# A new scenario generation-based method to solve the unit commitment problem with high penetration of renewable energies

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**Abstract**

Optimal operation of power systems with high integration of renewable power sources has become difficult as a consequence of the random nature of some sources like wind energy and photovoltaic energy. Nowadays this problem is solved using the Monte Carlo Simulation (MCS) approach, which allows the consideration of important statistical characteristics of wind and solar power production, such as the correlation between consecutive observations, the diurnal profile of the forecasted power production, and the forecasting error. In this paper, a new model of the unit scheduling of power systems with significant renewable power generation based on the Scenario Generation/Reduction method combined with the Priority List (PL) method is proposed that finds the probability distribution function (PDF) of a determined generator be committed or not. This approach allows the recognition of the role of each generation unit on the day-ahead unit commitment (UC) problem with a probabilistic point of view, which is important for acquiring a cost-effective and reliable solution. The capabilities and performance of the proposed approach are illustrated through the analysis of a study case, where the spinning reserve requirements are probabilistically verified with success.


*Keywords:* Greenhouse gas emissions, renewables integration, stochastic economic dispatch, system reliability, uncertainty.

## 1. Introduction

The impact of human activities on the environment as a consequence of the industrialization process, the rising trend of the price of fossil fuels, and the incentives offered by governments in many countries have driven the development and evolution of renewable energy sources. In the United States, it is predicted that energy consumption from renewable power sources is going to reach 4.5 quadrillion Btu in 2040, of which 39% and 7.5% will be obtained from wind farms and solar photovoltaic generators, respectively [1]. However, these types of power sources introduce variability and uncertainty in the control of the power system, making it difficult to obtain its optimal management, producing an increment in generation costs [2]. Luickx et al. [3], using the Belgian power system as a testing bench, have concluded that on the one hand, when the predicted wind power is higher than actual wind power, open cycle gas units are required in order to compensate for the forecasting error which increments the

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generation costs. On the other hand, when the predicted wind power is lower than actual wind power, about 30% of the potential cost savings could be lost due to the necessity of curtailing wind power production. This condition particularly affects in a critical manner those systems with low load profiles, because they obtain the least benefit. Katzenstein and Apt [4] studied the cost related to wind power variability using ERCOT market prices in hourly and sub-hourly time scales. Authors have concluded that the installation of a wind farm in a place with high resources can reduce the costs related to variability. Variability and uncertainty of wind energy can negatively affect greenhouse gas (GHG) emissions. In a general sense, the unit scheduling made from an economical perspective can increase GHG emissions due to the cycling of coal units which are more pollutant than gas units [3].

From the analysis of the Spanish power system, Gutiérrez-Martín et al. [5] have concluded that high integration of wind power drives conventional generators to an operational point of low efficiency, incrementing the predicted GHG emissions. To solve this problem, authors suggest increasing the amount of power generated from combined cycle gas units in order to avoid cycling of coal units.

In order to reduce the negative effects of wind power variability and uncertainty, several studies on the integration of energy storage systems (ESS), aggregation of wind power production and its relationship with geographic characteristics, the implementation of demand response programs, and incorporation of probabilistic models in the economic dispatch (ED) problem and unit commitment (UC) problem have been carried out [6]. The interest of this paper is in developing a methodology for the optimal management of power systems, taking into account wind power uncertainty.

In this sense, Liu and Xu [7] modeled the random behavior of the wind speed using a Weibull probability distribution function (PDF), while the power curve of the wind turbine has been modeled using a linear approximation. Under these assumptions, a probabilistic model is developed which incorporates the variability of wind power generation as a constraint in the optimization problem. Then, a threshold parameter is introduced to represent the degree at which load cannot be supplied. Adjusting this parameter to a small value allows a reduction of the risk of any energy shortage by incrementing the capacity of power to be supplied by thermal generators.

In a similar manner, in [8] analytical expressions are developed to determine the feasible ranges of optimal solution, cumulative distribution function (CDF) of optimal solution, the optimal value and its average in the ED problem. In [9] analytical expressions for the solution of the ED problem incorporating the emissions due to oxides of nitrogen ($NO_x$) are presented. A probabilistic analysis of the cases at which wind power generation is overestimated and underestimated is carried out, obtaining the expected value of the increment in the generation cost due to the forecasting error. Roy [10] proposed an approach that considers the characteristics of the aggregated wind power generation during a short duration interval, solving the ED problem by avoiding stochastic relations. Under these assumptions, a mathematical model using a short duration turbulence index and the short-duration-stable mean wind speed is presented.

Optimization problems, such as the ED and UC, subject to the influence of uncertain wind power generation could be solved by employing probabilistic methods or scenario generation-based methods. Probabilistic methods present important barriers to their development, which are mainly related to the availability of accepted models and the reliability of the results provided by these methods. Moreover, scenario generation-based methods have found the importance and the acceptance of the research community and power systems industry because it allows the representation of the temporal correlation of the wind power production time series, which directly influences the reserve requirements [11, 12].

In this context, Aghaei et al. [13] proposed a methodology for solving the multi-objective dynamic economic emission dispatch (DEED) problem by incorporating the uncertainty of wind power production through the scenario generation and reduction approach. Then, the stochastic optimization problem is solved by using an enhanced particle swarm optimization (PSO) algorithm. Constantinescu et al. [14] have integrated a numerical weather prediction (NWP) model and stochastic UC problem to consider wind power uncertainty. Results shown the importance of using weather research forecast (WRF) models and uncertainty information in order to improve the wind power accommodation using the minimum amount of reserves.

This integrated model allows overcoming of the limitations of using artificial forecasts and uncertainty information. Tuohy et al. [15] developed a methodology using mixed integer stochastic optimization. In this approach, several scenarios of wind power generation, energy demand and forced unit outages are obtained by applying scenario generation and reduction. Then, rolling planning is employed to solve the stochastic optimization problem, rolling time is displaced among the different stages in order to take into account the variability and uncertainty introduced by wind power production, load demand and system reliability. Decisions are divided into "Here-and-now" decisions and "Wait-and-see" decisions—"Here-and-now" decisions are taken in the first stage under the perfect forecasting assumption, while "Wait-and-see" decisions are taken in remaining stages by considering the uncertainty.

Ruiz et al. [16] analyzed the integration of stochastic programming and reserve requirement specification as a means to compensate all the situations not taken into account by the scenario generation and reduction process, concluding that this approach could lead to a robust solution of the stochastic UC problem. However, another important conclusion is that the computational time required in this analysis is high, so that authors have suggested the application of decomposition techniques and changes in the optimization model in order to reduce the computational efforts.

Driven by these conclusions, in [17] two models have been proposed to change the optimization model in order to reduce the computational time. One consists of the relaxation of the integrality constraint of fast-start units, while another one consists of representing the failure event of each unit as an increment in the load demand. Conclusions have shown that these changes can reduce computational time by preserving the quality of the obtained solution.

