

Optimising power generation efficiency for head-sensitive cascaded reservoirs in a competitive electricity market

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Abstract

This paper is on the problem of short-term hydro scheduling (STHS), particularly concerning head-sensitive reservoirs under competitive environment. We propose a novel method, based on nonlinear programming (NLP), for optimising power generation efficiency. This method considers hydroelectric power generation as a nonlinear function of water discharge and of the head. The main contribution of this paper is that the maximum water discharge, thus giving the maximum power generation, is also considered as head-dependent in order to obtain more realistic and feasible results. The proposed method has been applied successfully to solve a case study based on one of the Portuguese cascaded hydro systems, providing a higher profit at an acceptable computation time in comparison with classical optimisation methods based on linear programming (LP) that ignore head dependence. © 2007 Elsevier Ltd. All rights reserved.

Keywords: Short-term hydro scheduling (STHS); Electricity market; Head dependence; Nonlinear programming (NLP)

1. Introduction

The satisfaction of the demand for electric energy has been mainly achieved with hydro resources and thermal resources. Hydro resources particularly run-of-the river resources are considered to provide a clean and environmentally friendly energy option, while thermal resources particularly fossil fuel-based resources are considered to provide an environmentally aggressive energy option, but nevertheless still in nowadays a necessary option. Hence, promoting efficiency improvements in the exploitation of the hydro resources is increasingly important, reducing the reliance on fossil fuels and decreasing greenhouse emissions which are major contributors to climate change.

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The hydro scheduling problem is usually divided into different time horizons:

- Medium and long-term hydro scheduling, which encircle a time horizon of one or more years, discretised in weekly or monthly intervals. Stochastic models are used [1].
- Short-term hydro scheduling (STHS), which encircles a time horizon of one day to one week, usually discretised in hourly intervals. Deterministic models are used. Where stochastic quantities are included, such as hydro inflows or energy prices, the corresponding forecasts are used [2,3].

In a regulated environment, the main goal of the hydro scheduling problem is the minimisation of the deviation between total hydroelectric generation and electric energy demand, accomplishing the reservoir storage conditions at the beginning and at the end of the scheduling time horizon [4]. This problem could be a part of a traditional hydrothermal coordination problem, typically solved with methods based on decomposition approaches, determining the start-up and shut-down schedule of thermal plants, as well as the power output of thermal and hydro plants during the time horizon [5].

In a deregulated profit-based environment, such as the Norwegian case [6] or concerning Portugal and Spain given the forthcoming Iberian Electricity Market, the optimal management of the water available in the reservoirs for power generation, without affecting future operation use, represents a major advantage for generating companies (GENCOs) to face competitiveness given the economic stakes involved. The main goal in the profit-based hydro scheduling problem is to maximise the value of total hydroelectric generation throughout the time horizon, while satisfying all hydraulic constraints, aiming the most efficient and profitable use of the water [7]. Hence, the improvement of existing hydro scheduling models promoting a better exploitation efficiency of hydro resources is an important line of research [8], especially for head-dependent reservoirs in light of market conditions [9]. The efficiency characterizes the conversion of the potential energy contained in the water discharged through the turbines into the gross hydro energy output [10].

The hydro generation characteristics are mainly assumed as linear or piecewise linear in hydro scheduling models, neglecting head variations. For long-term time horizons, the linearity assumption is reasonable, since errors introduced by this assumption are expected to be small compared to uncertainties with respect, for instance, to hydro inflow [11]. For a particular configuration of the hydro system, the linearity assumption may be acceptable or not for short-term time horizons depending on how important is the head variation over the time horizon.

In hydro plants with a large storage capacity available, as it is the case in the Brazilian system for instance, head variation has negligible influence on power generation efficiency in the short-term [12], and the linearity assumption is acceptable.

In hydro plants with a small storage capacity available, also known as run-of-the-river hydro plants, the power generation efficiency can change significantly due to the head change effect. For instance, in the Portuguese system there are several hydro chains formed by many but small reservoirs. Hence, it is necessary to consider the head change effect on STHS in order to obtain more realistic and feasible results. The head change effect together with the cascaded hydro configuration, implying spatial-temporal coupling among reservoirs, increases the problem complexity.

Dynamic programming (DP) is among the earliest methods applied to the STHS problem [13–15]. Although, DP can handle the non-concavities and the nonlinear characteristics present in the hydro model, direct application of DP methods for hydro systems with many coupled plants is impractical due to the well-known DP curse of dimensionality, more difficult to avoid in short-term than in long-term optimisation without losing the accuracy needed in the model [16].

Artificial intelligence techniques have also been applied to the STHS problem, namely, neural networks [17,18] and genetic algorithms [19,20]. However, a significant computational effort is necessary to solve the problem for a time horizon of one week discretised in hourly intervals. Also, due to the heuristics used in the search process only sub-optimal solutions can be reached.

The network flow technique is especially effective for solving problems associated with the mathematical modelling of hydro resources [21], because of the underlying network structure subjacent in cascaded reservoirs [22–28]. A set of cascaded reservoirs, each one having just one downstream neighbour, can be represented by a tree. Their nodes represent the reservoirs and their arcs represent the water releases. The replication of this tree for each period results in a particular network. The arcs connecting the trees represent the water stored in the reservoirs [29]. For cascaded hydro systems, as there are water linkage and electric connections among plants, the advantages of the network flow technique are salient: for instance, it is more difficult for other approaches to consider the water travel delay between reservoirs in a river effectively, especially when the river is branched [30].

The network flow model is often simplified to a linear or piecewise linear one. Linear programming (LP) is a widely used method for STHS [31–33]. LP algorithms lead to extremely efficient codes,

implementations of which can be found commercially. Also, mixed-integer linear programming (MILP) is becoming frequently used for STHS [34–38], where binary variables allow modelling of start-up costs, which are mainly due to wear and tear of the windings and to malfunctions of the control equipment. However, LP algorithms imply that power generation is linearly dependent on water discharge, thus neglecting head dependence to avoid nonlinearities, leading to inaccuracy. Also, the discretisation of the nonlinear dependence between power generation, water discharge and head, used in MILP to model head variations, augment the computational burden required to solve this problem.

