75% and 4.5% for the 110-units system, depending on the selected significance level of the probabilistic analysis.

Acknowledgements

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Figures caption

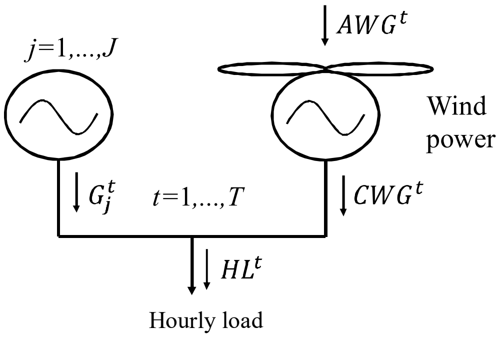

Fig. 1: Simplified system under study.

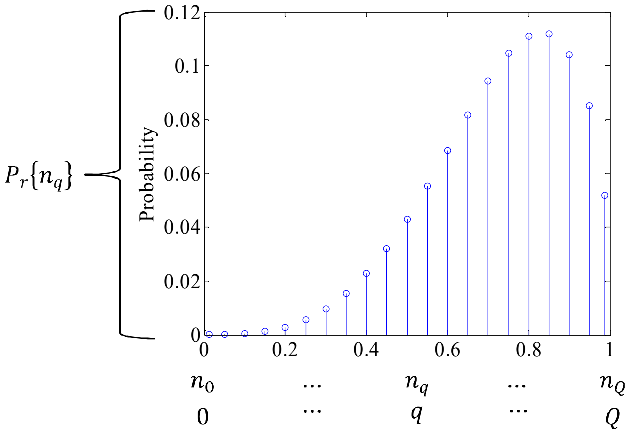

Fig. 2: Discretized beta distribution.

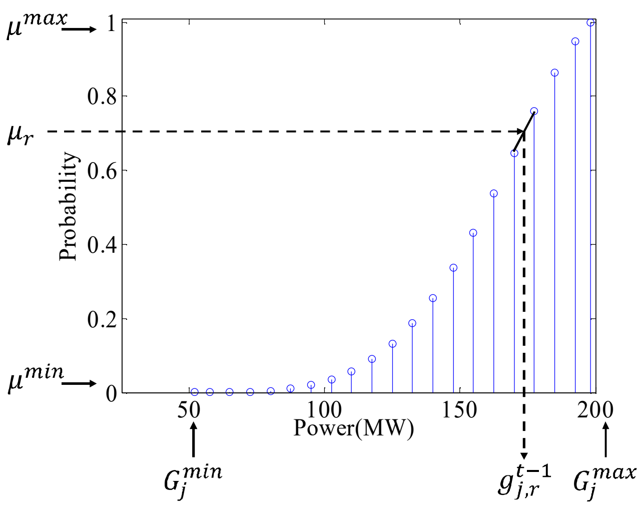

Fig. 3: Analysis of cumulative distribution of $\mathbf{G_j^{t-1}}$**.**

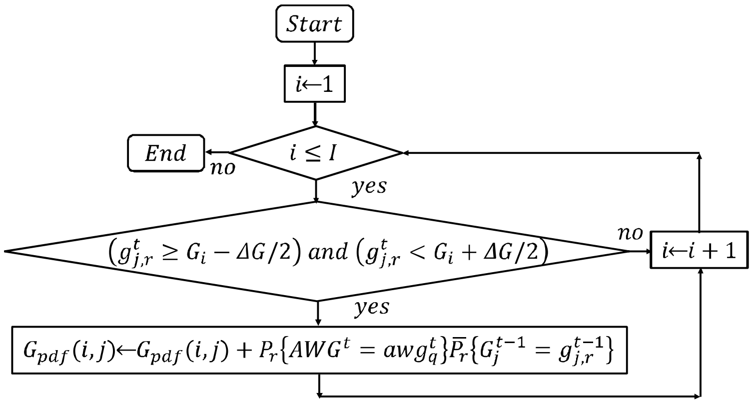

Fig. 4: Algorithm to fulfil the table $\mathbf{G_{pdf}}$.

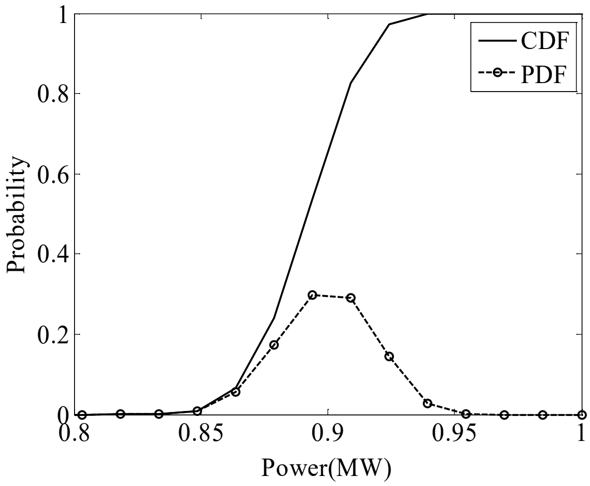

Fig. 5: Discretized PDF and CDF of spinning reserve when 3.2MW are committed.

