Multiobjective Ray Optimization Algorithm as a Solution Strategy for Solving Non-Convex Problems: a Power Generation Scheduling Case Study

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Abstract

Economic generation scheduling (EGS) is a non-convex optimization problem for allocating optimal generation among the committed units that can meet given real-world practical limits such as ramp rate limits, prohibited operating zones, valve loading effects, multi-fuel options, spinning reserve and transmission system losses at the minimum fuel cost. Moreover, considering environmental issues results in an environmental/economic generation scheduling (EEGS) problem that is a multiobjective optimization model with two non-commensurable and contradictory objectives. In this paper, a novel method has been presented in order to minimize production cost and emission of the steam power plants in short term periods. The obtained results showed that the proposed method can be used in short-term decision making of steam power plants which will be absolutely effective in long-term emission target oriented strategies. A framework is proposed for solving single objective EGS and multiobjective EEGS problems considering the aforementioned constraints. The problem is solved by a new meta-heuristic optimization called Ray Optimization (RO) to determine the optimal power generation. The performance of the proposed algorithm is investigated by applying it to solve diverse test systems having non-convex solution spaces. Numerical results have been comprehensively compared with some of the most recently published research works in the area in order to validate the results and confirm the potential of the proposed approach. The obtained results show the application of the proposed framework and effectiveness of the solutions.

Keywords: Optimal generation scheduling; fuel cost; emission; multiobjective optimization; ray optimization.

Nomenclature

a_p, b_p, c_p, e_p, f_p	coefficients of *p*th generator's fuel cost function	P_{p0}	power of *p*th generator in the previous hour
B_{gp}, B_{0p}, B_{00}	transmission power loss factors	$P_{p,min}$, $P_{p,max}$	lower and upper limits of the generators output power
b	unit vector that's direction is perpendicular to the line that connects the origin to the current position	PF_1, PF_2	penalty factors of POZs and spinning reserve constraints violations
C_p	fuel cost of *p*th generator ($/h)	P_{Load}	total load power
CV_1,CV_2	amount of POZs and spinning reserve constraints violations	P_{Loss}	transmission loss power
DR_p, UR_p	ramp down and up limits of the *p*th generator	P_{p,LB_Z}, P_{p,UB_Z}	lower and upper limits of the *z*th prohibited zone
d	unit vector that shows direction of current movement vector	*stoch*	a number between 0 and 1 showing probability of stochastic nature of algorithm
E_p	NO_x emission amount of the *p*th generator	S_p	spinning reserve of *p*th generator
F^{Fuel}, $F^{Emission}$	total fuel cost and total emission amount of the generators	SR	system required spinning reserve
		$S_{p,max}$	maximum spinning reserve contribution of *p*th generator
F_t	total fitness function with considering penalty values	s, a	Parameters for finding the next movement length
F_δ^c	value of *c*th objective function in the δth Pareto-optimal solution	T	total multiobjective fitness function value
GB	position of the so far best agent	ToL	maximum tolerance for transmission loss convergence in power balance equation
LB_i	best position of the *i*th agent	t	unit vector that shows direction of next movement vector
l	stands for 2D&3D group number	u	stands for the fuel option
m	number of variables	V_{ij}	*j*th variable of the *i*th agent's movement vector
maxit	maximum number of iterations	$V_{i,l}$	*i*th agent's movement vector that belongs to *l*th group
mp	number of Pareto-optimal solutions	w	weighting factor
N_G	number of the generating units	w_c	weight factor of the *c*th objective function
N_p^{POZ}	number of prohibited zones of *p*th generating unit	X_{ij}	*j*th variable of the *i*th agent's position
Npop	population size	$X_{l,\max}$, $X_{l,\min}$	maximum and minimum limits of variables that belongs to *l*th component of the movement vector
n_d, n_t	refraction index of the lighter and darker materials	α_p, β_p, γ_p, ζ_p, λ_p	coefficients of *p*th generator emission function
n	unit vector that shows direction of the line that connects the	ψ	set of generators which have POZs
		Ω_p	scaling factor

	origin to the current position		
O_i^k	ith agent's origin point's position in kth iteration	θ, φ	radiation and refraction angles
P_p	output power of the pth generator	μ_δ^c	membership function of cth objective function in the δth Pareto-optimal solution
P_{GS}	power of the generator connected to the slack bus	ξ, λ	unit vectors which are perpendicular to each other in the new coordinate system

1. Introduction

Today's consciousness is to produce electricity not only at the least possible cost, but also at the minimum level of gaseous pollution [1]. The main reason for this perception is the increased awareness about environmental protection and the passage of the clean air act amendments of 1990. Various kinds of solutions have been proposed for reducing gaseous emissions. For instance, substitution of pollutant thermal units with cleaner and more efficient ones, installation of emission control devices, modification of boiler burners, switching to low emission fuels and emission dispatch [2]. The first three options are long term solutions require significant investment cost. Although switching to low emission fuels may reduce emissions, but however because of the price and availability of low emission fuels it is not an attractive strategy [3]. To cope with this problem, economic and emission based generation scheduling (EEGS) is becoming more and more desirable [3]. This model satisfies emission quotas without any needs to fuel switching.

Because of the non-convex characteristics of cost function, emission function and physical constraints of generating units, the EEGS problem is a nonlinear, non-convex and non-differentiable optimization problem.

As indicated in [4] and [5], due to the physical limitations such as vibrations in a shaft bearing, a thermal or hydro generating unit cannot operate in the certain operating regions which are called prohibited operating zones (POZs). In addition, the ramp rate limits are the other limitation forcing the units to operate between two sequential hours [6]. Besides, in the multivalve steam turbine based generating units, the input-output curve is a non-convex curve [7, 8]. Actually, the valve-point effect present ripples in the cost function curve. Also, some generation units, especially those units supplied with the mixture of fuel sources, are dealt with the problem of determining which the most economical fuel to burn [9]. Moreover, power system is always facing unexpected events such as forced outage of generating units and load forecasting errors. Therefore, spinning reserve must be considered for generating units to keep power system ability to respond to these events [5]. In this regard, when the above-mentioned practical concerns of EGS and EEGS problems are considered, a complex nonlinear and non-convex problem is formed that need to be solved.

1.2. Literature review

Nonlinear equation systems play a vital role in science and engineering. Metaheuristic optimization methods are techniques designed for solving nonlinear equations more quickly when classic methods are too slow, or for finding an approximate solution when classic methods fail to find any exact solution. Some of this systems typically have more than one root [10], some's solution is a subset of data like feature selection problem (exploring the data in order to eliminate irrelevant, noisy and redundant ones from them) [11] and some are handling more than one object, simultaneously. There has been found ways based on heuristic methods to solve any of this problems like Fuzzy Neighbourhood-based Differential Evolution with Orientation (FNODE) [10], to effectively and reliably find the multiple roots of Nonlinear equation systems simultaneously, ϵ-constrained method [12] for multiobjective problems.

Over the last years, a wide range of mathematical and heuristic approaches has been presented in the literature to solve EGS and EEGS problems. In [4], these methods classified into two main groups: classical methods such as integer programming [13], quadratic programming [14], direct search method [15], dynamic programming [16], and meta-heuristic methods such as genetic algorithm [7, 17], particle swarm optimization [6], Hopfield neural network [9], differential evolution [18], Tabu search algorithm [19], evolutionary programming [20], Fuzzy adaptive chaotic ant swarm optimization [21], Fuzzy Adaptive Modified Particle Swarm Optimization [22], self-organizing migrating algorithm [23], combination of modified subgradient and harmony search [24], and cuckoo search algorithm [25, 26]. The main disadvantage of classical methods is that because of non-smooth and non-convex features of the EGS problem, they cannot guarantee to reach the global or near global solution. Moreover, computational burden and execution time for these methods is also considerable. On the contrary, meta-heuristic methods can consider real-world limitations of the EGS problem and can solve optimization problems with any kind of complexity in a reasonable time. The success of the meta-heuristic methods is partly due to their natural capability of processing a population of possible solutions, which lets them carry out an expanding exploration in the search space of the optimization problem. Therefore, there have been great interests in these methods. Also some Hybrid methods such as hybrid differential evolution and dynamic programming [27], hybrid PSO and sequential quadratic programming [28], fuzzy adaptive PSO [29] and hybrid Hopfield neural network and quadratic programming [30] have been reported in the literature to solve EGS problem.

Most of the practical engineering optimization problems are multiobjective. For example, an airplane design problem might require maximizing fuel efficiency and payload, while minimizing the weight of the structure [31]. In energy production systems, designing the wind turbine blade geometry in order to maximize the energy production of wind turbines and minimize the mass of the blade itself [32]. Multi Objective optimization algorithms like Free Search approach combined with Differential Evolution (MOFSDE) [33], NSGA II, MOPSO, and Predator-Prey [34] has been employed to economical design, considering maximum efficiency of heat transfer in heat exchangers which are commonly used in steam power plants, cooling systems, heating systems, air conditioning and petrochemical industries. With the increasing demand of multiobjective optimization in engineering problems, researches of multiobjective optimization algorithms are necessary and valuable. In the power system engineering, also economic generation scheduling for thermal power plants with minimum emission (EEGS) [35, 36], EEGS considering market clearing of electrical energy in the presence of wind turbines [37], Congestion Management Using Generation Rescheduling and Load Shedding [38], are other multiobjective optimization problems that EEGS has been the target of present work.

The EEGS problem has been studied in some research works reported in the literature. Especially, with the rapid development of multiobjective evolutionary algorithms, application of these methods has received much more attention. Tabu search [35], Niched Pareto genetic algorithm (NPGA) [39], nondominating sorting genetic algorithm (NSGA) [40], NSGA-II [41], multiobjective evolutionary algorithm (MOEA) [42], multiobjective particle swarm optimization [43], multiobjective stochastic search technique (MOSST) [44], strength Pareto evolutionary algorithm (SPEA) [45], and fuzzy clustering-based particle swarm optimization (FCPSO) [36] represent the recent EEGS problem.