Hydro scheduling is in nature a nonlinear optimisation problem. A nonlinear programming (NLP) method is proposed in this paper to solve the STHS problem considering head dependence. This method expresses hydro generation characteristics more accurately and the head change effect can be taken into account. Although there were considerable computational difficulties in the past to directly use NLP methods to this sort of problem [39–41], with the drastic advancement in computing power and the development of more effective nonlinear solvers in recent years this disadvantage seems to be eliminated. The nonlinear dependence between the power generation, the water discharge and the head is taken into account in our study through a novel nonlinear formulation, which represents one of the main difficulties associated with the STHS problem. In our earlier formulation [42], the maximum water discharge in each plant was considered constant. As a new contribution, the maximum water discharge and thus the maximum power generation is also considered head-dependent in this formulation, in order to obtain more realistic and feasible results.

The Portuguese fossil fuels energy dependence is among the highest in the European Union. Portugal does not have endogenous thermal resources, which has a negative influence on Portuguese economy. Moreover, the Portuguese greenhouse emissions are already out of Kyoto target and must be reduced in the near future. Hence, promoting efficiency improvements in the exploitation of the Portuguese hydro resources reduces the reliance on fossil fuels and decreases greenhouse emissions. In this paper, we report our research concerning efficiency improvements applied on a case study based on one of the Portuguese cascaded hydro systems, thus providing a higher profit for the GENCO.

The paper is structured as follows. Section 2 provides the notation used throughout the paper along with the mathematical formulation of the STHS problem. Section 3 develops the proposed NLP method

for solving the STHS problem considering head dependence. Section 4 presents a case study, illustrating the numerical simulation results. Section 5 provides conclusions.

2. Problem formulation

Notation

K	total number of hours in the scheduling time horizon.
J	total number of hydro resources.
l_{kj}	water level in reservoir j during period k .
l_j^{\max}	maximum water level in reservoir j .
l_j^{\min}	minimum water level in reservoir j .
h_{kj}	head of plant j during period k .
h_j^{\max}	maximum head of plant j .
h_j^{\min}	minimum head of plant j .
v_{kj}	water storage of reservoir j at end of period k .
v_j^{\max}	maximum storage of reservoir j .
v_j^{\min}	minimum storage of reservoir j .
v_{0j}	initial water storage of reservoir j .
v_{Kj}	final water storage of reservoir j .
q_{kj}	water discharge of plant j during the period k .
q_{kj}^{\max}	maximum water discharge of plant j during the period k .
q_j^{\min}	minimum water discharge of plant j .
s_{kj}	water spillage by reservoir j during the period k .
a_{kj}	natural inflow to reservoir j during the period k .
p_{kj}	power generation of plant j during period k .
η_{kj}	efficiency of plant j during period k .
η_j^{\max}	maximum efficiency of plant j .
η_j^{\min}	minimum efficiency of plant j .
τ_{mj}	water travel delay between reservoirs m and j .
λ_k	forecasted energy price during period k .
Ψ_j	future value of the water stored in reservoir j .
M_j	set of upstream reservoirs to reservoir j .
F	nonlinear function of variables.
A	constraint matrix.

- \mathbf{b}^{\max} upper bound vector on constraints.
 \mathbf{b}^{\min} lower bound vector on constraints.
 \mathbf{x} vector of the flux variables corresponding to the arcs of the network.
 \mathbf{x}^{\max} upper bound vector on variables.
 \mathbf{x}^{\min} lower bound vector on variables.

The STHS problem is formulated as a NLP problem. The objective function to be maximised can be expressed as:

$$F = \sum_{k=1}^K \sum_{j=1}^J \lambda_k p_{kj} + \sum_{j=1}^J \Psi_j (v_{Kj}) \quad (1)$$

The objective function in (1) is composed of two terms. The first term represents the profit with the hydro system during the short-term time horizon, where λ_k is the forecasted energy price during period k and p_{kj} is the power generation of plant j during period k . The last term expresses the water value, Ψ_j , for the future use of the water stored in the reservoirs at the last period, v_{Kj} . A representation when this term is explicitly taken into account can be seen in [43,44]. The storage targets for the short-term time horizon, which are established by medium-term planning studies, may be represented either by a penalty on water storage or by a previously determined ‘future cost function’.

The optimal value of the objective function is determined subject to constraints: equality constraints and inequality constraints or simple bounds on the variables. The following equations represent the set of constraints for the plants over the short-term time horizon.

1) *Water Balance Equation:*

$$v_{kj} = v_{k-1,j} + a_{kj} + \sum_{m \in M_j} (q_{k-\tau_{mj},m} + s_{k-\tau_{mj},m}) - q_{kj} - s_{kj}; \quad \forall k \in K, \quad \forall j \in J \quad (2)$$

2) *Power Generation Equation:*

$$p_{kj} = q_{kj} \eta_{kj} (h_{kj}); \quad \forall k \in K, \quad \forall j \in J \quad (3)$$

3) *Head Equation:*

$$h_{kj} = l_{kf(j)}(v_{kf(j)}) - l_{kt(j)}(v_{kt(j)}); \quad \forall k \in K, \quad \forall j \in J \quad (4)$$

4) *Water Storage Constraints:*

$$v_j^{\min} \leq v_{kj} \leq v_j^{\max}, \quad \forall k \in K, \quad \forall j \in J \quad (5)$$

5) *Water Discharge Constraints:*

$$q_j^{\min} \leq q_{kj} \leq q_{kj}^{\max}(h_{kj}); \quad \forall k \in K, \quad \forall j \in J \quad (6)$$