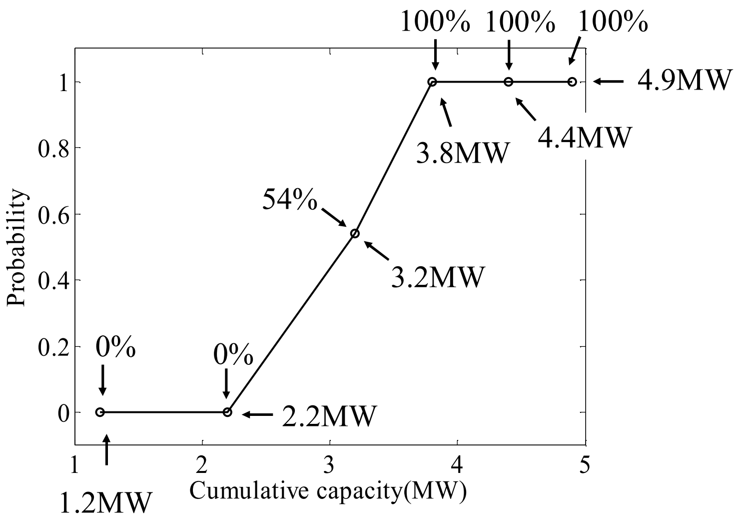

Fig. 6: Evaluation of $P_r\{SR^t \geq SR_{req}^t\}$ for case study 1.

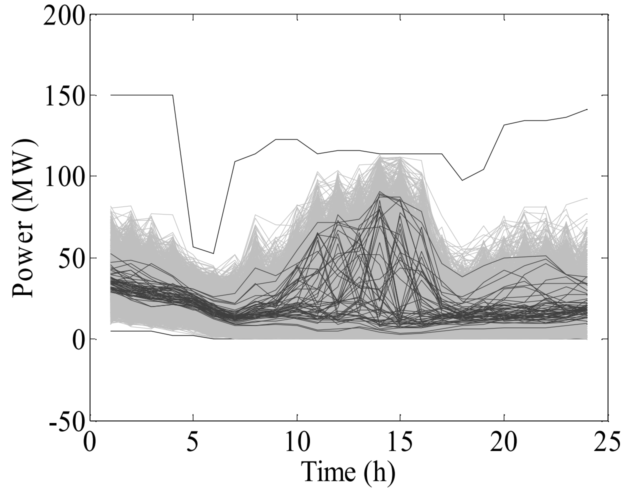

Fig. 7: Wind power scenarios for 10-units system.

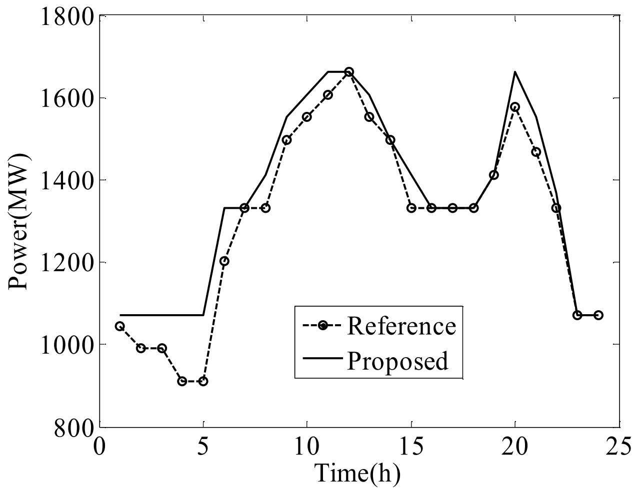

Fig. 8: Comparison of committed capacity for 10-units system ($\gamma = 1\%$).