Jiang et al. [18] have proposed a two-stage robust optimization model that assumes that the sources of uncertainty belong to a polyhedral uncertainty set, while the optimization problem is solved by using Benders' decomposition by including feasibility and optimality cuts. Wang et al. [19] have proposed an optimization model that incorporates sub-hourly variability and uncertainty of wind generation. The uncertainty of the UC problem is modeled by using stochastic mathematical formulation, while sub-hourly variability is modeled by using dispatch constraints. The computational time is reduced by using an improved Benders' decomposition algorithm.

In a similar manner, Zhao et al. [20] proposed a model that combines the advantages of stochastic UC and robust UC techniques. The unified approach developed by the authors overcomes the computational challenges related to the analysis of a large amount of scenarios required in stochastic programming and the conservativeness of the solution obtained from robust optimization. This approach incorporates weights in objective function so that the system operator can adjust these according to its necessities. Then, the optimization problem is solved by employing Benders' decomposition algorithm.

Ding et al. [21] have developed a methodology based on chance constraint programming, in which probabilistic constraints related to the wind power generation, load demand, system reliability, and energy prices are transformed into their equivalent deterministic ones. Then, this equivalent optimization problem is solved by using the Branch and Bound algorithm. Wang et al. [22] proposed a sample average approximation (SAA) algorithm that combines chance constraint programming and two-stage stochastic programming, each of which had been tested separately. This algorithm is composed of three parts:

scenario generation, convergence analysis, and solution validation. The optimization problem is solved by employing mixed integer linear programming (MILP).

Hargreaves et al. [23] have proposed a stochastic dynamic programming approach where load demand and wind power generation are discretized and represented by using a Markov transition matrix, while the size of the problem is reduced through a unit aggregation process. Finally, the optimization problem is represented as a two-stage one described by means of the recursive Bellman equation. In a similar manner, Luh et al. [24] have developed a methodology that represents wind power generation through a Markov transition matrix which has been adapted to capture rare events such as sudden changes in wind power production, while the optimization problem is solved employing the Branch and Cut algorithm.

Ji et al. [25] have presented a methodology that combines the scenario generation approach and quantum-inspired binary gravitational search algorithm (QBGSA) to solve the stochastic UC problem. In this approach, the decision process is carried out considering the security of the system, the generation costs, and the probability of occurrence of a determined wind power profile.

In this paper, a novel methodology to solve UC problem incorporating wind power variability and uncertainty is presented. The new proposed approach is based on scenario generation and reduction approach. Then, by a solution of a deterministic UC problem for each scenario, the PDF of commit a determined generator in a determined time is determined. In the next step, the definitive solution to the stochastic UC problem is carried out by selecting those generators with a probability of be committed higher than a predefined value. Finally, the obtained solution is probabilistically checked by evaluation of the selected UC solution, using the scenarios previously generated. The remaining of the paper is organized as follow: Section 2 describes the scenario generation process used, section 3 describes mathematical formulation of UC problem, section 4 presents the optimization method employed to solve UC problem, section 5 describes the proposed approach which is illustrated in section 6, while final conclusions are presented in section 7.

## 2. Scenario Generation Process

Recently, several methods for scenario generation and reduction have been developed. In [25] a methodology that combines Latin hypercube sampling (LHS) with Cholesky decomposition (LHS-CD) is proposed. In more detail, a joint PDF of wind power generation is modeled as a Gaussian one, assuming the forecasted values to be the mean values, while standard deviation depends on the forecasting error. Then, undesired correlations are reduced by means of the Cholesky decomposition method. In [26] a methodology was proposed that introduces forecasting error through empirical distributions, while assuming the PDF of wind power variability as a *t* location–scale distribution. Scenarios are generated by using an inverse transformation from the joint PDF, which is assumed to be a Gaussian-multivariate distribution.

The methodology used in this paper for the scenario generation is able to consider the most important features that describe the temporal behavior of the wind power time series, such as the autocorrelation that exists between consecutive observations, the hourly profile of the expected wind power production, and its corresponding forecasting error.

For the scenario generation, the first step consists in randomly generating a set of scenarios, taking into account the intrinsic autocorrelation of the hourly wind power production. Then in the second step, a sub-set of the scenarios previously generated is chosen according to the forecasting error. Finally, the scenarios to be used for the solution of the stochastic UC problem are selected by applying the k-means clustering algorithm to the set of scenarios obtained in the second step.

To reproduce the original forecasted wind power production, synthetically generated scenarios have to incorporate the correlated behavior of the wind power generation and its hourly profile. On the one hand, autocorrelation is introduced by generating a random series, assuming a first-order autoregressive Markov process according to equation (1):

$$x_m^t = \text{Ø} x_m^{t-1} + \epsilon \tag{1}$$

where $x_m^t$ is the time series which save the autocorrelation nature of the original wind power profile, index *m* refers to scenario generated (*m* = 1, 2, …, *M*) and index *t* refers to the time (*t* = 1, 2, …, *H*), Ø is the one-lag autocorrelation parameter, and  is a Gaussian white noise with mean zero and standard deviation of $\sqrt{1-\text{Ø}^2}$. On the other hand, the hourly wind power profile is introduced by normalizing the forecasted wind power production according to equation (2):

$$y^t = \frac{W^t - \mu}{\sigma} \tag{2}$$

where $y^t$ is the normalized wind power profile, $W^t$ is the time series of the total wind power generation, while $\mu$ and $\sigma$ are its mean and standard deviation, respectively.

Thus, a normalized time series of wind power generation that simultaneously incorporates the autocorrelation of the predicted wind power generation and its hourly profile is obtained with the addition of time series previously obtained in equations (1) and (2) [27]:

$$z_m^t = x_m^t + y^t \tag{3}$$

where $z_m^t$ is the normalized total wind power generation of scenario *m* at time *t*. Finally, total wind power generation ($W_m^t$) is obtained by applying the probability transformation described in equations (4) and (5) and Figure 1:

$$A(z_m^t) = h_m^t = A_W(W_m^t) \tag{4}$$

$$W_m^t = A_W^{-1}(A(z_m^t)) \tag{5}$$

where $A$ is the CDF of time series $z_m^t$ which has mean zero and standard deviation 1, and $A_W$ is the CDF of time series $W^t$. $A$ and $A_W$ are assumed to be Gaussian PDF. According to Figure 1, curve $A$ presented on the left side corresponds to the CDF of a normalized Gaussian PDF, which is the PDF of the time series obtained in equation (3), while curve $A_W$ presented on the right side corresponds to the CDF of the original predicted wind power profile modeled as a Gaussian PDF with mean $\mu$ and standard deviation $\sigma$. $h_m^t$ is an intermediate time series which has uniform PDF within the interval [0, 1] [28].