6) *Water Spillage Constraints:*

$$s_{kj} \geq 0; \quad \forall k \in K, \quad \forall j \in J \quad (7)$$

Eq. (2) corresponds to the water conservation equation, where v_{kj} is the water storage of reservoir j at end of period k , a_{kj} is the natural inflow to reservoir j during the period k , q_{kj} is the water discharge of plant j during the period k , s_{kj} is the water spillage by reservoir j during the period k , τ_{mj} is the water travel delay between reservoirs m and j , K is the total number of hours in the scheduling time horizon, J is the total number of hydro resources and M_j is the set of upstream reservoirs to reservoir j . The travel time between reservoirs must be taken into account if the transportation delays are not negligible. In (3) power generation, p_{kj} , is considered a function of water discharge, q_{kj} , and of efficiency, η_{kj} , expressed as the output-input ratio, which in turn depends on the head, h_{kj} . In (4) the head, h_{kj} , is considered a function of the water level in the upstream reservoir $f(j)$, $l_{kf(j)}$, and of the water level in the downstream reservoir $t(j)$, $l_{kt(j)}$, both levels depending on the water storages in the respectively reservoirs. Typically for a powerhouse with a reaction turbine, where the tail water elevation is not constant, the head is modelled as in (4), and for a powerhouse with an impulse turbine, where the tail water elevation remains constant, the head depends only on the upstream reservoir water level. In (5) water storage has lower and upper bounds. Here, for each reservoir j , v_j^{\min} is the minimum storage capacity and v_j^{\max} is the maximum storage capacity. In (6) water discharge has lower and upper bounds. The minimum water discharge, q_j^{\min} , in our case study is considered null, but may be required to be non-zero due to navigation, recreational or ecological reasons. As a new contribution to earlier studies, the maximum water discharge, q_{kj}^{\max} , is considered a function of the head. Hence, the maximum water

discharge may be different for each period k , which represents a real feature that is required on our case study in order to achieve better exploitation efficiency. In (7) a null lower bound is considered for water spillage. Water spillage by the reservoirs can occur only in normal schedule situations when without it the water storage exceeds its upper bound, so spilling is necessary due to safety considerations. The initial water storages, v_{0j} , and the inflows to reservoirs, a_{kj} , are assumed as known input data. Also, discharge ramping constraints [36] should be included for a reservoir with a task of navigation to keep a less and stead head variation.

3. The proposed NLP method

In order to solve the STHS problem, it is essential to use appropriate models, considering power generation as a function of water discharge and also of the head for run-of-the-river hydro plants. This function is represented by the unit performance curves, a family of nonlinear curves each for a specified value of the head, as shown in Fig. 1.

"See Fig. 1 at the end of the manuscript".

The main contribution of this paper is that the maximum water discharge is considered a function of the head as shown in (6), implying that the maximum power generation is also head-dependent. This is indicated by the dashed line labelled q_{kj}^{\max} in Fig. 1.

The STHS problem can be formulated as the following nonlinear optimisation problem:

$$\text{Max } F(\mathbf{x}) \tag{8}$$

Subject to:

$$\mathbf{b}^{\min} \leq \mathbf{A} \mathbf{x} \leq \mathbf{b}^{\max} \tag{9}$$

$$\mathbf{x}^{\min} \leq \mathbf{x} \leq \mathbf{x}^{\max} \tag{10}$$

where \mathbf{x} is the vector of the flux variables corresponding to the arcs of the underlying network structure in hydro chains, consisting of the water storages, the water discharges, and the water spillages, $F(\cdot)$ is a nonlinear function of the vector of the flux variables, \mathbf{A} is the constraint matrix, \mathbf{b}^{\max} is the upper bound vector on the constraints, \mathbf{b}^{\min} is the lower bound vector on the constraints, \mathbf{x}^{\max} is the upper bound

vector on the variables and \mathbf{x}^{\min} is the lower bound vector on the variables. The water balance in (2) is rewritten as in (9), setting the lower bound equal to the upper bound. The bounds on water storage, water discharge and water spillage in (5), (6) and (7), respectively, are rewritten as in the inequality constraints in (10). Also, due to the maximum water discharge head dependency, the upper bound on water discharge implies a new inequality constraint that will be rewritten as in (9).

In (3) the efficiency depends on the head. We consider it given by:

$$\eta_{kj} = \eta_j^0 + \alpha_j h_{kj}; \quad \forall k \in K, \quad \forall j \in J \quad (11)$$

where the parameters η_j^0 and α_j are respectively the offset and the slope given by:

$$\eta_j^0 = \eta_j^{\max} - \alpha_j h_j^{\max}; \quad \forall j \in J \quad (12)$$

$$\alpha_j = (\eta_j^{\max} - \eta_j^{\min}) / (h_j^{\max} - h_j^{\min}); \quad \forall j \in J \quad (13)$$

In (13) parameter α_j depends on the extreme values for efficiency and head, where η_j^{\max} is the maximum efficiency, η_j^{\min} is the minimum efficiency, h_j^{\max} is the maximum head and h_j^{\min} is the minimum head.

In (4) the water level depends on the water storage. We assume it given by:

$$l_{kj} = l_j^0 + \beta_j v_{kj}; \quad \forall k \in K, \quad \forall j \in J \quad (14)$$

where the parameters l_j^0 and β_j are respectively the offset and the slope given by:

$$l_j^0 = l_j^{\max} - \beta_j v_j^{\max}; \quad \forall j \in J \quad (15)$$

$$\beta_j = (l_j^{\max} - l_j^{\min}) / (v_j^{\max} - v_j^{\min}); \quad \forall j \in J \quad (16)$$

this assumption implies reservoirs with vertical walls, which is a good approximation for run-of-the-river reservoirs, due to its small storage capacity, as our data have shown for the case study.

In (16) parameter β_j depends on the extreme values for water level and storage, where l_j^{\max} is the maximum water level, l_j^{\min} is the minimum water level, v_j^{\max} is the maximum storage and v_j^{\min} is the minimum storage.