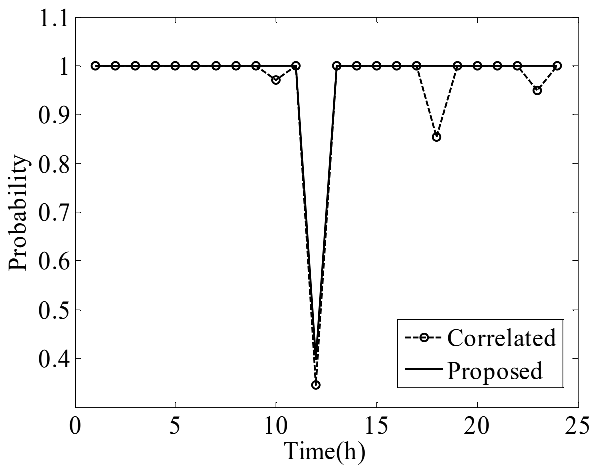

Fig. 9: Comparison of $P_r\{SR^t \geq SR^t_{req}\}$ between MCS and proposed approach for the case 2.

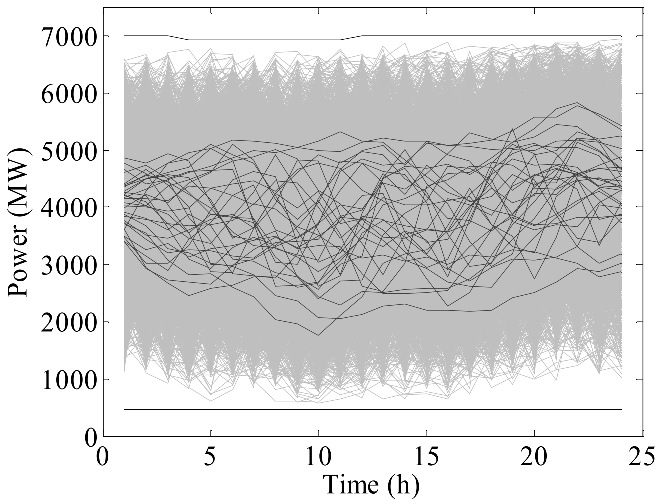

Fig. 10: Wind power scenarios for 110-units system.

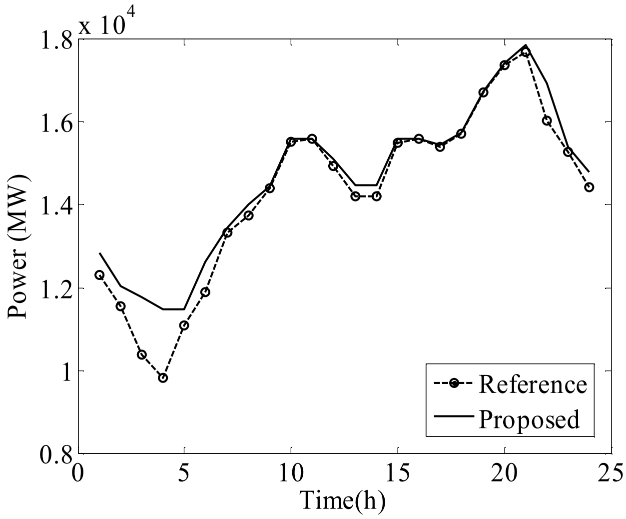

Fig. 11: Comparison of committed capacity for 110-units system ($\gamma = 1\%$).

Table caption

Table 1: data of 6-units power system.

j	A_j (\$/h)	B_j (\$/MWh)	C_j (\$/MW^{2}h)	G_j^{min} (MW)	G_j^{max} (MW)	$UR_j(MW)$	G_j^{t-1}
1	100	60	10	0.06	1.2	0.48	0.630
2	180	40	20	0.05	1.0	0.40	0.525
3	180	40	20	0.05	1.0	0.40	0.525
4	150	100	10	0.03	0.6	0.24	0.315
5	150	120	10	0.03	0.6	0.24	0.315
6	200	100	10	0.02	0.5	0.20	0.260

Table 2: Prediction of wind generation for 6-units system.

α^t	β^t	AWG_{max}^t (MW)	AWG_{min}^t (MW)
10	7	1.50	1.35

Table 3: Prediction of wind generation for 10-units system.

Time (h)	α^t	β^t	AWG_f^t (MW)	AWG_{max}^t (MW)	AWG_{min}^t (MW)
1	2.860350	10.42120	35.87775463	150	4.545437013
2	2.631970	11.21150	32.20319371	150	4.545437013
3	2.660710	13.07550	29.13805961	150	4.545437013
4	3.406050	17.10470	26.80320913	150	2.272724935
5	2.762190	4.30981	23.59278973	56.817994810	2.272724935
6	1.884270	3.81715	17.28766489	52.272622080	0
7	1.544660	11.0983	13.34824758	109.09061690	0
8	1.097010	6.22027	17.04852272	113.63598960	0
9	1.623870	9.47347	17.96137725	122.72686360	0
10	1.237900	5.75834	21.71822366	122.72686360	0
11	0.725405	2.76941	23.59664687	113.63598960	0
12	0.664855	2.29428	26.03692561	115.90874030	0
13	0.793996	2.48559	28.08763404	115.90874030	0
14	0.405002	1.09549	30.64876957	113.63598960	0
15	0.363353	1.19035	26.52163851	113.63598960	0
16	0.466796	2.13421	20.35279899	113.63598960	0
17	0.977628	6.42645	14.98367148	113.63598960	0
18	1.507940	8.80069	14.30353056	97.72699221	0
19	1.156170	6.44942	15.92866878	104.54511560	0
20	1.190020	8.13745	16.81838052	131.81748050	0
21	1.134190	7.52165	17.57052123	134.09061690	0
22	1.107700	7.02068	18.28280491	134.09061690	0
23	1.271600	8.05439	18.61066111	136.36375320	0
24	1.585750	9.93250	19.40137314	140.90874030	0