In this paper, the ray optimization as a new meta-heuristic method based on Snell's refraction law is proposed for solving the EGS and EEGS considering the practical constraints. Moreover, the above solution method for multiobjective EESG is equipped with a fuzzy decision making tool to consider the imprecise nature of the decision-maker's judgment. So, the proposed fuzzy decision making tool offers a better judgment among the Pareto-optimal solutions to select the best compromise one. The technique has been applied to the four standard systems having non-convex solution spaces. The obtained results (for different cases with different complexity) were widely compared with previously presented techniques to demonstrate the potential of the presented approach to handle the problem.

In brief the main contributions and aspects of this work with respect to the published papers in the area are as follows:

- Ray optimization as a new meta-heuristic method is formulated as a solution strategy for solving non-convex problems in power and energy systems;
- A multi-objective generation scheduling problem is formulated and solved;
- Using a fuzzy decision making tool the imprecise nature of the decision-maker's judgment is considered;
- To analyze the applicability of the model, four different test systems having complex and non-convex solution spaces are comprehensively studied.

The organization of the paper is as follows. In Section 2, the formulation of EGS and EEGS problems is presented and discussed in detail. The concepts of Ray algorithm as well as its procedure are presented in Section 3. Section 4 describes the constraint handling strategy of the EGS and EEGS problems and the implementation of RO approach for solving these problems. Obtained numerical results and the related comparisons with those from previously published literature, are given in Section 5. Finally, conclusion of the paper is provided in Section 6.

2. Problem Formulation

2.1. EGS problem

The EGS problem is an optimization problem to minimize operation costs while considering some technical constraints. In electric power systems, fuel cost of generation units is the main part of operation cost. The other costs are relatively small values can be added to the mentioned fuel cost. Typically, cost function of the generators is modeled by a quadratic function declared as follows:

$$C_p = a_p P_p^2 + b_p P_p + c_p \tag{1}$$

Therefore, the power system operator minimizes the summation of all generating units cost function:

$$F^{Fuel} = \sum_{p=1}^{N_G} C_p = \sum_{p=1}^{N_G} (a_p P_p^2 + b_p P_p + c_p) \tag{2}$$

The presence of valve loading effects into the fuel cost function leads to more practical and precise modeling. However, the inclusion of valve loading effects increases the non-linearity and as a result the solution approach may trap in the local optima. The total fuel cost function with valve loading effects is modeled as follows:

$$F^{Fuel} = \sum_{p=1}^{N_G} (a_p + b_p P_p + c_p P_p^2 + |d_p \sin\{e_p (P_{p,\min} - P_p)\}|) \tag{3}$$

In the real-world, units may be supplied with multiple fuel types. In these cases, unlike the conventional units, cost function should be modeled with some piecewise functions showing the effects of fuel type changes. The total fuel cost function considering the valve-point loadings and multiple fuel options is as follows:

$$F^{Fuel} = \sum_{p=1}^{N_G} \{a_{pu} + b_{pu} P_p + c_{pu} P_{pu}^2 + |d_{pu} \sin\{e_{pu} (P_{p,\min} - P_p)\}|) \quad \text{if} \quad P_{pu}^{\min} \le P_p \le P_{pu}^{\max} \tag{4}$$

where u stands for the fuel option.

In the EGS problem, there are two types of constraints which are considered, i.e. equality and inequality constraints. The equality constraint is the power balance constraint which means the total scheduled generation of the system must be equal to the total demand and active power losses of the transmission lines. The inequality constraints are minimum and maximum limits, prohibited operating zones, ramp-rate limit and spinning reserve of each generator. These constraints are concluded below.

1) Power balance

The total scheduled generation of the system must cover the total demand P_D and the active power losses in transmission lines P_{Loss}. Hence,

$$\sum_{g=1}^{N_G} P_g - P_{Load} - P_{Loss} = 0 \tag{5}$$

The active power losses P_{Loss} in Eq.(5) is calculated by Kron's loss formula:

$$P_{Loss} = \sum_{g=1}^{N_G}\sum_{p=1}^{N_G} P_g B_{gp} P_p + \sum_{p=1}^{N_G} B_{0p} P_p + B_{00} \tag{6}$$

where B_{gp}, B_{0p} and $B_{00}$are the loss coefficients [46].

2) Generation limits

$$P_{p,\min} \le P_p \le P_{p,\max} \tag{7}$$

3) POZs

Thermal units may have the certain operating regions called POZs which cannot operate in and indeed causes a discontinuity in input-output performance curve. POZs of a thermal generating unit are defined as follows:

$$P_p \in \begin{cases} P_{p,\min} \le P_p \le P_{p,LB_1} \\ \quad\vdots \\ P_{p,UB_{Z-1}} \le P_p \le P_{p,LB_Z} \\ P_{p,UB_Z} \le P_p \le P_{p,\max} \end{cases}, \; z = 2,3,\ldots,N_{POZ_p} \tag{8}$$

4) Ramp-rate limits

Consideration of ramp-rate limits affects the generation limits as follows:

$$\max(P_{p,\min}, P_{p0} - DR_p) \le P_p \le \min(P_{p,\max}, P_{p0} + UR_p) \tag{9}$$

5) Spinning reserve constraints

The spinning reserve constraints included as follows:

$$\sum_{p=1}^{N_G} S_p \ge SR \tag{10}$$

$$S_p = \begin{cases} \min[(P_{p,\min} - P_p), S_{p,\max}] & p \notin \psi \\ 0 & p \in \psi \end{cases} \tag{11}$$

As contribution of generating units with POZs may result in falling into certain prohibited zones, therefore, these units cannot contribute to the spinning reserve provision.

2.2. EEGS problem

As indicated in the previous section, in real-world power generation scheduling, absolute minimum cost is not any more the sole criterion to be met. Nowadays, significant emission control targets stated by different countries around the world, make environmental considerations as one of the major management concerns [47]. The EEGS is an optimization problem in which considering the environmental and economic issues simultaneously while satisfying various physical and operational constraints does generation scheduling.

The power industry, the biggest emitter of gaseous emissions among all industries, has got to take the largest responsibility for emission reduction. There are various pollutants produced in steam power plants like CO_X, NO_X, SO_X and methane gases, but NO_X and SO_X have the largest share among these pollutant gaseous. Therefore, in most of the studies in this area, these two gases have been considered and inserted to the EEGS models. Among the various pollutants emitted by power plants, SO_x and NO_x are the most important gaseous emissions considered due to their effects on the environment. A commonly used approach for modeling these emissions is to use a combination of the polynomial and exponential terms that associate emissions with power production for each generating unit [48]:

$$E_p = \alpha_p + \beta_p P_p + \gamma_p P_p^2 + \zeta_p \exp(\lambda_p P_p) \tag{12}$$

In this paper, the gaseous emissions emitted by thermal units are modeled as an environmental cost and added to the generation cost:

$$F^{Emission} = \sum_{p=1}^{N_G} \Omega_p E_p = \sum_{p=1}^{N_G} \Omega_p (\alpha_p + \beta_p P_p + \gamma_p P_p^2 + \zeta_p \exp(\lambda_p P_p)) \tag{13}$$

where Ω is the scaling factor and determined according to (14). This factor is used to coordinate the objective functions in optimization problem.

$$\Omega_p = \frac{C(P_{p,\max})}{E(P_{p,\max})} \tag{14}$$

Here, to model the relative preference of the objective functions, an appropriate weight value is used for each objective function. The EEGS objective function therefore becomes:

$$\text{Minimize } T = w \times F^{Fuel} + (1-w) \times F^{Emission} \tag{15}$$

where F^{Fuel} is the fuel cost function and $F^{Emission}$ is the emission function and w is the weight factor for showing the importance of two objective functions with respect to each other.

The constraints of the EEGS problem are the same as EGS problem constraints.

In order to obtain the Pareto-optimal front, starting from zero to one increases the value of *w*. Fuzzy set is applied in this part to choose the best compromise solution among the obtained Pareto-optimal solution according to the preferences of power systems operator [48]. In this regard, a linear membership function (μ^c) is defined for each of the objective functions, i.e. F^{Fuel} and $F^{Emission}$:

$$\mu_{\delta}^{c}_{c\in\{Fuel,Emission\}} = \begin{cases} 1 & F_{\delta}^{c} \le F_{\min}^{c} \\ \left[\dfrac{F_{\max}^{c} - F_{\delta}^{c}}{F_{\max}^{c} - F_{\min}^{c}}\right] & F_{\min}^{c} \le F_{\delta}^{c} \le F_{\max}^{c} \\ 0 & F_{\delta}^{c} \ge F_{\max}^{c} \end{cases} \quad (16)$$

where F_{δ}^{c} and μ_{δ}^{c} stand for the value and the membership function of the *c*th objective function in the *δ*th Pareto-optimal solution, respectively. Also, by assuming minimization for all the objective functions, $F_{\min}^{c}$ and $F_{\max}^{c}$ are the best (completely satisfactory) and the worst (clearly unsatisfactory) values of the objective function, respectively. For every non-dominated solution δ, the membership function can be normalized as follows:

$$\mu_{\delta} = \frac{\sum_{c} w_{c}\mu_{\delta}^{c}}{\sum_{\delta=1}^{mp}\sum_{c} w_{c}\mu_{\delta}^{c}} \quad (17)$$

where w_c is the weight factor of the *c*th objective function in the EEGS problem and *mp* is the number of obtained Pareto-optimal solutions. The decision maker may select the weight values w_c according to the importance of economic and environmental issues. The solution with the maximum membership function μ_{δ} is selected as the best Pareto-optimal solution or the final solution of the EEGS problem.