Substituting (11) into (3) we have:

$$p_{kj} = q_{kj} (\eta_j^0 + \alpha_j h_{kj}); \quad \forall k \in K, \quad \forall j \in J \quad (17)$$

By substituting (4) and (14) into (17) power generation becomes a nonlinear function of water discharge and water storage, given by:

$$p_{kj} = \eta_j^0 q_{kj} + \alpha_j l_{f(j)}^0 q_{kj} - \alpha_j l_{t(j)}^0 q_{kj} + \alpha_j \beta_{f(j)} q_{kj} v_{kf(j)} - \alpha_j \beta_{t(j)} q_{kj} v_{kt(j)}; \\ \forall k \in K, \quad \forall j \in J \quad (18)$$

In our model, the maximum water discharge is considered head-dependent and it is given by:

$$q_{kj}^{\max} = q_j^0 + \delta_j h_{kj}; \quad \forall k \in K, \quad \forall j \in J \quad (19)$$

where the parameters q_j^0 and δ_j are respectively given by:

$$q_j^0 = q_j^1 - \delta_j h_j^{\max}; \quad \forall j \in J \quad (20)$$

$$\delta_j = (q_j^1 - q_j^{\max}) / (h_j^{\max} - h_j^{\min}); \quad \forall j \in J \quad (21)$$

In (21) parameter δ_j depends on the extreme values for head and the corresponding maximum water discharge values, where h_j^{\max} is the maximum head, h_j^{\min} is the minimum head, q_j^1 is the maximum water discharge achieved at h_j^{\max} and q_j^{\max} is the maximum water discharge achieved at h_j^{\min} . Since q_j^1 is inferior to q_j^{\max} , δ_j is never greater than zero.

Substituting (4) and (14) into (19) we have:

$$q_{kj}^{\max} = q_j^0 + \delta_j l_{f(j)}^0 - \delta_j l_{t(j)}^0 + \delta_j \beta_{f(j)} v_{kf(j)} - \delta_j \beta_{t(j)} v_{kt(j)}; \quad \forall k \in K, \quad \forall j \in J \quad (22)$$

The maximum water discharge becomes a function of water storage, given by:

$$q_{kj}^{\max} = \gamma_j^0 + \gamma_j^1 v_{kf(j)} - \gamma_j^2 v_{kt(j)}; \quad \forall k \in K, \quad \forall j \in J \quad (23)$$

where the parameters γ_j^0 , γ_j^1 and γ_j^2 are respectively given by:

$$\gamma_j^0 = q_j^0 + \delta_j l_{f(j)}^0 - \delta_j l_{t(j)}^0; \quad \forall j \in J \quad (24)$$

$$\gamma_j^1 = \delta_j \beta_{f(j)}; \quad \forall j \in J \quad (25)$$

$$\gamma_j^2 = \delta_j \beta_{t(j)}; \quad \forall j \in J \quad (26)$$

Hence, the new inequality constraint to be rewritten as in (9) is given by:

$$q_{kj} - \gamma_j^1 v_{kf(j)} + \gamma_j^2 v_{kt(j)} \leq \gamma_j^0; \quad \forall k \in K, \quad \forall j \in J \quad (27)$$

4. Case study

The proposed NLP method has been applied on a case study based on one of the Portuguese hydro systems, consisting of three head-sensitive cascaded reservoirs. The spatial coupling among reservoirs is shown in Fig. 2.

"See Fig. 2 at the end of the manuscript".

Only the first reservoir has inflow. This inflow is due to an upstream watershed belonging to a different company and is shown in Fig. 3.

"See Fig. 3 at the end of the manuscript".

Our model was implemented on a 600-MHz-based processor with 256 MB of RAM using the optimisation solver package Xpress-MP under MATLAB. The scheduling time horizon chosen is one week divided into 168 hourly periods.

The energy price profile over the time horizon is shown in Fig. 4 (where \$ is a symbolic economic quantity).

"See Fig. 4 at the end of the manuscript".

Energy prices are important input data to achieve a successful schedule based on profit maximisation. This data has uncertainty due to the deregulation of the electricity markets. Hence, an accurate forecast of energy prices has become a very important tool for a GENCO to develop an appropriate bidding strategy in the market and to optimally schedule its hydro resources. Several techniques have been tried out for energy prices forecasting, mainly based on time series and ARIMA models [45,46], or on artificial neural networks [47–49]. These energy prices are considered as deterministic input data for the STHS problem.

In our case study the final water storage in reservoirs is constrained to be equal to the value at the beginning of the scheduling time horizon. Hence, the future value of the water stored in reservoirs is not considered.

The computation time for solving a nonlinear program is highly dependent on the starting point. A considerable reduction in the overall solution time may be obtained if a good guessed starting point is obtained. We consider for the optimisation procedure a starting point given by the solution of an LP problem and using NLP in our case study we always arrive at convergence to a superior solution.

The computed 168-hours optimal reservoir storage and head trajectories are shown respectively in Figs. 5 and 6.

"See Fig. 5 at the end of the manuscript".

"See Fig. 6 at the end of the manuscript".

Considering the head change effect, the reservoirs should operate at an appropriated high storage level in order to achieve the most benefiting point of the overall efficiency for the conversion of potential energy of the water into electric energy. The storage trajectories of the first and second reservoirs are pulled up, opposing to the change in the storage trajectory of the third reservoir. Nevertheless, due to the constraint on final water storage, the storage trajectory of the third reservoir is pulled up near the final hours of the time horizon, implying a decrease on the storage trajectory of the second reservoir. This behaviour is in favour of the overall power generation efficiency thereby yielding an increase on total profit for the GENCO. The data for this case study satisfies the following relations between parameters: $\alpha_1 \beta_1 > \alpha_2 \beta_2 > \alpha_3 \beta_3$. Different watershed data giving different parameterisation and its effect on the behaviour of a head-sensitive hydro chain can be seen in our earlier work [42], with no model consideration for head-dependent maximum water discharge.

In Fig. 7 the computed 168-hours optimal plant discharge trajectories are shown.

"See Fig. 7 at the end of the manuscript".

The water discharge and consequently the hydro production tend to follow the shape of the price profile in Fig. 4, but due to the consideration of the head change effect some shape adaptation is imposed. The effect on maximum water discharge for the first plant implies that there is a slope shape at the most favourable price hours of each day, instead of the normal flat shape when the maximum water discharge was considered constant. This effect is less important due to the numerical data in the other plants. Hence, in Fig. 7 the normal flat shape is seen for the second and third plant.

A comparison of the power generation per water discharge between the LP method and the proposed NLP method for plant 1, 2 and 3 is shown respectively in Figs. 8, 9 and 10.

"See Figs. 8, 9 and 10 at the end of the manuscript".

This comparison is in favour of the proposed NLP method, achieving a higher total profit with an increase of 4.94% as shown in the Table 1. Moreover, when there is a need to consider the maximum water discharge as a function of the head, the proposed NLP method is more adequate than our earlier NLP method [42].