*"See Figure 1"*

Scenarios obtained from the implementation of the procedure described previously could lead to unrealistic situations, in which scenarios with extremely high or low values are obtained. To deal with this problem, an algorithm to select those scenarios with reliable values is introduced. Assuming a determined PDF for the hourly forecasting error, a determined value for the significance level ($\alpha$) is fixed and the corresponding confidence interval is calculated for each hour.

Then, a vector of $H$ binary elements ($F_m$) is created, as a storage vector if the corresponding scenario *m* at time *t* is within the corresponding confidence interval. In the case that $W_m^t$ is inside, the confidence interval value of one is assigned and if it is outside a value of zero is assigned. Once vector $F_m$ has been built for each scenario, an index ($I_m$) that reflects the degree at which the scenario under analysis (*m*) fulfills the hourly forecasting error is calculated. This index is defined according to equation (6)

$$I_m = \left( \sum_{t=1}^{t=H} F_m \right) \Big/ H \tag{6}$$

If $I_m$ is equal to one, it means that during all hours each value of scenario *m* is within the confidence interval. On the other hand, values of this index lower than 1 means that during some hours the scenario generated is out of the corresponding confidence interval. In the next step, by establishing a determined limit to this index ($\beta$) all scenarios that correspond to the specified forecasting error are selected. As an example, if a value of $\beta$ = 0.9 is chosen, those scenarios with $I_m$ higher than $\beta$ should be selected. Finally, the required scenarios to be used in the solution of the stochastic UC problem are found by applying the k-means clustering algorithm [29] on the set of scenarios previously selected by using the parameter $\beta$.

## 3. Unit Commitment Problem

In this section the mathematical formulation of the UC problem integrating the uncertainty related to the net load is presented. Net load is defined as the subtraction between load demand and wind power generation. Solving the stochastic UC problem consists of finding out the optimal combination of generators that should be committed and their corresponding power production in order to minimize the generation costs over the scheduling horizon, considering the possible fluctuations of the different sources of uncertainty (wind power generation and load demand, among others.). An important barrier to the successful solution of this optimization problem and the accommodation of wind power generation is the set of constraints that characterize the operation of the thermal generation units, such as generation limits, operating ramp rate constraints, start up and shut down ramp rate constraints, reserve constraints and minimum up and down time constraints.

### 3.1. Objective Function

As was stated before, UC is an optimization problem that consists of minimizing the expected operating cost. This cost could be divided into fuel-consumption cost and starting-up cost. Traditionally, fuel-consumption cost has been modeled by using a quadratic expression in terms of the corresponding power production, while starting-up cost has been modeled by using a piecewise expression that depends on the number of hours that a specific generator has been de-committed. The mathematical expression for generation cost is presented in equation (7):

$$f = \sum_{m=1}^{M} P_r\{m\} \left\{ \sum_{t=1}^{t=H} \sum_{n=1}^{n=N} a_n U_{n,m}^t + b_n P_{n,m}^t U_{n,m}^t + c_n (P_{n,m}^t)^2 U_{n,m}^t + SUC_n^t (1 - U_{n,m}^{t-1}) U_{n,m}^t \right\} \tag{7}$$

where $f$ is the expected value of total operating cost, $P_r\{m\}$ is the probability of occurrence of a determined scenario ($m$), and $P_{n,m}^t$ is the power production of generator $n$, at time $t$, and in scenario $m$. $U_{n,m}^t$ is a binary variable to represent if generator $n$, at time $t$, and in scenario $m$ is committed or de-committed, and $SUC_{n,m}^t$ is the starting-up cost of generator $n$, parameters $a_n$, $b_n$, and $c_n$ correspond to the fuel-consumption of generator $n$. The ED problem is solved by means of a quadratic programming approach, an approximation of the starting-up cost is presented in equation (8):

$$SUC_n^t = \begin{cases} HSU_n; & OFF_{n,m}^t \leq MDT_n + CST_n \\ CSU_n; & OFF_{n,m}^t > MDT_n + CST_n \end{cases} \tag{8}$$

where $HSU_{n,m}^t$ is hot startup cost, $CSU_{n,m}^t$ is cold startup cost, and $CST_{n,m}^t$ is cold startup time of generator $n$. $OFF_{n,m}^t$ is an integer variable that saves the cumulative account of the number of hours that generator *n* has been de-committed. In a similar manner, $ON_{n,m}^t$ saves the number of hours that generator *n* has been committed. The definition of these variables is presented in equations (9) and (10):

$$ON_{n,m}^t = \begin{cases} ON_{n,m}^{t-1} + 1; & U_{n,m}^t = 1 \\ 0; & U_{n,m}^t = 0 \end{cases} \tag{9}$$

$$OFF_{n,m}^t = \begin{cases} OFF_{n,m}^{t-1} + 1; & U_{n,m}^t = 0 \\ 0; & U_{n,m}^t = 1 \end{cases} \tag{10}$$

### 3.2. Generation Limit Constraints

If the generator *n* is committed, its power production should be limited by its minimum ($P_n^{min}$) and maximum ($P_n^{max}$) production. This is mathematically expressed in equation (11):

$$P_n^{min} \leq P_{n,m}^t \leq P_n^{max}; \; U_{n,m}^t = 1 \tag{11}$$

### 3.3. Operating Ramp Rate Constraints

Many of the technologies used nowadays have important limitations to change their power production suddenly. These limitations are expressed through the set of constrains of equations (12) and (13):

$$P_{n,m}^t - P_{n,m}^{t-1} \leq UR_n; \; U_{n,m}^t = 1 \; U_{n,m}^{t-1} = 1 \tag{12}$$

$$P_{n,m}^{t-1} - P_{n,m}^t \leq DR_n; \; U_{n,m}^t = 1 \; U_{n,m}^{t-1} = 1 \tag{13}$$

where $UR_n$ and $DR_n$ are the ramp up and ramp down rates of generator *n*.

### 3.4. Startup and Shutdown Ramp Rate Constraints

The effects of the ramping limitations during starting process are considered by inclusion of the constraints of equations (14) and (15) in the optimization problem:

$$P_{n,m}^t \leq SUR_n + P_n^{min}; \; U_{n,m}^t = 1 \; U_{n,m}^{t-1} = 0 \tag{14}$$

$$P_{n,m}^t \leq SDR_n + P_n^{min}; \; U_{n,m}^t = 1 \; U_{n,m}^{t+1} = 0 \tag{15}$$

where $SUR_n$ and $SDR_n$ are the startup and shutdown ramp rates.