3. Ray optimization theory

a. Background

Ray optimization (RO) [49] is a method that uses Snell's refraction law of light for finding the global solution. According to this law, as the light travels through transparent materials so-called dielectric, its path is changed by the materials refraction index. If the index of the refraction of the lighter and darker materials denoted by n_d and n_t, respectively, the Snell's law can be expressed as:

$$n_d.\sin(\theta) = n_t.\sin(\phi). \tag{18}$$

where as shown in Fig.1, θ and ϕ are the radiation and refraction angles, respectively. Therefore, by using the direction of incoming ray vector and the index of refraction of the materials, the direction of refracted ray vector ***t*** is achieved.

b. Ray in a 2D (*bi-dimensional*) and 3D *spaces*

Vector ***t*** is calculated using the direction of *n*, *b*, *d*, the angle between the *n* and *d* (*i.e.* θ) and the index of the refraction (n_d/n_t). Here, *n*, *b* and *d* are considered as unit vectors, for the sake of simplicity.

Place of Fig.1

Place of Table 1

By using (18), *t* is expressed in terms of *n*, *d* and θ as follows:

$$t = -n.\sqrt{1-\frac{n_d^2}{n_t^2}.\sin^2\theta} + \frac{n_d}{n_t}.(d-(d.n).n) \tag{19}$$

In tracing a ray in a 2D space, *d*, *t*, *n* are placed in *z*=0 plane. The ray tracing in a 3D spaces is a special state of ray tracing in 2D spaces that occurs in a plane with an arbitrary orientation. In a 3D space, *n* and *d* are stated in a new coordinate system as:

$$n^* = (1,0), \qquad d^* = (d.\xi^*, d.\lambda^*) \tag{20}$$

where ξ^* and λ^* are normalized vectors that are perpendicular to each other (see

Table 1). These vectors state *n* and *d* in the new 2D space. Consequently, by calculating $t^* = (t_1^*, t_2^*)$ in a 2D space, *t* in a 3D space is obtained as:

$$t = t_1^* . \xi^* + t_2^* . \lambda^* \tag{21}$$

c. Optimization procedure

The RO, like other meta-heuristic methods has a number of agents including the variables of the optimization problem. This part, provides the step-by-step implementation process of the RO.

Step 1: *Initialization and save bests*

The initial positions of the agents in the search space are determined randomly using Eq.(22). Then the objective function is evaluated for each agent and the position of the best agent is saved as the global best and the position of each agent is saved as its local best.

$$X_{ij} = X_{j,\min} + rand.(X_{j,\max} - X_{j,\min}). \tag{22}$$

Step 2: *First movement*

The variables vector must be divided into two variables and three variables groups. Then each group moved to its new position in 2D or 3D spaces using Eq.(23) as the first movement.

$$V_{ij} = -1 + 2.rand. \tag{23}$$

where V_{ij} is the jth variable of the ith agent and it may belong to a 2D or 3D group.

Step 3: *Refinement of the movement vectors and update bests.*

Now by adding the movement vector of each agent, they move to their new position, but there is a possibility of boundary violation, so the movement vector of the violating agent must be refined. Now, a vector with a length equal to 0.9 times of the length between the position of current agent and the boundary intersection caused by the prior movement vector and with the same direction is selected as the new movement vector. After motion refinement, the 2D and 3D groups of agents must be joined together. Then the goal function of agents evaluated and the so-far best agent at this stage is selected as the global best and for each agent, the so-far best position by this stage, is selected as its local best.

Step 4: *Origin making*

After the first motion, the new point to which each particle must be moved, need to be determined. This point that is named origin is determined as:

$$O_i^k = \frac{(maxit + k).GB + (maxit - k).LB_i}{2.maxit} \tag{24}$$

Step 5: *Next movement based on Snell's law*

The normal vector *n* is selected as a vector whose direction is from origin to the current position of the agent. Now, the direction of the new movement vector can be created because *n* and incoming ray vector, *d*, which is the last movement vector, are obtained based on Eq.(19). This is a normalized vector and it requires a logical coefficient. Thus the final form of the movement vector after finding the new direction is given by:

$$V_{i,l} = V'_{i,l}.norm(X_{i,l} - O_{i,l}). \tag{25}$$

where $X_{i,l}$, $V'_{i,l}$, $O_{i,l}$ and $V_{i,l}$ are the agent's current position, normalized movement vector, the origin and refined movement vector of the agent *i*, respectively, that belong to group *l*.

Place of Fig.2

Fig.2 represents how a 2D agent moves to the origin (origin supposed fixed during the search for simplicity in representation) according to Snell's law.

Provided that origin and its current position become the same, accordingly the direction of the normal cannot be achieved. In order to solve this problem, the direction of the movement can be attained using the next equation:

$$V_{i,l}^{k+1} = \frac{V_{i,l}^k}{norm(V_{i,l}^k)}.rand.0.001. \tag{26}$$

where $V^k_{i,l}$ is the movement vector of the iteration *k* that belongs to group *l* of the agent *i*, and $V^{k+1}_{i,l}$ is the movement vector of the iteration (*k*+*1*).

Step 6: *Next movement based on stochastic nature.*

For adding a stochastic nature to find the best answer, a random change is added to the movement vector. In this regard, a random number is determined by *stoch* to decide about the determination of the movement vector by Snell's law and/or Eq.(27).

$$V_{ijl}^{(k+1)} = -1 + 2.rand. \tag{27}$$

where $V_{ijl}^{(k+1)}$ is the component j of the group l that belongs to the agent i in iteration ($k+1$). Though, the following equation is used to refine the length of this vector:

$$V_{il}^{k+1} = \frac{V_{il}^{(k+1)'}}{norm(V_{il}^{(k+1)'})}.\frac{a}{s}.rand. \tag{28}$$

in which a is calculated as follows:

$$a = \sqrt{\sum_{l=1}^{q} (X_{l,\max} - X_{l,\min})^2} \quad q = \begin{Bmatrix} 2 & \text{for two variable groups} \\ 3 & \text{for three variable groups} \end{Bmatrix} \tag{29}$$

In this equation, for effective search, a is divided into smaller parts.

Interested readers may refer to [49] for further details.

Step 7: *Check search termination criterion.*

If the iterations aren't finished, return to step 3.

4. RO-based EEGS

The pseudo code of the proposed RO-based solution methodology for EGS and EEGS problems is summarized as follows:

Level 1: Initialization

Step 1. *Initialization.* Generate a random array of agents in the search space.

Step 2. *Agents ranking.* Balance generation with load power and losses by handling slack generator power, then evaluate the goal function for the agents based on Fuel cost and emission amount. Then penalize the agents those violate the POZs and spinning reserve by converting the constrained EGS and EEGS problems into the unconstrained one with penalty factors PF_1 and PF_2 [4]:

$$CV_1 = (\max[0, \min(P_p - P_{p,\min}, P_{p,LB_1} - P_p), \ldots$$
$$, \min(P_p - P_{p,UB_{z-1}}, P_{p,LB_z} - P_p) \qquad (30)$$
$$, \min(P_p - P_{p,UB_z}, P_{p,\max} - P_p)])^2 \qquad z = 2,3,\ldots,N_{POZ_p}$$

$$CV_2 = (\max[0, SR - \sum_{p \notin N_G} S_p])^2 \qquad (31)$$

$$F_t = T + PF_1 \times CV_1 + PF_2 \times CV_2 \qquad (32)$$

CV_1 stands for slack generator (usually the first generator) power violation from its limits and CV_2 stands for units spinning reserve violation. T is the goal function without considering slack generator limits and spinning reserve.

The ramp rate limits constraint is handled on lower and upper limits of the generators' power (the powers change from their older state is limited). At the end save the agent's global best and the local bests in the memory. This step is illustrated in Fig.3.

Place of Fig.3

Step 3. *First movement.* Divide the solution vectors into 2-variable and 3-variable groups to move in the 2D and 3D spaces. Then Move all groups according to (23).

Step 4. *Refinement the solution vectors.* If there is any generation boundary violation in a group, refine it.

Step 5. *Rebuild the agents.* Mix the 2D and 3D groups and rebuild the agents.

Step 6. *Update the memory.* Do the same as step 2 for constraint handling, evaluating goal function and updating bests.

Level 2: Search

Step 1. *Calculating origin point.* Determine the point that each particle must be moved to, by (24).

Step 2. *Next movement.* Split the particles to 2D and 3D groups and Find the next movement vector for each group of particles utilizing the Snell's refraction law, or stochastic nature with probability of *stoch*, then move the agents to the new positions.

Step 3. *Refinement the solution vectors.* If there is any boundary violation in a group, refine it.

Step 4. *Rebuild the agents.* Mix the 2D and 3D groups and rebuild the agents.

Step 5. *Update the memory.* Do the same as step 2 for constraint handling. Then evaluate fitness function for the new agents and update the global best and the local bests.

Level 3: stop search

Step 1. Repeat search level steps until the max iteration reached.

Finally, the flowchart of the proposed RO-based solution methodology for EEGS problem is depicted in Fig.4.

Place of Fig.4

5. Numerical results

In this section, in order to assess the efficiency of the proposed RO-based solution method for EGS and EEGS problems, it is applied to five case studies. In this studies, error tolerance value in Fig.3 has been taken as $ToL = 1\times10^{-3}$. RO parameters total number of population has been taken as $npop = 100$, index of the refraction as $n_d / n_t = 0.5$, constant parameter in Eq.(28) as $s = 7.5$ and stochastic nature probability has been taken as $stoch = 0.35$. The maximum iteration number for five test cases is considered as 2000, 3000, 100, 100 and 200, respectively.

Place of Table 2

5.1. Description of the test cases

The characteristics of the five test cases are presented in Table 2. Here, Case I and Case II are considered for testing the proposed method to solve EGS problem. In addition, Case III, Case IV and Case V are used to examine the proposed RO method to solve bi-objective EEGS problem.

Case I: In this case a 10-generator network is considered. The cost functions of fuel consumption of these units include both valve loading effects and multi-fuel option. Total load power of the system is 2700 MW [50].