"See Table 1 at the end of the manuscript".

The computation time for this case study was about 0.88 s, showing that the proposed NLP method is not only more accurate but also computationally acceptable.

5. Conclusion

The new environment of competitive electricity markets for energy requires new computing tools to allow generating companies to achieve a better short-term hydro schedule, more realistic and feasible, improving on power generation efficiency which is crucial to face competitiveness. A generating company should not ignore the head change effect for head-sensitive cascaded reservoirs in order to improve power generation efficiency. This effect implies not only a nonlinear dependence between the power generation, the water discharge and the head, but also implies that the maximum water discharge giving the maximum power generation is a function of the head. This paper proposes a nonlinear programming method for head-sensitive cascaded reservoirs in order to consider the head change effect on hydroelectric power generation. As a new contribution to earlier studies, we report the consideration of a slope shape for water discharge at the most favourable price hours of each day, instead of the normal flat shape when the maximum water discharge was considered with no head change effect. The proposed method has been successfully tested on a case study based on one of the Portuguese cascaded hydro systems with head-sensitive reservoirs, providing a higher profit in comparison with classical optimisation methods based on linear programming that ignore head dependence. Numerical simulation results show that the proposed method is both accurate and computationally acceptable, providing a novel and better approach to optimise power generation efficiency for head-sensitive cascaded reservoirs.

References

- [1] Pereira MVF. Optimal stochastic operations scheduling of large hydroelectric systems. *Electr Power Energy Syst* 1989;11(3):161–9.
- [2] Ferreira LAFM, Andersson T, Imparato CF, Miller TE, Pang CK, Svoboda A, et al. Short-term resource scheduling in multi-area hydrothermal power systems. *Electr Power Energy Syst* 1989;11(3):200–12.
- [3] Mendes VMF, Ferreira LAFM, Roldão P, Pestana R. Optimal short-term scheduling in large hydrothermal power systems. In: *Proceedings of the 11th Power Systems Computation Conference, Avignon, France; 1993*, p. 1297–303.
- [4] García-González J, Parrilla E, Barquín J, Alonso J, Sáiz-Chicharro A, González A. Under-relaxed iterative procedure for feasible short-term scheduling of a hydro chain. In: *IEEE Power Tech Conference Proceedings, Bologna, Italy; 2003*.
- [5] Jiménez-Redondo N. On centralized power pool auction: a novel multipliers stabilization procedure. *Electr Power Energy Syst* 2005;27(2):83–90.
- [6] Fosso OB, Gjelsvik A, Haugstad A, Mo B, Wangensteen I. Generation scheduling in a deregulated system. The Norwegian case. *IEEE Trans Power Syst* 1999;14(1):75–80.
- [7] Catalão JPS, Mariano SJPS, Mendes VMF, Ferreira LAFM. Scheduling of head-sensitive cascaded hydro systems: a comparison based on numerical simulation results. In: *Proceedings of the Fourth IASTED International Conference on Power and Energy Systems, Rhodes, Greece; 2004*, p. 418–423.
- [8] Ponrajah RA, Witherspoon J, Galiana FD. Systems to optimise conversion efficiencies at Ontario Hydro's hydroelectric plants. *IEEE Trans Power Syst* 1998;13(3):1044–50.
- [9] Mariano S, Catalão J, Mendes V, Ferreira L. Power generation efficiency improvement in cascaded and head-dependent reservoirs. In: *Proceedings of the 15th Power Systems Computation Conference, Liège, Belgium; 2005*.
- [10] Cohen AI, Sherkat VR. Optimization-based methods for operations scheduling. *Proc IEEE* 1987;75(12):1574–91.
- [11] Nordlund P, Sjelvgren D, Pereira MVF, Bubenko JA. Generation expansion planning for systems with a high share of hydro power. *IEEE Trans Power Syst* 1987;PWR-2(1):161–7.
- [12] Soares S, Ohishi T. Hydro-dominated short-term hydrothermal scheduling via a hybrid simulation-optimisation approach: a case study. *IEE Proc-Gener Transm Distrib* 1995;142(6):569–75.
- [13] Amado SM, Ribeiro CC. Short-term generation scheduling of hydraulic multi-reservoir multi-area interconnected systems. *IEEE Trans Power Syst* 1987;PWR-2(3):758–63.
- [14] Arce A, Ohishi T, Soares S. Optimal dispatch of generating units of the Itaipú hydroelectric plant. *IEEE Trans Power Syst* 2002;17(1):154–8.
- [15] Lyra C, Ferreira LRM. A multiobjective approach to the short-term scheduling of a hydroelectric power system. *IEEE Trans Power Syst* 1995;10(4):1750–5.
- [16] Pursimo JM, Antila HK, Vilkkö MK, Lautala PA. A short-term scheduling for a hydropower plant chain. *Electr Power Energy Syst* 1998;20(8):525–32.
- [17] Liang R-H, Hsu Y-Y. Short-term hydro-scheduling using Hopfield neural network. *IEE Proc-Gener Transm Distrib* 1996;143(3):269–75.
- [18] Naresh R, Sharma J. Short term hydro scheduling using two-phase neural network. *Electr Power Energy Syst* 2002;24(7):583–90.
- [19] Cau TDH, Kaye RJ. Evolutionary optimisation method for multistorage hydrothermal scheduling. *IEE Proc-Gener Transm Distrib* 2002;149(2):152–6.
- [20] Chen P-H, Chang H-C. Genetic aided scheduling of hydraulically coupled plants in hydro-thermal coordination. *IEEE Trans Power Syst* 1996;11(2):975–81.
- [21] Sjelvgren D, Brannlund H, Dillon TS. Large-scale non-linear programming applied to operations planning. *Electr Power Energy Syst* 1989;11(3):213–7.
- [22] Feltenmark S, Lindberg PO. Network methods for head-dependent hydro power scheduling. *Lect Notes Econ Math Syst* 1997;450:249–64.
- [23] Franco PEC, Carvalho MF, Soares S. A network flow model for short-term hydro-dominated hydrothermal scheduling problems. *IEEE Trans Power Syst* 1994;9(2):1016–22.
- [24] Johannesen A, Gjelsvik A, Fosso OB, Flatabo N. Optimal short term hydro scheduling including security constraints. *IEEE Trans Power Syst* 1991;6(2):576–83.
- [25] Oliveira ARL, Soares S, Nepomuceno L. Short term hydroelectric scheduling combining network flow and interior point approaches. *Electr Power Energy Syst* 2005;27(2):91–9.