### 3.5. Reserve Requirements Constraint

Reserve is a specification that allows system operator face unexpected situations and failure events; this specification is incorporated through the variable *SR* in the constraint (16):

$$\sum_{n=1}^{n=N} P_{n,m}^{t,max} U_{n,m}^{t} - \sum_{n=1}^{n=N} P_{n,m}^{t} U_{n,m}^{t} \geq (SR) L^{t}\ ;\ \ U_{n,m}^{t} = 1 \qquad (16)$$

where $L^t$ is the value of load demand at time $t$, and $P_{n,m}^{t,max}$ is maximum power that could be generated taking into account the effects of the ramp constraints.

### 3.6. Power Balance

This constraint guarantees the balance between total power production and its consumption. This idea is mathematically expressed in equation (17):

$$\sum_{n=1}^{n=N} P_{n,m}^{t} U_{n,m}^{t} + W_{m}^{t} = L^{t};\ \ U_{n,m}^{t} = 1 \qquad (17)$$

Note that wind power generation is assumed to be completely integrated, so that all wind power generation is assumed to be consumed.

### 3.7. Minimum Up/Down Time Constraint

Another important limitation of the generators used for electricity generation is that they have to be on-line for at least a determined number of hours. On the contrary, generation units have to be off-line for at least another determined number of hours. These required times are known as minimum up time ($MUT_n$) and minimum down time ($MDT_n$) of generator *n*. These constraints are presented in equations (18) and (19):

$$ON_{n,m}^{t} \geq MUT_n \qquad (18)$$

$$OFF_{n,m}^{t} \geq MDT_n \qquad (19)$$

## 4. Priority List Method to the Unit Scheduling

Among the methodologies developed to solve the UC problem, mixed integer linear programming (MILP) has been strongly accepted due to the fact that, in a determined number of steps, it is able to find solutions that are guaranteed to converge to the global-optimal solution [30]. However, recent papers have been found that under high integration of renewable resources, and consequently low values of net load, MILP method has difficulties finding a feasible solution in a reasonable computational time [31].

Moreover, Priority List (PL) is a methodology for solving the UC problem which is able to give a near-optimal solution in a reduced computational time. This method has undergone important developments. In [32] a stochastic PL method is introduced. In this approach, generators are committed according to a determined PDF that depends on the characteristics of the system under analysis. In [33] the PL method has been adapted to the management of power systems with ESS. In [34] the combination of an improved PL and an augmented Hopfield Lagrange (AHL) neural network was proposed. In [35] improved pre-prepared power demand (IPPD) was combined with the Muller method. In [36] a combination of improved Lagrangian relaxation (ILR) and augmented Lagrange Hopfield (ALH) embedded in the PL method has been proposed.

The PL method is composed of several processes that jointly arrive at a feasible and cost-effective solution to the UC problem. The processes involved are primary unit scheduling, minimum up/down time repairing, spinning reserve repairing, shutdown repairing, unit substitution, and shutdown excess of power generation. All these processes are detailed in the next sub-sections.

### 4.1. Primary Unit Scheduling

The order in which each generator is committed depends on its average production cost ($G_n$) which is defined according to equations (20) and (21) [34]:

$$G_n = \frac{a_n + b_n q_n + c_n (q_n)^2}{q_n} \tag{20}$$

$$q_n = \frac{P_n^{max}}{2}\left(1 + \frac{P_n^{min}}{P_n^{max}}\right) \tag{21}$$

where $q_n$ is the average power production of generator $n$. The procedure for developing a primary approximation to the solution is as follow:

- Step 1: Create the matrix of primary unit scheduling ($PUS_{n,m}^{t}$). Set $PUS_{n,m}^{t} = 0$, for $n = 1, 2, \ldots, N$ and $t = 1, 2, \ldots, H$.
- Step 2: Using the values obtained from equations (20) and (21), build the priority list.
- Step 3: Set $t \leftarrow 1$.
- Step 4: Select the first generator of the priority list built in Step 2, i.e. set $n \leftarrow 1$.
- Step 5: Set $PUS_{n,m}^{t} \leftarrow 1$.
- Step 6: If the committed capacity is not enough to fulfill the reserve requirements and $n \leq N$, set $n \leftarrow n + 1$ and go back to Step 5, else if $t \leq H$ set $t \leftarrow t + 1$ and go to Step 4; else stop.

### 4.2. Minimum Up/Down Time Repairing

The solution obtained from primary unit scheduling should fulfill minimum up/down time constraints. To solve this problem an additional process is applied. An example of the repairing process is shown in Figure 2 where the first approximation resulting from primary scheduling (mathematically modeled by the matrix $PUS_{n,m}^{t}$) is repaired by committing generator $n$ to two additional hours to fulfill the condition

$MUT_n$ = 3. In the contrary situation, Figure 3 shows the repairing process for the situation in which minimum down time constraint is violated, and the repairing algorithm commits generator $n$ during three hours in order to fulfill the condition $MDT_n$ = 4.

*"See Figure 2"*

*"See Figure 3"*

The algorithm to the minimum up/down time constraint presented in ref. [34] has been used in this paper; this algorithm consists of the next steps:

- Step 1: Using the results of primary unit scheduling, calculate $ON_{n,m}^{t}$ and $OFF_{n,m}^{t}$ matrices according to equations (9) and (10). Then, create the matrix scheduling for each scenario ($U_{n,m}^{t}$) and set it to $U_{n,m}^{t} = 0$.
- Step 2: Set $t \leftarrow 1$.
- Step 3: Set $n \leftarrow 1$.
- Step 4: If ($PUS_{n,m}^{t} = 0$) and ($PUS_{n,m}^{t-1} = 1$) and ($ON_{n,m}^{t-1} < MUT_n$), set $U_{n,m}^{t} \leftarrow 1$.
- Step 5: If ($PUS_{n,m}^{t} = 0$) and ($PUS_{n,m}^{t-1} = 1$) and ($t + MDT_n - 1 \leq H$) and ($OFF_{n,m}^{t+MDT_n-1} < MDT_n$), set $U_{n,m}^{t} \leftarrow 1$.
- Step 6: If ($PUS_{n,m}^{t} = 0$) and ($PUS_{n,m}^{t-1} = 1$) and ($t + MDT_n - 1 > H$) & ($\sum_{j=t}^{H} PUS_{n,m}^{j} > 0$), set $U_{n,m}^{t} \leftarrow 1$.
- Step 7: Calculate the elements of the matrices calculate $ON_{n,m}^{t}$ and $OFF_{n,m}^{t}$ that corresponds to generator $n$ using equations (9) and (10).
- Step 8: If $n < N$, set $n \leftarrow n + 1$ and go back to step 4.
- Step 9: If $t < H$, set $t \leftarrow t + 1$ and go back to Step 3, else stop.