Case II: The system considered in this case, is a thirteen generating units network with the ramp rate limits, POZs and valve loading effects. Moreover this case comprises spinning reserve constraints. Here, 5 generation units out of thirteen ones include POZs, and the remaining 8 units provide the required spinning reserve. Total load power of this system is 2520 MW, and the system contributes in the provision of a required spinning reserve of at least 180 MW. In this case, transmission losses also considered. System's data is reported in [51].

Case III: The IEEE 30-bus 6-generator network with NO_x emission and total load power of 283.4 MW is considered in this case. System's data containing cost and emission functions coefficients and the generators output power limits are reported in [52].

Case IV: The system used in this case, is the same 6 units system used in Case III, but here transmission losses considered also. Transmission loss coefficients for this system are obtained from [52].

Case V: In the last case study, a little complex system with ten generators with valve loading effects and NO_x emission is considered. The data of this system is reported in [1].

Place of Table 3

5.2. EGS test cases

In this part, the computational results of the best, average, and the worst fuel costs among the 30 trial runs of solutions for EGS problem of the systems of Case I and Case II, are presented. For the sake of results comparison, the achieved results from several recently published EGS and EEGS solution methods are also represented in these Tables. It is worthy of being mentioned that results of the other optimization methods have been directly quoted from their respective references.

The results of RO method for Case I in comparison of the results of other algorithms, including TSA [19], improved GA with multiplier updating (IGA-MU) [50], PSO [19], hybrid harmony search (HHS) [53], modified harmony search algorithm (MHSA) [4] and real coded GA (RGA) [54] are provided in Table 3. The RO provided a solution with 624.0922 \$/h better than TSA, IGA-MU, PSO and RGA, except HHS and MHSA. Besides, the best, average and worst solution of RO for this case, is presented in Table 4 in comparison of stochastic weight trade-off PSO (SWT_PSO) [55], differential evolution (DE) [54], DSPSO-TSA [19], conventional GA with multiplier updating (CGA-MU) [50] and the methods reported in Table 3. The resulted average and worst solutions compared with the other reported methods, shows the reliable performance of the proposed RO in most of runs. According to Table 3 and Table 4, the proposed RO method is effective in solving EGS problem with valve loading effects and multi-fuels.

Place of Table 4

Place of Table 5

In Table 6 and 7, the results of the application of RO for Case II are reported. As can be seen from these tables, the RO result is acceptable and comparable with the other methods. According to Table 6, all of the system constraints including generators' ramp rate limits, POZs and spinning reserve of system are satisfied. In Table 7, there just MHSA [4] has a better worst solution than RO among reported methods, which shows reliable performance of this method in multiple runs.

The aforementioned results and comparisons through the different test cases reveal the capability of RO method to solve practical EGS problems.

In order to investigate POZs impress over the solution provided by RO, another run has been done, without considering POZs but the results were as with as considering POZs. As it can be seen from Fig.6, in this special case the provided generations by RO when considering POZs, are far from the POZs. Besides that, the results without considering POZs showed that the RO provided solution for Case II is not restricted by POZ constraint. Among the works reported in Table 6, just the generation of P_2 provided by GA [42] has stuck to POZ boundary

305.0000 MW. So it can be said the solution provided by GA [42] for Case II, is restricted by POZ constraint.

Place of Fig.5

Place of Table 6

Place of Table 7

Place of Fig.6

Place of Table 8

Place of Fig.7

Place of Table 9

5.3. EEGS test cases

In the EEGS two competing objectives are simultaneously solved to obtain the Pareto-optimal solutions. In this paper for obtaining the best compromise solution, we select $w_c = 0.5$ for both objectives. In other words, the same importance is considered for the economic and emission aspect in the EEGS problem.

The obtained Pareto-optimal solutions, with increasing the value of *w* starting from 0 towards 1 with 0.1 intervals and solving the EEGS for Case III, are presented in Table 8 illustrated in Fig.7. According to the results of Fig.7, the solutions found for different values of *w* are uniformly distributed and completely covered the whole Pareto front. In this case, the best compromise solution is obtained at $w = 0.4$ with the cost of 610.2756 \$/h and the emission of 0.2004 ton/h. Out of them, two non-dominated solutions that represent the best cost ($w = 1$) and best emission ($w = 0$) are given in Table 9, as compared with the solutions reported using LP [56], SPEA [45], MOSST [44], NPGA [39], NSGA [40], NSGA-II [41], SBF [57], FCPSO [36], and EC [48]. As can be seen from Table 9, the fuel cost given by RO for the economic dispatch problem ($w = 1$), is the same as EC [48] method (600.1114 \$/h) and it is less than those of other methods reported in Table 9. Also for the emission dispatch problem ($w = 0$), RO gives the same minimum emission amount that reported in Table 9 (0.1942 ton/h), with a better fuel cost than other methods (638.261 \$/h). The obtained Pareto-optimal front of RO for Case IV is shown in Fig.8. Results of RO for this case are presented in Table 10. The best compromise solution that is extracted by fuzzy decision making, for this case is obtained at $w = 0.4$ with the fuel cost of 615.5268 \$/h and the emission value of 0.2007 ton/h. The best fuel cost and best emission of RO for Case IV are compared with those of other methods in Table 11. As can be seen, for the economic dispatch problem ($w = 1$) of Case IV, between 36 methods those are reported in Table 11, total fuel cost obtained from RO (605.8582 \$/h) is better than other methods except EC [48] and MHSA [4]. Also the obtained best emission value ($w = 0$) of RO for this case is competitive with those of the previously reported approaches.

Place of Table 10

Place of Table 11

Place of Fig.8

Place of Fig.9

Place of Fig.10

Place of Table 12

Place of Table 13

Fig.9 shows the Pareto-optimal front of RO for Case V. this case is a little complex test case and in order to obtain an even Pareto-optimal front for this case, the weighting factor w is varied with non-equal intervals between 0 and 1. The best compromise solution for this case by fuzzy decision making, is obtained at $w = 0.72$ with total fuel cost of 1.12959×10^5 \$/h and total emission of 4087.52 lb/h. Among the obtained non-dominated solutions in the Pareto-optimal set, two non-dominated solutions that represent the best cost and best emission are compared with those of MHSA [4] and DE [1] algorithms in Table 12. It is obvious that fuel cost given by RO for economic dispatch ($w = 1$) is less than those of both algorithms. Also for emission dispatch problem ($w = 0$) of Case V, RO gives better emission than of both methods (i.e. bold numbers). The best compromise solution of RO for EEGS problem of Case V, is compared with MODE [1], MHSA [4], SPEA2 [1], PDE [1], NSGA-II [1] and GSA [58] algorithms in Table 13. As can be seen, the results achieved by RO algorithm, are considerably better than other methods reported in Table 13. And also comparison of the computing time of the methods in Table 13 shows that RO finds a better solution in a competitive time. In order to illustrate the RO fast performance, another run with less population has been done. In this scenario, the number of max iteration has been selected to be 50 iterations, to terminate within a time lower than other methods. This run took about 1.39 sec to find a solution with 1.13014×10^5 \$/h cost and 4082.26 lb/h emission which is still better than the reported works and faster computing time. When the RO algorithm has been applied to Case V, the changes of the total fuel cost and total emission amount are depicted in Fig.10 for economic dispatch ($w = 1$), emission dispatch ($w = 0$) and best compromise solution ($w = 0.72$). In this case because of complexity, the weight value w isn't varied uniformly with 0.1 intervals, in order to get a plain Pareto front.

6. Conclusion

In this paper, a new optimization method based on Snell's refraction law called Ray optimization has been used for EGS and multiobjective EEGS problem, considering practical constraints of real-world power systems including valve loading effects, ramp rate limits, POZs, multi-fuel options, transmission losses and spinning reserve. Ray Optimization which is a multi-agent method considers agents as rays of light. Based on the Snell's light refraction law when light travels from a lighter medium to a darker medium, it refracts and it's direction changes. This behaviour helps the agents to explore the search space in the early stages of the optimization process and to make them converge in the final stages. This law is the main tool of the Ray Optimization algorithm to find the near to optimal answer. In the EEGS problem, the RO method has tried to reach the best result for each value of weighting coefficient *w*. in the solution transaction, the weight factor *w* has been changed between 0 and 1. Furthermore, a decision making approach is used to extract the best compromise solution over the Pareto-optimal set. The numerical results and relative comparisons with several recently published research works show that RO is an efficient method for solving EGS and EEGS problems with respect to all abovementioned practical constraints.