- [26] Rakic MV, Markovic ZM. Hydraulically coupled power-plants commitment within short-term operation planning in mixed hydro-thermal power systems. *Eur Trans Electr Power* 1997;7(5):323–30.
- [27] Sjelvgren D, Andersson S, Andersson T, Nyberg U, Dillon TS. Optimal operations planning in a large hydro-thermal power system. *IEEE Trans Power App Syst* 1983;PAS-102(11):3644–51.
- [28] Wakamori F, Masui S, Morita K, Sugiyama T. Layered network model approach to optimal daily hydro scheduling. *IEEE Trans Power App Syst* 1982;PAS-101(9):3310–4.
- [29] Oliveira GG, Soares S. A second-order network flow algorithm for hydrothermal scheduling. *IEEE Trans Power Syst* 1995;10(3):1635–41.
- [30] Wang C, Shahidehpour SM. Power generation scheduling for multi-area hydro-thermal systems with tie line constraints, cascaded reservoirs and uncertain data. *IEEE Trans Power Syst* 1993;8(3):1333–40.
- [31] Hreinsson EB. Optimal short term operation of a purely hydroelectric system. *IEEE Trans Power Syst* 1988;3(3):1072–7.
- [32] Piekutowski MR, Litwinowicz T, Frowd RJ. Optimal short-term scheduling for a large-scale cascaded hydro system. *IEEE Trans Power Syst* 1994;9(2):805–11.
- [33] Wood AJ, Wollenberg BF. *Power generation, operation and control*. 2nd ed. New York: Wiley; 1996.
- [34] Chang GW, Aganagic M, Waight JG, Medina J, Burton T, Reeves S, et al. Experiences with mixed integer linear programming based approaches on short-term hydro scheduling. *IEEE Trans Power Syst* 2001;16(4):743–9.
- [35] Conejo AJ, Arroyo JM, Contreras J, Villamor FA. Self-scheduling of a hydro producer in a pool-based electricity market. *IEEE Trans Power Syst* 2002;17(4):1265–72.
- [36] Guan X, Svoboda A, Li C-A. Scheduling hydro power systems with restricted operating zones and discharge ramping constraints. *IEEE Trans Power Syst* 1999;14(1):126–31.
- [37] Nilsson O, Sjelvgren D. Mixed-integer programming applied to short-term planning of a hydro-thermal system. *IEEE Trans Power Syst* 1996;11(1):281–6.
- [38] Parrilla E, García-González J. Improving the B&B search for large-scale hydrothermal weekly scheduling problems. *Electr Power Energy Syst* 2006;28(5):339–48.
- [39] Brannlund H, Bubenko JA, Sjelvgren D, Andersson N. Optimal short term operation planning of a large hydrothermal power system based on a nonlinear network flow concept. *IEEE Trans Power Syst* 1986;PWRS-1(4):75–82.
- [40] Ikura Y, Gross G. Efficient large-scale hydro system scheduling with forced spill conditions. *IEEE Trans Power App Syst* 1984;PAS-103(12):3502–12.
- [41] Ni E, Guan X, Li R. Scheduling hydrothermal power systems with cascaded and head-dependent reservoirs. *IEEE Trans Power Syst* 1999;14(3):1127–32.
- [42] Catalão JPS, Mariano SJPS, Mendes VMF, Ferreira LAFM. Parameterisation effect on the behaviour of a head-dependent hydro chain using a nonlinear model. *Electr Power Syst Res* 2006;76(6-7):404–12.
- [43] Gil E, Bustos J, Rudnick H. Short-term hydrothermal generation scheduling model using a genetic algorithm. *IEEE Trans Power Syst* 2003;18(4):1256–64.
- [44] Uturbey W, Simões Costa A. Dynamic optimal power flow approach to account for consumer response in short term hydrothermal coordination studies. *IET Gener Transm Distrib* 2007;1(3):414–21.
- [45] Contreras J, Espínola R, Nogales FJ, Conejo AJ. ARIMA models to predict next-day electricity prices. *IEEE Trans Power Syst* 2003;18(3):1014–20.
- [46] Nogales FJ, Contreras J, Conejo AJ, Espínola R. Forecasting next-day electricity prices by time series models. *IEEE Trans Power Syst* 2002;17(2):342–8.
- [47] Catalão JPS, Mariano SJPS, Mendes VMF, Ferreira LAFM. Short-term electricity prices forecasting in a competitive market: a neural network approach. *Electr Power Syst Res* 2007;77(10):1297–304.
- [48] Szkuta BR, Sanabria LA, Dillon TS. Electricity price short-term forecasting using artificial neural networks. *IEEE Trans Power Syst* 1999;14(3):851–7.
- [49] Yamin HY, Shahidehpour SM, Li Z. Adaptive short-term electricity price forecasting using artificial neural networks in the restructured power markets. *Electr Power Energy Syst* 2004;26(8):571–81.

Figure captions

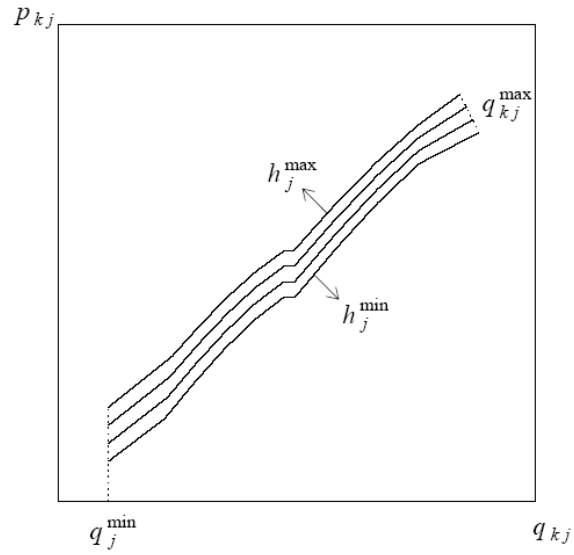


Fig. 1. Unit performance curves.