### 4.3. Spinning Reserve Repairing

Total capacity generation of the system could be considerably reduced by incorporation of operating ramp rate constraints and startup and shutdown ramp rate constrains; as a consequence, these limitations reduce the spinning reserve estimated previously in the primary unit scheduling process. To deal with this problem, using the results obtained from the primary unit scheduling and minimum up/down time repairing processes, more generation capacity is committed following the next algorithm:

- Step 1: For each time instant ($t$ = 1, 2,…, $H$) the reserve requirements are checked by using equation (16).
- Step 2: Then, those hours at which spinning reserve requirements are insufficient are determined. These hours (in combination with the priority list) are used to determine those points ($n$, $t$ in $U_{n,m}^{t}$) at which generation capacity should be added. All these points are saved in a list of two columns; the first column saves the generators, while the second column saves the time intervals.
- Step 3: If the list created in Step 2 is not empty, go to Step 4, in other case stop.

- Step 4: Then, the list developed in Step 2 is sorted according to its second column in ascending order.
- Step 5: In this step, the first point of the sorted list developed in Step 4 is selected; the status of the generator *n* at hour *t* corresponding to this point is changed from 0 to 1.
- Step 6: Apply minimum up/down time repairing in order to avoid the violation of these constraints.
- Step 7: Go to Step 1.

### 4.4. Shutdown Repairing Process

In order to fulfill the shutdown ramp rate constraint, it is likely that additional hours be required so that the generator *n* may have enough time to be effectively de-committed. In order to overcome this situation those generators in problems are committed during more time in order to get the adequate level of generation. This is done following the next algorithm:

- Step 1: For each generator ($n$ = 1, 2, …, $N$) and time interval ($t$ = 1, 2, …, $H$), the shutdown ramp rate constraint is checked by application of equation (15).
- Step 2: Then, a list of all those points at which this constraint is violated is created. All those hours at which the operation of the corresponding generators should be extended are saved in a list of two columns; the first column saves the generators, while the second column saves the time intervals.
- Step 3: If the list created in Step 2 is not empty, go to Step 4, in other case stop.
- Step 4: Then, the list created in Step 2 is sorted according to its second column in ascending order.
- Step 5: In this step, the first point of the sorted list developed in Step 4 is selected; the status of the generator *n* at hour *t* corresponding to this point is changed from 0 to 1.
- Step 6: Apply minimum up/down time repairing in order to avoid the violation of these constraints.
- Step 7: Go to Step 1.

### 4.5. Unit Substitution Process

After the minimum up/down time repairing process has been carried out, some generators are committed during more hours than those required. This situation is illustrated in Figure 2, where generator *n* is required during only one hour; however, due to minimum up time constraint it is committed during three hours. In order to achieve a cost-effective scheduling, this generator with $MUT_n$ = 3 is substituted by another one with a lower $MUT_n$.

To recognize the generators under this situation, i.e., generators to be substituted, a matrix ($CH_{n,m}^{t}$) that store the changes in the primary scheduling owed to minimum up/down time repairing is created. This matrix is obtained by the subtraction of the matrices $U_{n,m}^{t}$ and $PUS_{n,m}^{t}$. The matrix $D_{n,m}^{t}$ is created to save the generators and the times at which they are going to be substituted. The elements of this matrix are binary so that $D_{n,m}^{t} = 1$ means that generator *n* should be substituted at hour *t*, while the contrary situation is represented by using $D_{n,m}^{t} = 0$.

Figure 4 extends the example previously described in Figure 2. In Figure 4, the row of generator *n* of the matrices $PUS_{n,m}^{t}$, $U_{n,m}^{t}$, $CH_{n,m}^{t}$, $ON_{n,m}^{t}$, and $D_{n,m}^{t}$ between $t = 1$ and $t = 7$ are shown. From the analysis of this figure, the reader can note that in $t = 3$, the matrix element $CH_{n,m}^{3} = 0$; this means that during the initial moment any change in the scheduling can be found. Otherwise, $ON_{n,m}^{3} = 1$ and $ON_{n,m}^{6} = 0$, which means that effectively generator *n* is committed only during its $MUT_n$, and $\sum_t^6 CH_{n,m}^{t} = 2 > 0$, which means that there is a change in the scheduling due to minimum up/down time repairing. As was stated before, $D_{n,m}^{t}$ indicates the generators and times to be used in the unit substitution process so that, for our example, the elements of $D_{n,m}^{t}$ become 1 between $t = 3$ and $t = 5$.

*"See Figure 4"*

From the analysis of this situation, an algorithm to recognize the generators that could be substituted and their corresponding times is presented as follow:

- Step 1: Estimate the matrix $CH_{n,m}^{t}$ as the subtraction between $U_{n,m}^{t}$ and $PUS_{n,m}^{t}$.
- Step 2: Create and initialize the matrix $D_{n,m}^{t}$ by assigning $D_{n,m}^{t} = 0$ for $n = 1, 2, \ldots, N$ and $t = 1, 2, \ldots, H$.
- Step 3: Set $n \leftarrow 1$.
- Step 4: Set $t \leftarrow 1$.
- Step 5: If $(CH_{n,m}^{t} = 0)$ and $(ON_{n,m}^{t} = 1)$ and $(t + MUT_n < H)$ and $(ON_{n,m}^{t+MUT_n} = 0)$ and $(MUT_n > 1)$ and $(\sum_t^{t+MUT_n-1} CH_{n,m}^{t} > 0)$, the elements of $D_{n,m}^{t}$ from $t$ to $t + MUT_n - 1$ become 1. Else if $(CH_{n,m}^{t} = 0)$ and $(ON_{n,m}^{t} = 1)$ and $(t + MUT_n - 1 = H)$ and $(ON_{n,m}^{t+MUT_n-1} = MUT_n)$ and $(MUT_n > 1)$ and $(\sum_t^{t+MUT_n-1} CH_{n,m}^{t} > 0)$, the elements of $D_{n,m}^{t}$ from $t$ to $t + MUT_n - 1$ become 1; else go to Step 6.
- Step 6: If $t < H$, set $t \leftarrow t + 1$ and go to Step 5, else go to Step 7.
- Step 7: If $n < N$, set $n \leftarrow n + 1$ and go to Step 4, else stop.

Once the matrix $D_{n,m}^{t}$ has been created, the generators to be substituted can be easily recognized. Then, considering one by one each of these generators, all processes described in the previous sections are repeated. If the substitution of a determined generator leads to an increment in the generation cost, the unit substitution process is stopped.

### 4.6. Shutdown Excess of Generation

Minimum up/down time repairing and spinning reserve repairing could lead to an excess of spinning reserve in some hours, which increase the generation costs. In order to get cost-effective unit scheduling, shutdown of excess of generation is carried out following the next algorithm:

- Step 1: Using equation (16), the excess of spinning reserve is checked over the entire horizon scheduling and a list is created by saving the corresponding hours. This list is assumed to have *R* elements.
- Step 2: Set $r \leftarrow 1$.
- Step 3: The point *r* of the list created in Step 1 is chosen. To this hour the most expensive generator is selected. Then, if $ON_{n,m}^{t}$ is higher than the corresponding $MUT_n$, the status of this generator is changed from 1 to 0.
- Step 4: Using the scheduling obtained in the Step 3, minimum up/down time repairing is carried out in order to get a feasible solution.
- Step 5: Using the scheduling obtained in the Step 4, startup/shutdown ramp rate constraints and spinning reserve requirements are checked by using equations (15) and (16), respectively. If both of these constraints are not violated, the element $U_{n,m}^{t}$ becomes 0, in other case it becomes 1.
- Step 6: If ($r<R$), set $r \leftarrow r+1$ and go back to Step 3, else stop.