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Figures

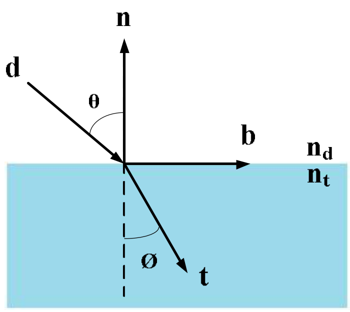

Fig.1. Demonstration of a typical incident and refracted rays

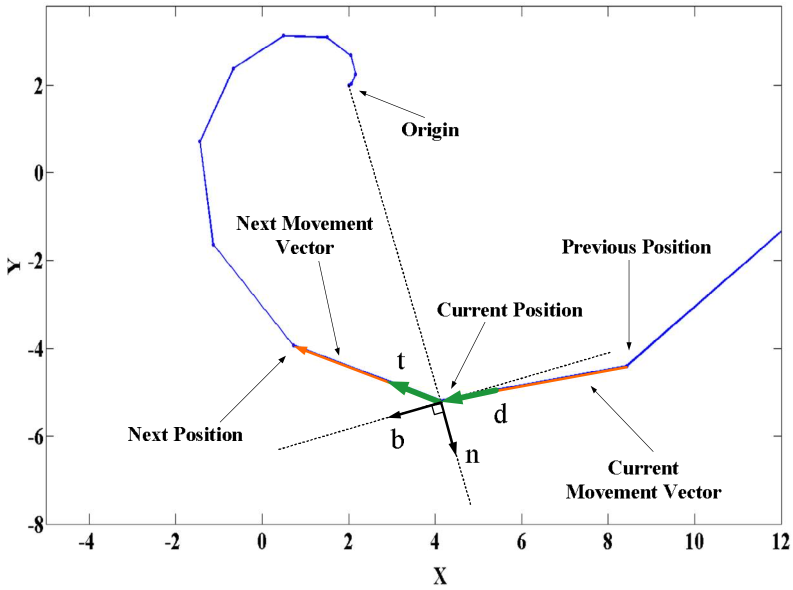

Fig.2. Movement of a 2D agent to the origin

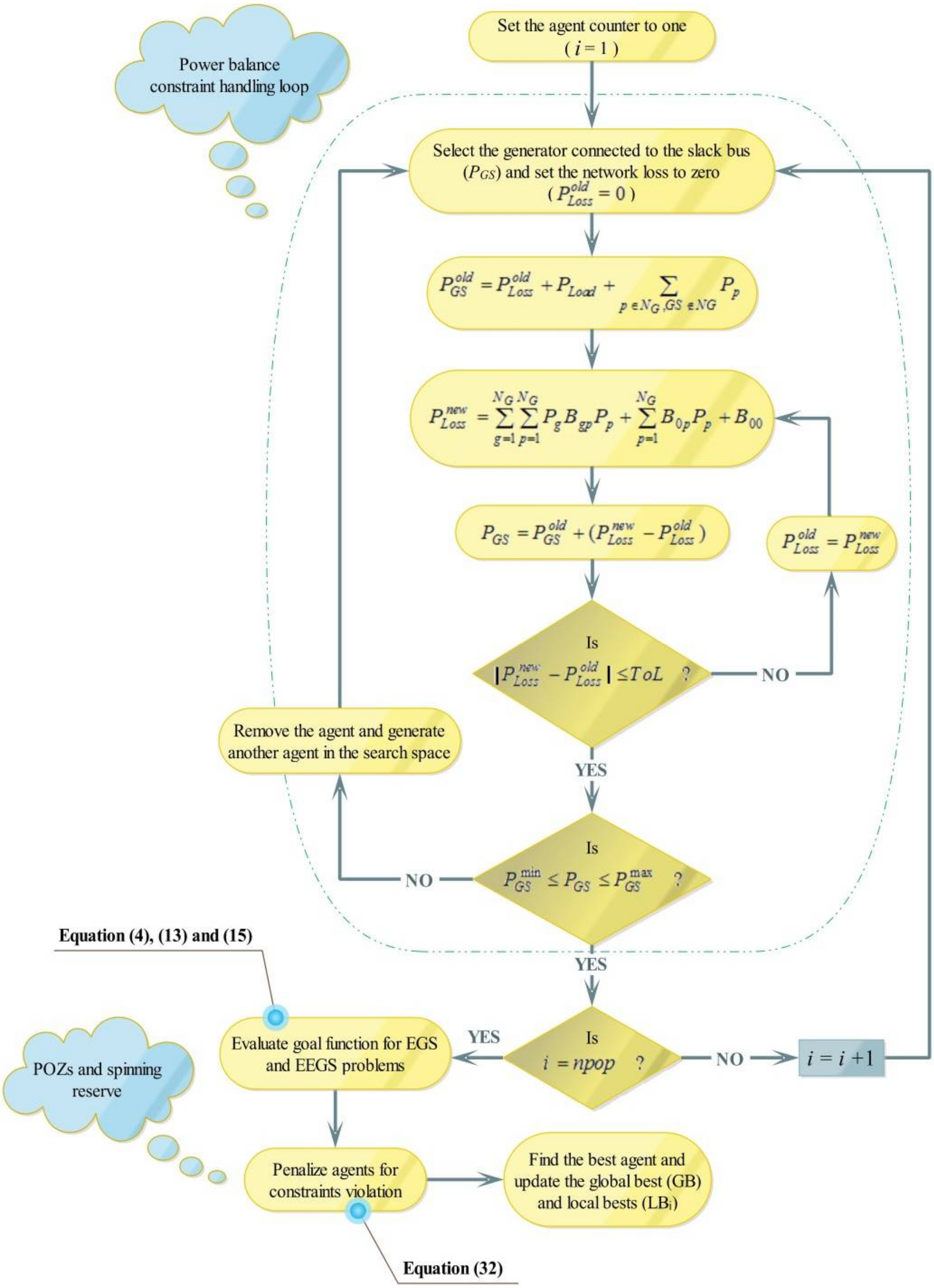

Fig.3. Constraint handling strategy of EGS and EEGS problems

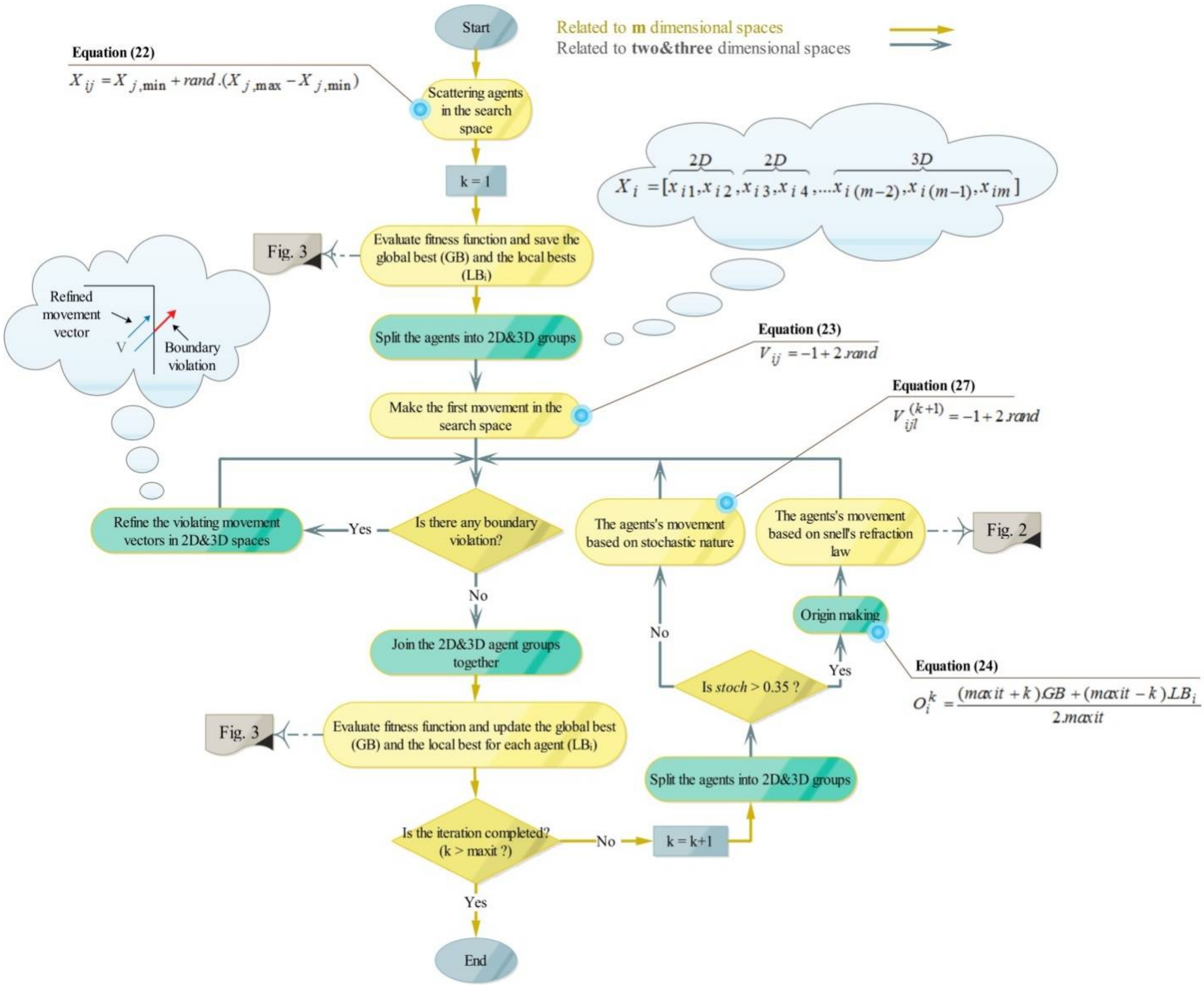

Fig.4. RO-based EEGS method flowchart

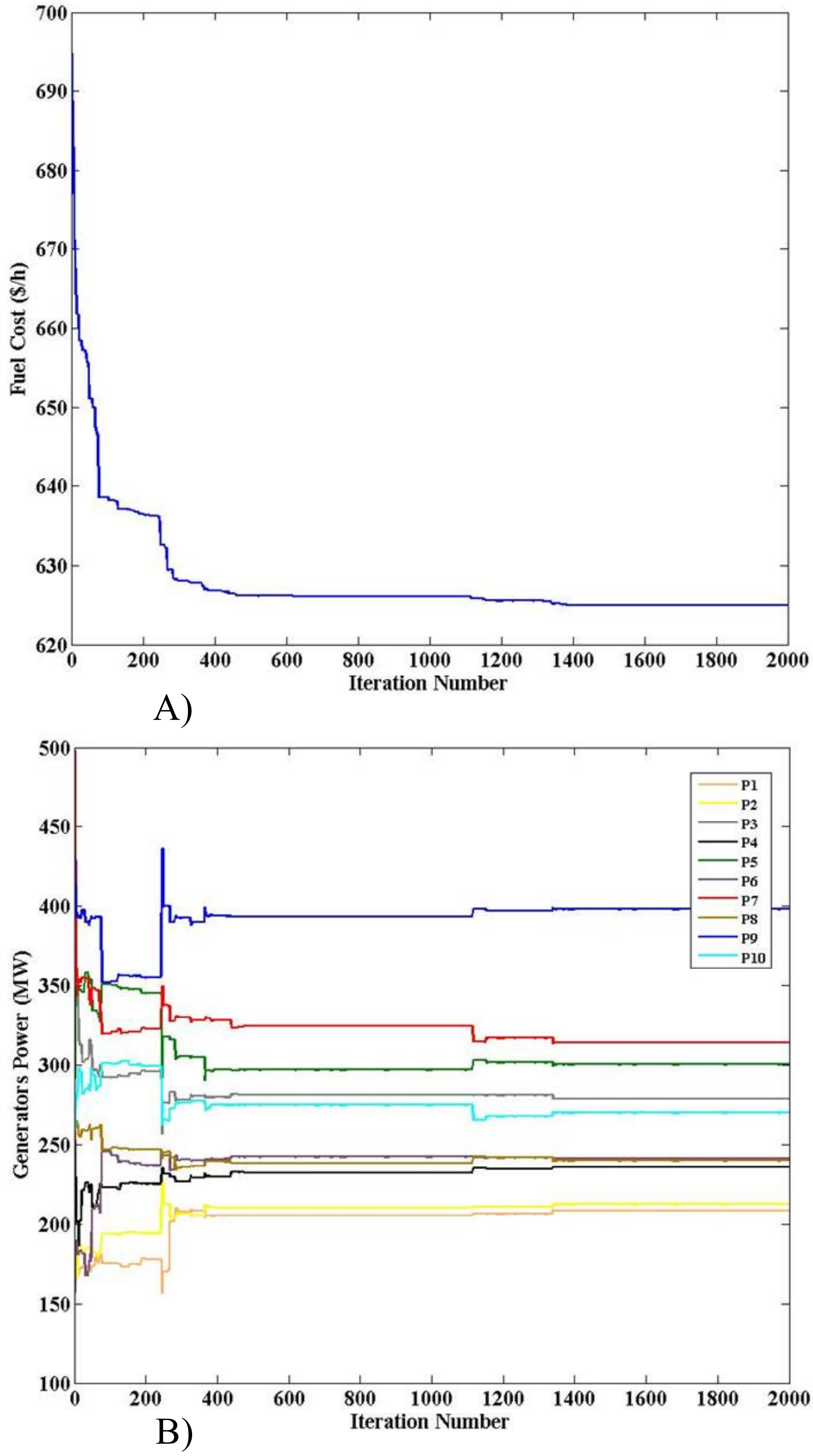