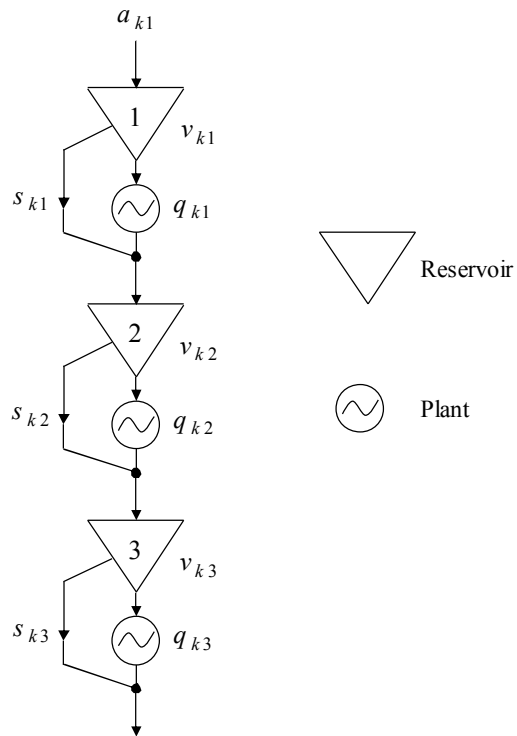


Fig. 2. Hydro system with three cascaded reservoirs.

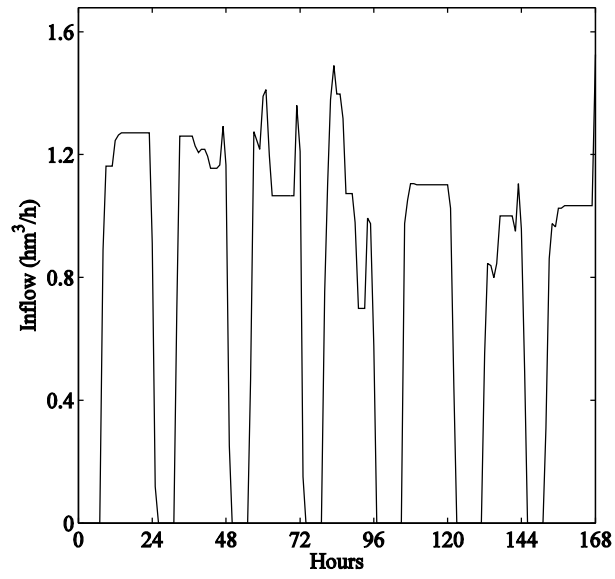


Fig. 3. Inflow on the first reservoir.

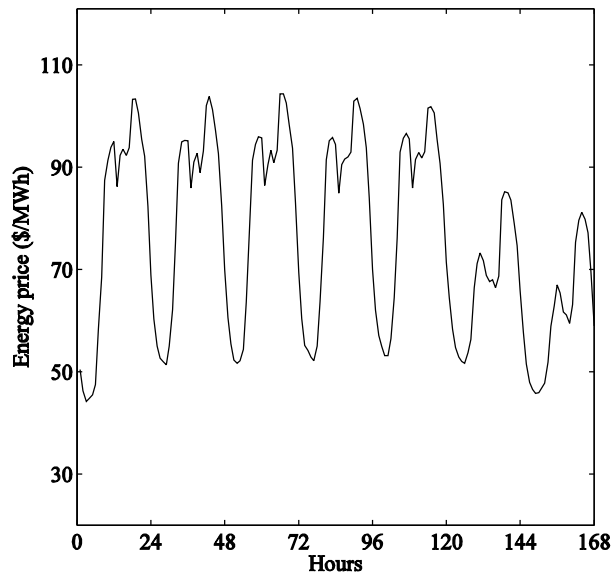


Fig. 4. Energy price profile considered.

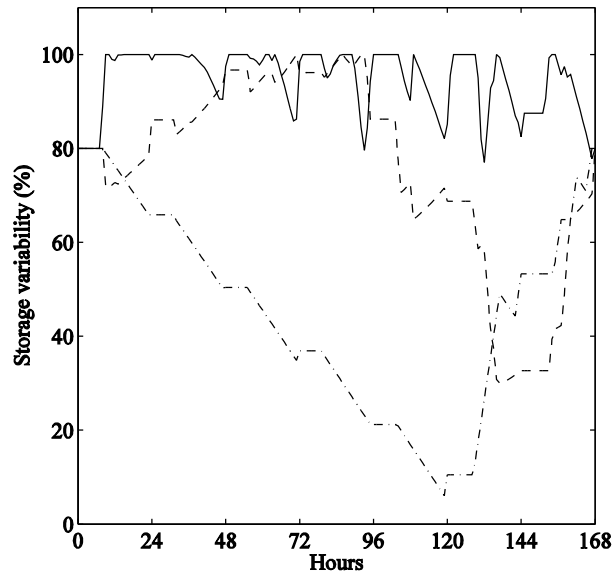


Fig. 5. Optimal reservoir storage trajectories over minimum storage. The solid line denotes reservoir 1 results, the dashed line denotes reservoir 2 results and the dash-dot line denotes reservoir 3 results.

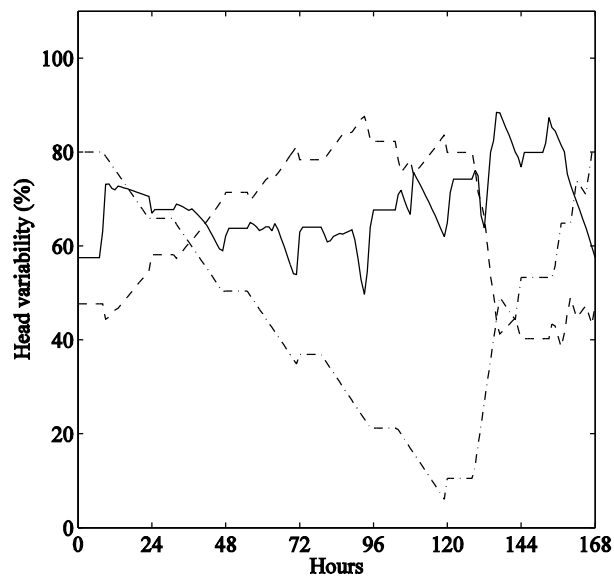


Fig. 6. Optimal plant head trajectories over minimum head. The solid line denotes plant 1 results, the dashed line denotes plant 2 results and the dash-dot line denotes plant 3 results.