## 5. Proposed Approach

The proposed approach consists of building the PDF of the situation at which a determined generator (*n*) be committed or not at a determined time (*t*). Then, those generators and hours (*n*, *t* in $U_{n,m}^{t}$) with a high probability of being committed are selected. However, the scheduling obtained from this procedure could be unfeasible due to the violation of minimum up/down time constraints, so that this solution is repaired by means of the corresponding process. The methodology proposed in this paper to the solution of stochastic UC problem is implemented by following the next algorithm:

- Step 1: In this step *M* scenarios of wind power production and load demand are built following the methodology presented in section 2.
- Step 2: Solve UC problem for each scenario (*m*) using the mathematical formulation presented in section 3 and the PL method described in section 4.
- Step 3: Estimate histogram of frequency of unit scheduling ($HF_{n,m}^{t}$) and its corresponding PDF ($PDF_{n}^{t}$) using equations (22) and (23). The matrices $HF_{n,m}^{t}$ and $PDF_{n}^{t}$ have the same dimensions that matrix $U_{n,m}^{t}$.

$$HF_{n}^{t} = \sum_{m=1}^{m=M} U_{n,m}^{t}\,; \qquad t = 1,2,\dots,H \tag{22}$$

$$PDF_{n}^{t} = \frac{HF_{n}^{t}}{M} \tag{23}$$

- Step 4: Create the probabilistic primary scheduling, which is a matrix ($PPUS_n^t$) with $N$ rows and $H$ columns. Set all elements of this matrix to zero ($PPUS_n^t = 0$, $n$ = 1, 2,…, $N$ and $t$ = 1, 2, …, $H$). Then, according to a determined significance level ($\alpha$), those generators and hours so that $PDF_n^t \geq \alpha$ are chosen and their status is changed from 0 to 1.
- Step 5: Solution obtained in Step 4 ($PPUS_n^t$) could be infeasible due to the violation of minimum up/down time constraint. For this reason minimum up/down time repairing is carried out, obtaining the solution to the stochastic UC problem $U_n^t$ (Note that variable $U_{n,m}^t$ represents the deterministic solution of UC problem for the scenario $m$, while $U_n^t$ represents the scheduling suggested to solve stochastic UC problem taking into account all scenarios previously generated).

## 6. Case Study

The proposed approach to the solution of the UC problem, incorporating the uncertainty related to wind power generation, is illustrated analyzing the power system whose characteristics are presented in Table 1 and Table 2, while Table 3 presents hourly load and wind power forecasting [25, 30, 33].

*"See Table 1"*

*"See Table 2"*

*"See Table 3"*

In our illustrative study case, spinning reserve requirements of 10% (*SR* = 0.1) have been considered in order to guarantee the power system reliability against any failure event. Results from the scenario generation and reduction process described in section 2 are shown in Figure 5. Initially, 2,000 scenarios were randomly generated. Thus, considering a forecasting error of 20%, $\alpha$ = 0.05 and $\beta$ = 0.05, 250 scenarios were used in the optimization process (*M*=250) obtained from the application of the k-means clustering algorithm.

*"See Figure 5"*

Table 4 presents PDF of unit scheduling ($PDF_n^t$) for the case under analysis, while Table 5 presents the average power production of each generator along the horizon of scheduling. In Table 4, the probability that corresponds to the selected scheduling is in bold, which are those generators and hours with probabilities higher than $\alpha$ = 0.05.

The reader can note how those generators that are in base and cycling condition are committed in all the scenarios and consequently they have the probability of being committed equal to 1. Moreover, peak units have a probability lower than 1 according to the requirements for supplied sudden changes in wind power generation. These results could be understood as those decision variables that correspond to stages 1 and 2 in the stochastic programming framework, i.e., those generators with probabilities equal to 1 could be understood as those generators to be committed before the uncertainty is realized, while those generators with probabilities lower than 1 could be understood as those generators whose decision of being committed is taken in stage 2 (fast start generators).

From these results, it is possible to observe how the proposed approach offers a probabilistic perspective of the role of each generation unit in the solution of the stochastic UC problem. The expected value of generation cost is $525,220.604. This value is higher than that obtained by evaluation of the scheduling suggested in [25], i.e. ($516,115.05). It is important to take into account that the mathematical formulation used here to check and measure the reserve requirements is different from that used in [25]; the formulation used in [25] is expressed in terms of $P_n^{max}$, while the expression used in this paper was carried out in terms of maximum power generation considering the ramp rate constraints (see equation (16)), which requires more generation capacity and consequently more generation costs.

*"See Table 4"*

*"See Table 5"*

Figure 6 presents the CDF for fulfilling the spinning reserve requirements for hours 1 and 17, which correspond to the situation of low load. For these hours, the specified spinning reserve requirements are guaranteed. On the other hand, Figure 7 shows the CDF for hours 12 and 20, each of which corresponds to the hours of high energy demand. For hour 12, all the generation capacity of the system has been committed, but the required reserve requirements cannot be totally guaranteed due to the effects of ramp rate constraints. This result shows the negative effects of the ramp rate constraints on the accommodation of wind power generation. However, for $t$ = 20 the committed specified reserve level can be guaranteed.

*"See Figure 6"*

*"See Figure 7"*

Table 6 shows the probability of obtaining a spinning reserve higher than 10% for the entire horizon scheduling. It could be noted that except for $t$ = 12 (which was discussed before), the probability of fulfilling this constraint is higher than 95%.

The proposed approach was implemented in MATLAB programming language. The computer used has an i7-3630QM CPU at 2.40 GHz with 8 GB of memory and 64 bit operating system. The computational time required to solve this illustrative example was 1403 s.

*"See Table 6"*

## 7. Conclusions

This paper presents a methodology for solving the UC problem to be applied in those systems with a high integration of renewable power sources. The proposed methodology consists of the generation of some representative scenarios which are selected considering the auto-correlated nature of wind power production, its hourly profile and its forecasting error. Then, the probability of occurrence of each scenario is estimated by solving the deterministic UC problem for each scenario previously generated. Finally, according to a determined probability level ($\alpha$), those hours with a probability of occurrence equal to or higher than $\alpha$ are selected and the minimum up/down time repairing is applied in order to obtain a feasible solution. The capabilities and performance of the proposed approach were illustrated through the analysis of a case study, where the spinning reserve requirements were probabilistically verified.