Fig.5. Evolution of RO method for Case I. A) Fuel cost convergence B) generators output power convergence.

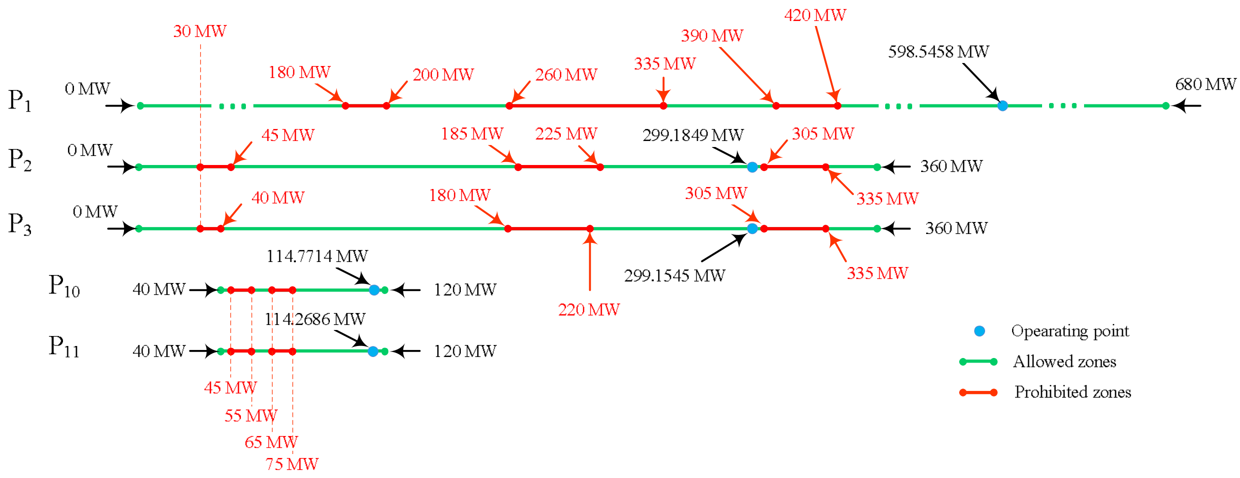

Fig.6. Representation of Case II units' power constrained by POZs and the RO provided solution

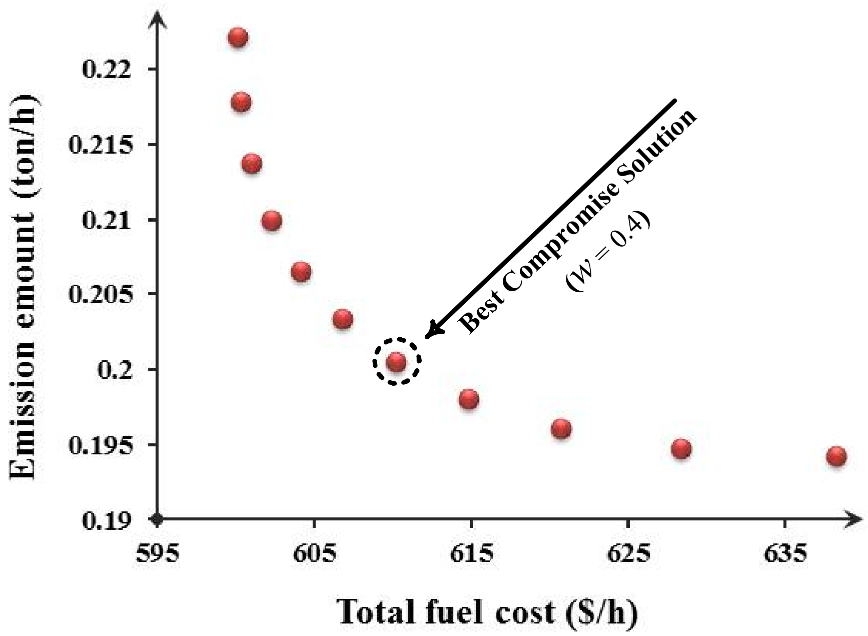

Fig.7. Pareto-optimal front of RO method for Case III.

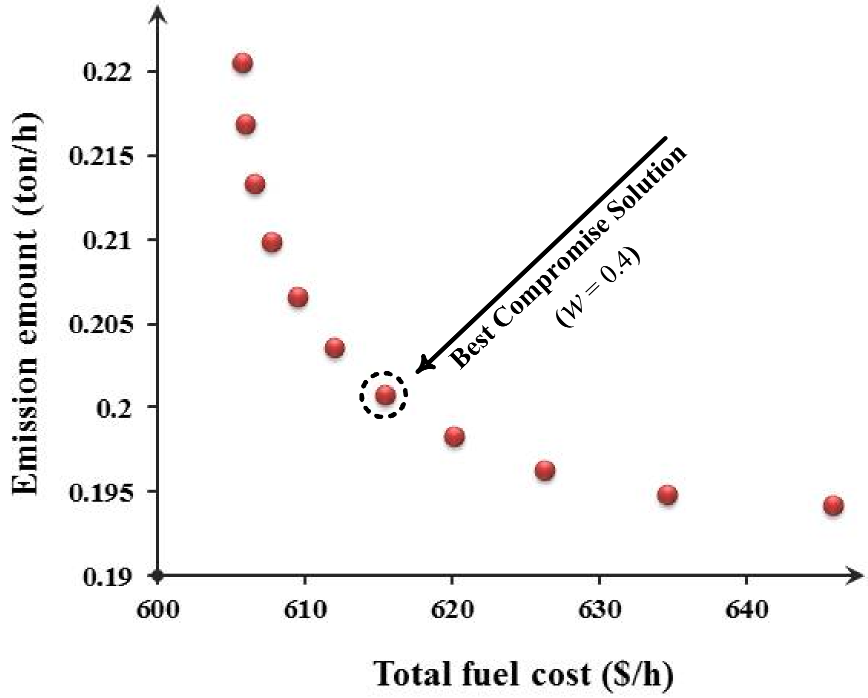

Fig.8. Pareto-optimal front of RO for Case IV.

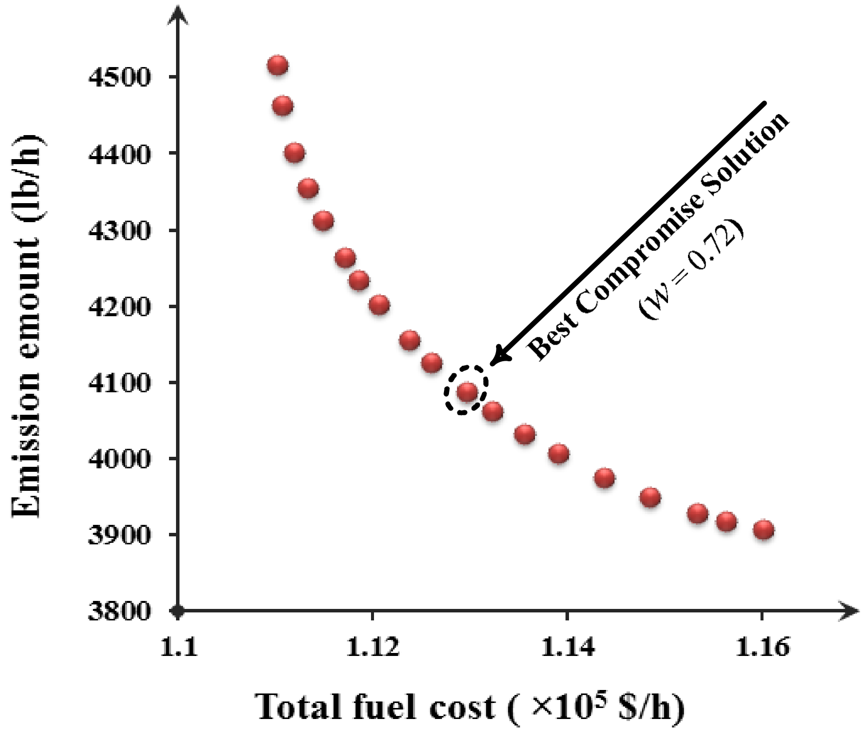

Fig.9. Pareto-optimal front of RO method for Case V.