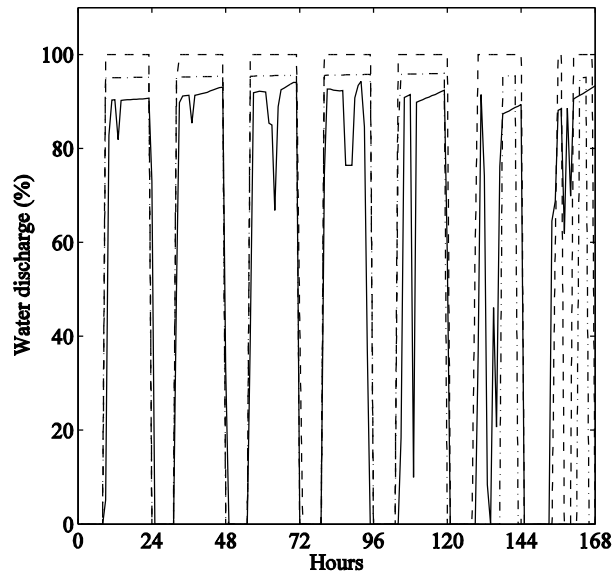


Fig. 7. Optimal plant discharge trajectories. The solid line denotes plant 1 results, the dashed line denotes plant 2 results and the dash-dot line denotes plant 3 results.

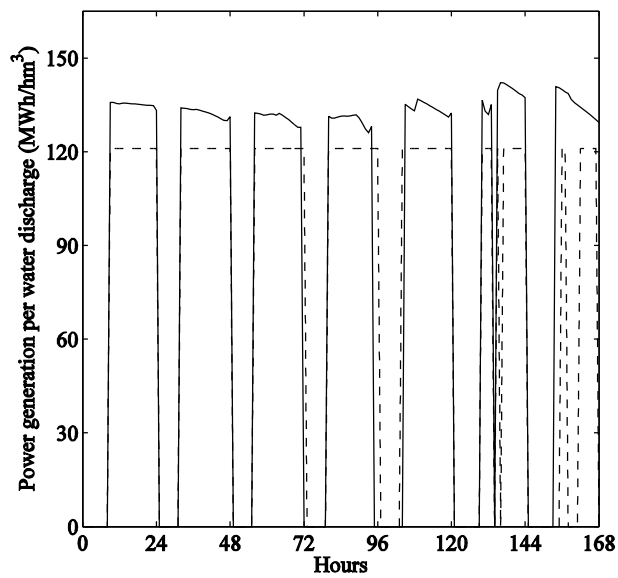


Fig. 8. Power generation per water discharge at plant 1 — LP method (dashed line) versus proposed NLP method (solid line).

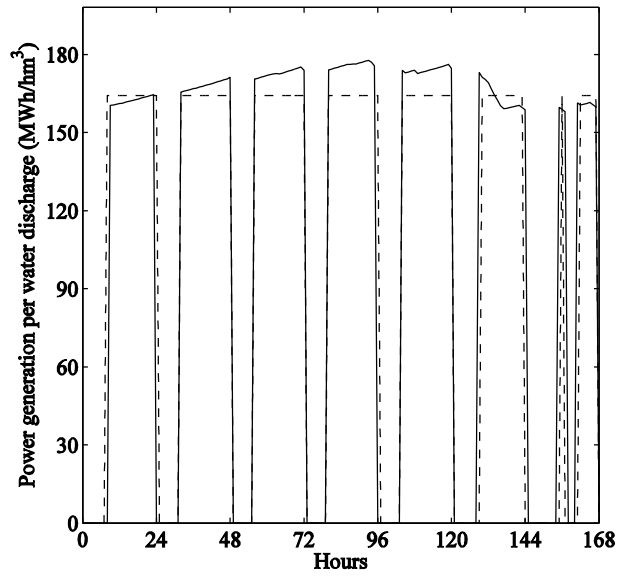


Fig. 9. Power generation per water discharge at plant 2 — LP method (dashed line) versus proposed NLP method (solid line).

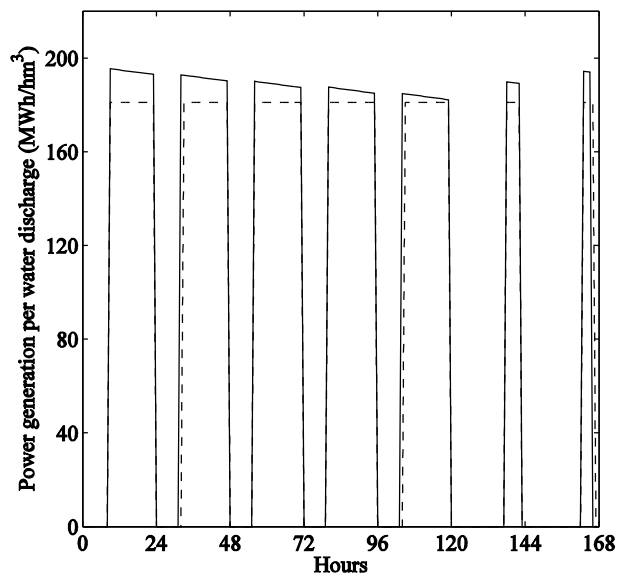


Fig. 10. Power generation per water discharge at plant 3 — LP method (dashed line) versus proposed NLP method (solid line).

Tables

Table 1

Comparison of LP with the NLP method

	Profit (\$)	% Increase	CPU time (s)
LP method	5,259,872	-	0.21
Proposed NLP method	5,519,738	4.94	0.88