## Acknowledgements

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**Figure Captions**

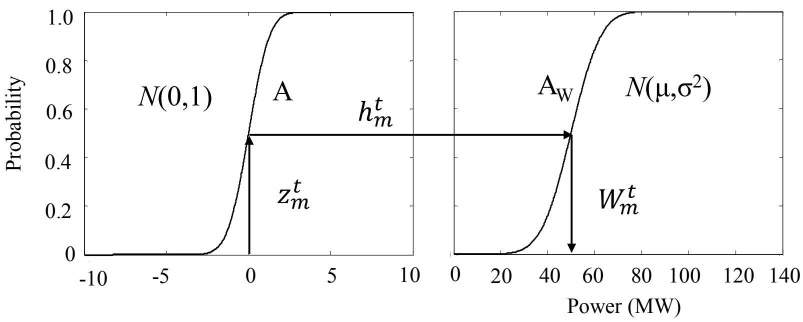


**Figure 1**
Probability transformation.

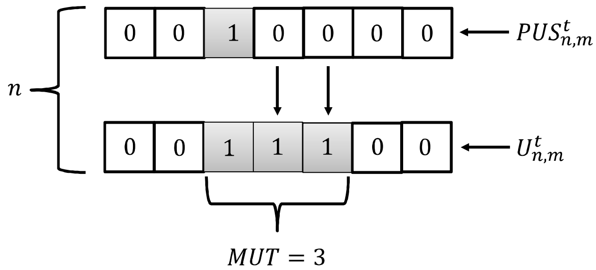


**Figure 2**
Repairing process of minimum up time constraint.

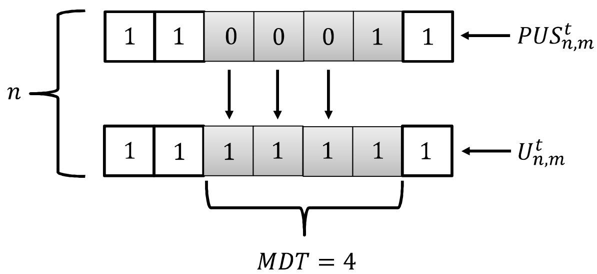


**Figure 3**
Repairing process of minimum down time constraint

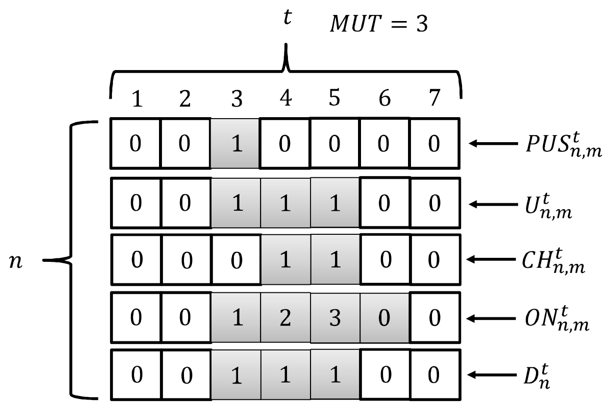


**Figure 4**

Selection of generators in unit substitution process.

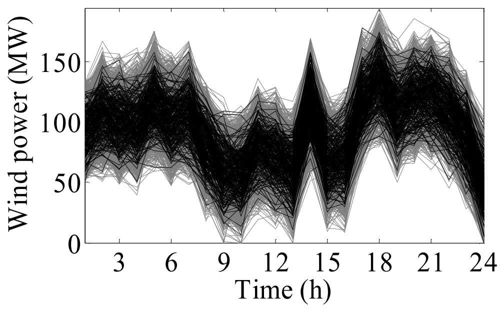


**Figure 5**

Results from the scenario generation and reduction process.

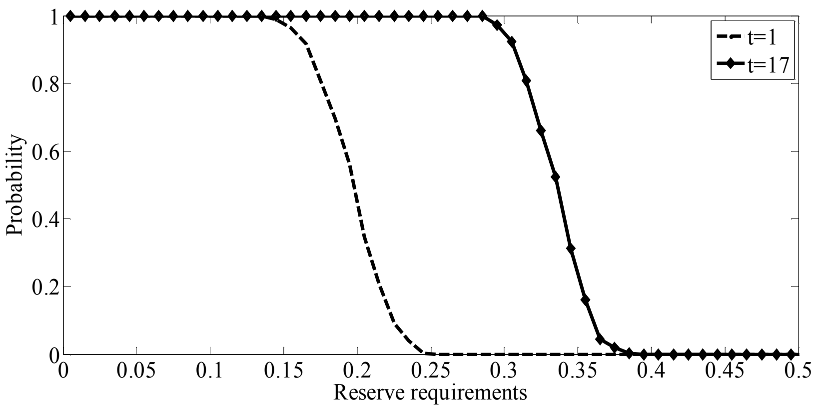


**Figure 6**

CDF of supply reserve requirements for $t = 1$ and $t = 17$.

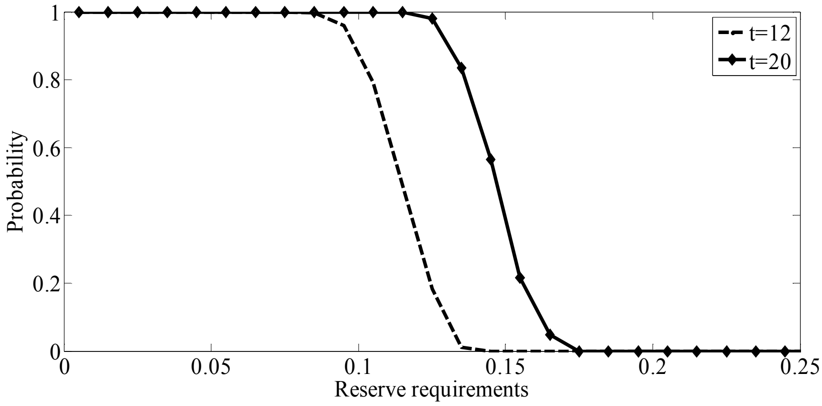


**Figure 7**

CDF of supply reserve requirements for $t = 12$ and $t = 20$.