Table 4: Probability of fulfil the required spinning reserve requirements for 10-units system.

Time (h)	Significance level (γ)		Time (h)	Significance level (γ)	
	10%	1%		10%	1%
1	1	1	13	1	1
2	1	1	14	1	1
3	1	1	15	1	1
4	1	1	16	1	1
5	1	1	17	1	1
6	0.946693	1	18	0.853649	0.853649
7	1	1	19	1	1
8	1	1	20	1	1
9	1	1	21	1	1
10	0.970404	0.970404	22	1	1
11	1	1	23	0.950022	0.950022
12	0.345073	0.345073	24	1	1

Table 5: Evaluation of proposed approach for 10-units system (the solution obtained by reference method is 551,060.065$ in 48.049 seconds).

γ	$E\{K\}$ ($)	ΔK (%)	Time (s)	Iterations
10%	558,840.0	1.411808	75.560	17
1%	558,989.4	1.438929	78.895	18

Table 6: Prediction of wind generation for 110-units system.

Time (h)	α^t	β^t	AWG_f^t (MW)	AWG_{max}^t (MW)	AWG_{min}^t (MW)
1	2.99616	2.45200	4,054.917418	7,000	456.524172
2	2.96971	2.53426	3,986.994699	7,000	456.524172
3	2.94002	2.60665	3,924.796773	7,000	456.524172
4	2.85717	2.55981	3,867.893203	6,923.942051	456.524172
5	2.83312	2.54650	3,862.64189	6,923.942051	456.524172
6	2.80922	2.53345	3,857.261447	6,923.942051	456.524172
7	2.78557	2.52058	3,851.881003	6,923.942051	456.524172
8	2.59041	2.48979	3,754.258237	6,923.942051	456.524172
9	2.39065	2.42182	3,669.118099	6,923.942051	456.524172
10	2.20859	2.34557	3,592.629715	6,923.942051	456.524172
11	2.34426	2.36178	3,678.028114	6,923.942051	456.524172
12	2.52819	2.44768	3,780.902193	7,000	456.524172
13	2.67248	2.39877	3,904.566306	7,000	456.524172
14	2.62938	2.43037	3,856.658837	7,000	456.524172
15	2.58686	2.45653	3,812.539200	7,000	456.524172
16	2.54458	2.47736	3,771.733917	7,000	456.524172
17	2.74820	2.48496	3,892.600199	7,000	456.524172
18	2.93763	2.43562	4,033.783036	7,000	456.524172
19	3.10541	2.33074	4,194.421556	7,000	456.524172
20	3.20148	2.24473	4,302.977384	7,000	456.524172
21	3.30078	2.15577	4,414.847564	7,000	456.524172
22	3.40902	2.07074	4,527.449485	7,000	456.524172
23	3.29546	2.12700	4,433.356290	7,000	456.524172
24	3.17969	2.17268	4,343.825711	7,000	456.524172

Table 7: Probability of fulfil the required spinning reserve requirements for 110-units system.

Time (h)	Significance level (γ)		Time (h)	Significance level (γ)	
	10%	1%		10%	1%
1	1	1	13	1	1
2	1	1	14	1	1
3	1	1	15	0.999002	1
4	1	1	16	0.999002	0.999763
5	1	1	17	1	1
6	1	1	18	0.999002	1
7	0.999002	1	19	0.999002	1
8	1	1	20	0.999002	1
9	0.999002	1	21	0.999002	1
10	1	1	22	1	1
11	0.999002	0.999763	23	0.999002	1
12	1	1	24	0.999002	1

Table 8: Evaluation of proposed approach for 110-units system (the solution obtained by reference method is 2,436,638.424$ in 36,000 seconds).

γ	$E\{K\}$ (\$)	ΔK (%)	Time (s)	Iterations
10%	2,528,236	3.759163	777.181	82
1%	2,547,217	4.538180	956.706	100