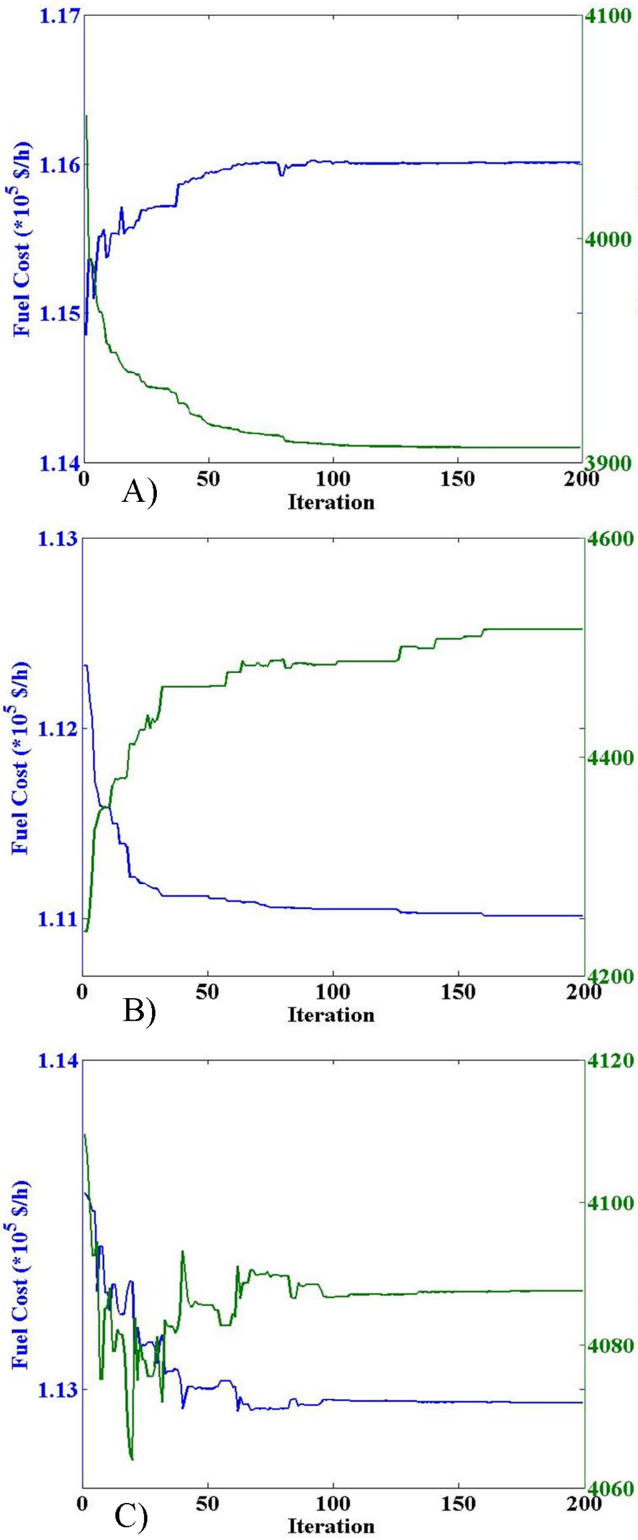

Fig.10. Changes of the fuel cost and emission amount according to iteration numbers for Case V. A) $w = 0$. B) $w = 1$. C) $w = 0.72$.

Tables

Table 1
The designed elements of the new coordinate system

	$-0.05 \le n.d \le 0.05$	$0.05 \le n.d \le 1$	$-1 \le n.d \le -0.05$
ξ^*	n	n	n
λ^*	d	$\lambda^* = \dfrac{(n - \frac{d}{n.d})}{norm(n - \frac{d}{n.d})}$	$\lambda^* = \dfrac{(n - \frac{d}{-n.d})}{norm(n - \frac{d}{-n.d})}$

Table 2
The values used for tuning RO parameters

parameter	value
ToL	1×10^{-3}
npop	100
n_d/n_t	0.5
s	7.5
stoch	0.35

Table 3
The characteristics of the five test systems

	EGS test systems		EEGS test systems		
	Case I	Case II	Case III	Case IV	Case V
Number of units	10	13	6	6	10
Transmission losses		✓		✓	✓
Valve loading effects	✓	✓			✓
Ramp rate limits		✓			
POZs		✓			
Multi-fuel	✓				
Spinning reserve		✓			

Table 4
The results of the application of RO for Case I.

	TSA [19]		IGA-MU [50]		PSO [19]		HHS [53]		MHSA [4]		RGA [54]		RO	
	P (MW)	FT	P (MW)	FT	P (MW)	FT	P (MW)	FT	P (MW)	FT	P (MW)	FT	P (MW)	FT
P_1	219.4959	2	219.1261	2	225.5792	2	218.25	2	218.5895	2	220.9376	2	217.5902	2
P_2	206.7093	1	211.1645	1	208.2240	1	211.66	1	211.4642	1	212.6096	1	207.5031	1
P_3	291.3532	1	280.6572	1	278.8078	1	280.72	1	280.6562	1	283.5811	1	283.9859	1
P_4	237.6731	3	238.4770	3	238.0062	3	239.63	3	239.2362	3	240.0089	3	238.5645	3
P_5	279.2478	1	276.4179	1	282.4136	1	278.53	1	279.8499	1	282.8920	1	285.1790	1
P_6	237.3793	3	240.4672	3	239.6464	3	239.63	3	239.7738	3	240.4739	3	240.1769	3
P_7	277.9598	1	287.7399	1	285.4269	1	288.58	1	287.7299	1	292.9792	1	292.4011	1
P_8	238.9435	3	240.7614	3	239.1045	3	239.63	3	240.4457	3	240.1989	3	236.4145	3
P_9	429.9256	3	429.3370	3	425.5856	3	428.51	3	426.3877	3	406.9988	3	425.0234	3
P_{10}	281.3126	1	275.8518	1	277.2121	1	274.86	1	275.8668	1	279.3199	1	273.1127	1
Total cost ($/h)	624.3078		624.5178		624.3046		623.739		623.834		624.5081		624.0922	

Table 5
Best, average and worst cost for Case I.

Method	Best cost ($/h)	Worst cost ($/h)	Average cost ($/h)
CGA-MU [50]	624.7193	633.8652	627.6087
IGA-MU [50]	624.5178	630.8705	625.8692
PSO [19]	624.3045	625.9252	624.5054
TSA [19]	624.3078	635.0623	624.8285
DSPSO-TSA [19]	623.8375	623.8625	623.9001
HHS [53]	623.739	-	-
RGA [54]	624.5081	624.5088	624.5079
DE [54]	624.5146	624.5458	624.5246
RCGA [59]	623.8281	623.8814	623.8495
SWT-PSO [55]	623.8274	626.4755	624.1246
MHSA [4]	623.8340	625.1302	624.0412
RO	**624.0922**	**627.1189**	**625.2564**

Table 6
The results of the application of RO for Case II.

Output power of Generators (MW)	DE [51]	MHSA [4]	GA [51]	HDE [51]	RO
P_1	628.0117	628.3080	628.4311	628.3290	598.5458
P_2	300.2498	300.2387	305.0000	299.3286	299.1849
P_3	348.2995	299.7709	302.6497	304.5139	299.1545
P_4	159.0591	159.9158	158.9094	159.7930	159.7261
P_5	159.7318	159.7459	160.4743	159.8114	159.1135
P_6	159.7324	159.7519	159.7312	159.8572	158.9151
P_7	159.7330	159.7773	160.1004	159.9505	159.5829
P_8	147.6877	109.9736	159.6400	109.8658	158.8117
P_9	160.7340	159.8171	109.6715	159.7405	156.5171
P_{10}	77.29379	114.8350	114.5156	114.8171	114.7714
P_{11}	115.6040	114.8483	116.2229	115.7702	114.2686
P_{12}	55.01118	98.90760	92.08722	94.97113	92.26748
P_{13}	91.19282	92.86890	92.43267	92.40933	92.32242
Reserve power (MW)	198.3218	187.084	198.3218	190.8761	180.0662
Transmission losses (MW)	42.3412	38.9448	39.8664	39.1582	43.18139
Total fuel cost ($/h)	24819.32	24585.361	24632.42	24591.76	24610.216

Table 7
Best, average and worst cost for Case II.