**Table Captions**

**Table 1**
Description of the power system under analysis

| $n$ | $P_{min}^{n}$ (MW) | $P_{max}^{n}$ (MW) | $a_i$ ($/h) | $b_i$ ($/MWh) | $c_i$ ($/MW$^2$h) | $DR$ (MW/h) | $UR$ (MW/h) |
|---|---|---|---|---|---|---|---|
| 1 | 150 | 455 | 1000 | 16.19 | 0.00048 | 130 | 130 |
| 2 | 150 | 455 | 970 | 17.26 | 0.00031 | 130 | 130 |
| 3 | 25 | 162 | 450 | 19.7 | 0.00398 | 90 | 90 |
| 4 | 20 | 130 | 680 | 16.5 | 0.00211 | 60 | 60 |
| 5 | 20 | 130 | 700 | 16.6 | 0.002 | 60 | 60 |
| 6 | 20 | 80 | 370 | 22.26 | 0.00712 | 40 | 40 |
| 7 | 20 | 80 | 370 | 22.26 | 0.00712 | 40 | 40 |
| 8 | 25 | 85 | 480 | 27.74 | 0.00079 | 40 | 40 |
| 9 | 25 | 85 | 480 | 27.74 | 0.00079 | 40 | 40 |
| 10 | 10 | 55 | 660 | 25.92 | 0.00413 | 40 | 40 |

**Table 2**
Description of the power system under analysis (continued)

| $n$ | $P_0$ (MW) | $IS$ (h) | $MUT_n$ (h) | $MDT_n$ (h) | $CSC$ ($) | $HSC$ ($) | $CST$ (h) |
|---|---|---|---|---|---|---|---|
| 1 | 455 | 8 | 8 | 8 | 9000 | 4500 | 5 |
| 2 | 163 | 8 | 8 | 8 | 10000 | 5000 | 5 |
| 3 | 0 | -6 | 6 | 6 | 1800 | 900 | 4 |
| 4 | 0 | -5 | 5 | 5 | 1120 | 560 | 4 |
| 5 | 0 | -5 | 5 | 5 | 1100 | 550 | 4 |
| 6 | 0 | -3 | 3 | 3 | 340 | 170 | 2 |
| 7 | 0 | -3 | 3 | 3 | 340 | 170 | 2 |
| 8 | 0 | -3 | 3 | 3 | 520 | 260 | 2 |
| 9 | 0 | -3 | 3 | 3 | 520 | 260 | 2 |
| 10 | 0 | -1 | 1 | 1 | 60 | 30 | 0 |

**Table 3**
Load demand and wind power forecasting

| Time (h) | Wind (MW) | Load (MW) | Time (h) | Wind (MW) | Load (MW) |
|---|---|---|---|---|---|
| 1 | 93 | 700 | 13 | 60 | 1400 |
| 2 | 107 | 750 | 14 | 115 | 1300 |
| 3 | 100 | 850 | 15 | 68 | 1200 |
| 4 | 100 | 950 | 16 | 70 | 1050 |
| 5 | 117 | 1000 | 17 | 117 | 1000 |
| 6 | 103 | 1100 | 18 | 135 | 1100 |
| 7 | 108 | 1150 | 19 | 110 | 1200 |
| 8 | 80 | 1200 | 20 | 121 | 1400 |
| 9 | 60 | 1300 | 21 | 123 | 1300 |
| 10 | 57 | 1400 | 22 | 110 | 1100 |
| 11 | 78 | 1450 | 23 | 88 | 900 |
| 12 | 72 | 1500 | 24 | 47 | 800 |

**Table 4**
PDF of unit scheduling

| $n$ | Time (h) | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 1 | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** |
| 2 | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** |
| 3 | 0 | **0.06** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **0.30** | 0 |
| 4 | 0 | 0 | 0 | **0.08** | **0.08** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **0.98** | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | **0.56** | **0.68** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **0.24** | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0.15** | **1.00** | **1.00** | **1.00** | **1.00** | **1.00** | **0.20** | 0.01 | 0 | 0 | 0 | **0.94** | **0.98** | **0.98** | 0.04 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0.92** | **1.00** | **1.00** | **1.00** | **1.00** | 0 | 0 | 0 | 0 | 0 | 0 | 0.01 | 0.01 | 0.01 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.01 | **1.00** | **1.00** | **1.00** | **0.89** | 0 | 0 | 0 | 0 | 0 | 0.01 | **0.99** | **0.05** | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0.88** | **0.67** | **1.00** | 0.02 | 0 | 0 | 0 | 0 | 0 | 0 | **0.95** | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.01 | 0 | **0.91** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0.10** | 0 | 0 | 0 | 0 |

**Table 5**
Average power production (MW)

| *n* | Time (h) | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 1 | 449.8 | 453.2 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 | 455.0 |
| 2 | 157.2 | 165.6 | 269.3 | 291.1 | 275.3 | 308.6 | 303.2 | 360.1 | 447.7 | 455.0 | 455.0 | 455.0 | 455.0 | 417.5 | 391.7 | 262.1 | 154.9 | 225.3 | 329.5 | 450.1 | 407.7 | 351.3 | 331.7 | 297.5 |
| 3 | 0 | 25.0 | 25.3 | 25.0 | 25.0 | 25.0 | 25.0 | 25.0 | 32.1 | 106.2 | 135.7 | 161.4 | 114.8 | 32.0 | 25.0 | 25.0 | 25.0 | 25.0 | 25.4 | 63.6 | 25.0 | 25.0 | 25.0 | 0 |
| 4 | 0 | 0 | 0 | 80.0 | 130.0 | 130.0 | 130.0 | 130.0 | 130.0 | 130.0 | 130.0 | 130.0 | 130.0 | 130.0 | 130.0 | 125.8 | 128.5 | 130.0 | 130.0 | 130.0 | 130.0 | 80.0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 80.0 | 130.0 | 130.0 | 130.0 | 130.0 | 130.0 | 130.0 | 130.0 | 130.0 | 130.0 | 111.5 | 119.6 | 130.0 | 130.0 | 130.0 | 130.0 | 80.0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 20.0 | 20.0 | 21.1 | 20.4 | 40.7 | 20.2 | 20.0 | 0 | 0 | 0 | 0 | 20.0 | 20.0 | 20.0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 25.0 | 25.0 | 25.0 | 25.0 | 25.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 10.0 | 10.0 | 11.0 | 10.0 | 0 | 0 | 0 | 0 | 0 | 0 | 10.0 | 10.0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 10.0 | 10.0 | 10.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 10.0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 10.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 10.0 | 0 | 0 | 0 | 0 |

**Table 6**
Probability of supply the required reserve

| Time (h) | $P_r\{SR \geq 0.1\}$ | Time (h) | $P_r\{SR \geq 0.1\}$ |
|---|---|---|---|
| 1 | 1 | 13 | 0.974 |
| 2 | 1 | 14 | 1 |
| 3 | 0.954 | 15 | 0.954 |
| 4 | 1 | 16 | 1 |
| 5 | 1 | 17 | 1 |
| 6 | 1 | 18 | 0.986 |
| 7 | 1 | 19 | 0.956 |
| 8 | 1 | 20 | 1 |
| 9 | 0.96 | 21 | 1 |
| 10 | 0.956 | 22 | 1 |
| 11 | 0.96 | 23 | 1 |
| 12 | 0.876 | 24 | 1 |