Algorithm	Best cost ($/h)	Worst cost ($/h)	Average cost ($/h)
HDE [51]	24591.76	25074.90	24739.53
Self-tuning HDE [51]	24560.08	24872.44	24706.63
MHSA [4]	24585.36	24711.30	24638.36
DE [51]	24819.32	25656.40	25217.64
GA [51]	24632.42	25188.59	24874.93
RO	**24610.22**	**24765.11**	**24692.37**

Table 8
Total fuel cost and emission amount for different values of *w* (Case III).

w	Generators output power (MW)						Total fuel cost ($/h)	Total emission (ton/h)
	P_1	P_2	P_3	P_4	P_5	P_6		
0	40.6131	45.9125	53.8018	38.3044	53.8018	50.9664	638.2613	**0.1942**
0.1	36.5429	43.4439	53.8771	46.9715	53.8753	48.6892	628.3793	0.1947
0.2	32.8143	41.2678	53.9367	54.7691	53.9382	46.674	620.7457	0.1960
0.3	29.3624	39.3263	53.9682	61.9172	53.9499	44.8759	614.8118	0.1980
0.4	26.1867	37.5963	53.9483	68.4451	53.9679	43.2556	**610.2756**	**0.2004**
0.5	23.2316	36.0358	53.8853	74.5803	53.8850	41.7820	606.7963	0.2033
0.6	20.4658	34.6139	53.7572	80.3664	53.7587	40.4381	604.1946	0.2065
0.7	17.8790	33.3155	53.5527	85.9020	53.5509	39.1999	602.3188	0.2099
0.8	15.4448	32.1193	53.2708	91.2431	53.2707	38.0513	601.0615	0.2137
0.9	13.1477	31.0108	52.9007	96.4604	52.9007	36.9797	600.3432	0.2178
1	10.9720	29.9763	52.4314	101.620	52.4286	35.9717	**600.1114**	0.2221

Table 9
Comparison of the best fuel cost and emission amount for Case III.

Different Methods	W=1.0 (Best Fuel cost)		W=0.0 (Best emission)	
	TFC ($/h)	TEA (ton/h)	TFC ($/h)	TEA (ton/h)
LP [56]	606.314	0.22330	639.600	0.19424
SPEA [45]	600.15	0.2215	638.51	0.1942
MOSST [44]	605.889	0.22220	644.112	0.19418
NPGA [39]	600.259	0.22116	639.182	0.19433
NSGA [40]	600.572	0.22282	639.231	0.19436
NSGA-II [41]	600.155	0.22188	638.269	0.19420
FCPSO [36]	600.1315	0.2223	638.3577	0.1942
FSBF [57]	600.1141	0.222	638.2835	0.1942
EC [48]	600.1114	0.2221	638.2703	0.1942
RO	**600.1114**	0.2221	638.261	**0.1942**

Table 10
Obtained best fuel cost, best emission and best compromise solution of RO for Case IV.

Output power (MW)	Emission dispatch ($w=0$)	Economic dispatch ($w=1$)	Best compromise solution ($w=0.4$)
P_1	41.0789	12.5225	25.3014
P_2	46.352	28.6735	37.3308
P_3	54.4217	58.1623	56.4985
P_4	39.015	99.1127	68.6705
P_5	54.4254	52.0955	54.8383
P_6	51.4874	35.3174	43.2731
Power losses (MW)	3.3802	2.4987	2.5126
Total fuel cost ($/h)	645.8519	605.8582	615.5268
Total emission (ton/h)	0.1942	0.2206	0.2007

Table 11
Comparison of best fuel cost and emission amount for Case IV.

Different Methods	*w*=1 (Best Fuel cost)			*w*=0 (Best emission)		
	TFC ($/h)	TEA (ton/h)	P_{Loss} (MW)	TFC ($/h)	TEA (ton/h)	P_{Loss} (MW)
BB-MOPSO [60]	605.9817	0.2201	2.5620	646.4847	0.1941	3.5370
MO-DE/PSO [61]	606.0073	0.2208	2.5550	646.0243	0.1941	3.5350
EC [48]	605.8363	0.2208	2.4600	646.2203	0.1942	3.6200
BBO [62]	606.2600	0.2187	2.4654	642.9250	0.1942	3.4990
FSBF [57]	607.5081	0.2196	3.1900	645.3981	0.1942	3.0300
NSBF [57]	607.5901	0.2211	3.3100	647.7413	0.1944	2.9200
MOCASO [63]	607.8500	0.2227	3.0500	644.2700	0.1932	3.0500
PSO [64]	607.7800	0.2198	3.3100	645.2300	0.1942	3.1100
MOPSO-II [64]	607.7900	0.2193	3.3300	644.7400	0.1942	3.0900
PSO [65]	607.8400	0.2192	3.2900	642.9000	0.1942	3.0800
MOPSO [66]	608.1000	0.2227	3.0500	644.2700	0.1935	3.0500
MOCPSO [66]	607.7600	0.2221	3.0500	663.3100	0.1908	3.0500
MODE [67]	606.4160	0.2221	2.6034	643.5190	0.1942	3.3699
GA [42]	607.7800	0.2199	3.3200	645.2200	0.1942	3.1100
NSGA [40]	607.9800	0.2191	3.4600	638.9800	0.1947	2.9700
NPGA [39]	608.0600	0.2207	3.3700	644.2300	0.1943	3.1400
NSGA-II (1)[68]	611.8392	0.2215	5.0200	646.9075	0.1944	5.2100
NSGA-II (2) [41]	607.8010	0.2189	3.3000	644.1330	0.1941	3.1000
NSGA-II(3) [69]	613.6759	0.2223	5.9500	648.7090	0.1942	6.0400
NSGA-II/CAO [69]	613.5488	0.2205	5.9500	650.7343	0.1942	6.1900
MNSGA-II [70]	608.1248	02199	3.4658	645.4787	0.1942	3.3313
MNSGA-II + DCD [70]	608.1283	0.2199	3.4548	645.3998	0.1942	3.2894
MNSGA-II + DCD + CE [70]	608.1247	0.2198	3.4709	645.6472	0.1942	3.3173
CMOPSO [61]	606.0472	0.2204	2.5600	645.9985	0.1941	3.5170
SMOPSO [61]	605.9909	0.2206	2.5970	648.5035	0.1942	3.4950
TV-MOPSO [61]	606.4028	0.2197	2.6040	642.7938	0.1942	3.3920
ε-GA [71]	606.4533	0.2028	2.3200	642.8976	0.1882	4.0800
SPEA [45]	607.8600	0.2176	3.3200	644.7700	0.1943	3.0000
DE [72]	608.0658	0.2193	3.4180	645.0850	0.1942	3.0403
MBFA [73]	607.6700	0.2198	3.2600	644.4300	0.1942	3.2800
FCPSO [73]	607.7860	0.2201	3.3500	642.8964	0.1942	3.0900
MA θ-PSO [62]	605.9984	0.2206	2.5562	649.2070	0.1942	3.5330
HSA [68]	606.2858	0.2148	1.7500	647.4345	0.1951	2.2600
CSS [52]	605.9865	0.2204	2.5417	645.6639	0.1941	3.2915
MHSA [4]	605.6440	0.2203	2.5638	645.6172	0.1941	3.5190
RO	**605.8582**	0.2206	2.4987	645.8519	**0.1942**	3.3802

Table 12
Comparison of the best fuel cost and the best emission for Case V.

output power (MW)	MHSA [4]		DE [1]		RO	
	Best fuel cost ($w=1$)	Best emission ($w=0$)	Best fuel cost ($w=1$)	Best emission ($w=0$)	Best fuel cost ($w=1$)	Best emission ($w=0$)
P_1	55.0000	55.0000	55.0000	55.0000	54.4870	55.0000
P_2	80.0000	80.0000	79.8063	80.0000	79.3060	79.9970
P_3	106.0998	82.0444	106.8253	80.5924	100.8430	80.6940
P_4	100.3628	80.8657	102.8307	81.0233	97.9080	80.8710
P_5	82.6482	160.0000	82.2418	160.0000	83.2320	160.0000
P_6	82.8328	240.0000	80.4352	240.0000	85.0050	240.0000
P_7	300.0000	293.1646	300.0000	292.7434	299.8880	292.4880
P_8	340.0000	300.4258	340.0000	299.1214	339.0110	295.4280
P_9	470.0000	395.2394	470.0000	394.5174	470.0000	394.8770
P_{10}	470.0000	394.7599	469.8975	398.6383	469.3710	395.0620
Total fuel cost ($/h)	111491.954	116410.358	111500	116400	**111014.4**	116005.9
Total emission (lb/h)	4566.955	3932.192	4581.00	3923.40	4515.91	**3906.534**

Table13
Comparison of the best compromise solution for Case V.

Unit	MODE [1]	MHSA [4]	SPEA 2 [1]	PDE [1]	NSGA-II [1]	GSA [58]	**RO (*maxit* = 200)**	**RO (*maxit* = 50)**
P1 (MW)	54.9487	54.4132	52.9761	54.9853	51.9515	54.9992	**55.0000**	**54.9999**
P2 (MW)	74.5821	70.6736	72.8130	79.3803	67.2584	79.9586	**80.0000**	**79.8867**
P3 (MW)	79.4294	97.0719	78.1128	83.9842	73.6879	79.4341	**84.3498**	**84.1302**
P4 (MW)	79.4294	86.4019	83.6088	86.5942	91.3554	85.0000	**82.9006**	**83.5377**
P5 (MW)	80.6875	138.0141	137.2432	144.4386	134.0522	142.1063	**141.8027**	**143.0821**
P6 (MW)	136.8551	162.4903	172.9188	165.7756	174.9504	166.5670	**161.6252**	**162.9784**
P7 (MW)	172.6393	283.6421	287.2023	283.2122	289.4350	292.8749	**298.0125**	**296.2670**
P8 (MW)	283.8233	311.5283	326.4023	312.7709	314.0556	313.2387	**314.5186**	**311.8496**
P9 (MW)	316.3407	439.0945	448.8814	440.1135	455.6978	441.1775	**427.0716**	**428.3075**
P10 (MW)	448.5923	440.7168	423.9025	432.6783	431.8054	428.6306	**431.0606**	**431.2747**
Cost ($\times 10^5$ $/h)	1.1348	1.1329	1.1352	1.1351	1.1354	1.1349	**1.12959**	**1.13014**
Emission (lb/h)	4124.9	4153.3	4109.1	4111.4	4130.2	4111.4	**4087.52**	**4082.26**
CPU time (s)	**3.82**	**-**	**7.53**	**4.23**	**6.02**	**-**	**5.61**	**1.39**