

# Scheduling of head-dependent cascaded reservoirs considering discharge ramping constraints and start/stop of units

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## Abstract

This paper is on the problem of short-term hydro scheduling (STHS), particularly concerning head-dependent reservoirs under competitive environment. We propose a novel method, based on mixed-integer nonlinear programming (MINLP), for optimising power generation efficiency. This method considers hydroelectric power generation as a nonlinear function of water discharge and of the head. The main contribution of this paper is that discharge ramping constraints and start/stop of units are also considered, in order to obtain more realistic and feasible results. The proposed method has been applied successfully to solve two case studies based on Portuguese cascaded hydro systems, providing a higher profit at an acceptable computation time in comparison with classical optimisation methods based on mixed-integer linear programming (MILP).

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*Keywords:* Short-term hydro scheduling (STHS); Electricity market; Head dependence; Mixed-integer nonlinear programming (MINLP)

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## 1. Introduction

The satisfaction of the demand for electric energy has been mainly achieved with hydro resources and thermal resources. Hydro resources particularly run-of-the river resources are considered to provide a clean and environmentally friendly energy option, while thermal resources particularly fossil fuel-based resources are considered to provide an environmentally aggressive energy option [1]. Hence, promoting efficiency improvements in the exploitation of the hydro resources is increasingly important, reducing the reliance on fossil fuels and decreasing greenhouse emissions which are major contributors to climate change [2].

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The hydro scheduling problem is usually divided into different time horizons:

- Medium and long-term hydro scheduling, which encircle a time horizon of one or more years, discretised in weekly or monthly intervals. Stochastic models are used [3].
- Short-term hydro scheduling (STHS), which encircles a time horizon of one day to one week, usually discretised in hourly intervals. Deterministic models are used. Where stochastic quantities are included, such as hydro inflows or energy prices, the corresponding forecasts are used [4].

In a regulated environment, the main goal of the hydro scheduling problem is the minimisation of the deviation between total hydroelectric generation and electric energy demand, accomplishing the reservoir storage conditions at the beginning and at the end of the scheduling time horizon [5]. This problem could be a part of a traditional hydrothermal coordination problem, typically solved with methods based on decomposition approaches, determining the start-up and shut-down schedule of thermal plants, as well as the power output of thermal and hydro plants during the time horizon [6].

In a deregulated profit-based environment, such as the Norwegian case [7] or concerning Portugal and Spain given the forthcoming Iberian Electricity Market, the optimal management of the water available in the reservoirs for power generation, without affecting future operation use, represents a major advantage for generating companies (GENCOs) to face competitiveness given the economic stakes involved. The goal is to maximise the value of total hydroelectric power generation throughout the time horizon, satisfying all physical and operational constraints, and consequently to maximise the profit of the GENCO from selling energy into the electricity market [8]. Hence, the improvement of existing hydro scheduling models promoting a better exploitation efficiency of hydro resources is an important line of research [9].

The hydro generation characteristics are mainly assumed as linear or piecewise linear in hydro scheduling models, neglecting head variations. For long-term time horizons, the linearity assumption is reasonable, since errors introduced by this assumption are expected to be small compared to uncertainties with respect, for instance, to hydro inflow [10]. For a particular configuration of the hydro system, the linearity assumption may be acceptable or not for short-term time horizons depending on how important is the head variation over the time horizon.

In hydro plants with a large storage capacity available, as it is the case in the Brazilian system for instance, head variation has negligible influence on power generation efficiency in the short-term [11], and the linearity assumption is acceptable. In hydro plants with a small storage capacity available, also known as run-of-the-river hydro plants, the power generation efficiency can change significantly due to the head change effect. For instance, in the Portuguese system there are several hydro chains formed by many but small reservoirs. Hence, it is necessary to consider the head change effect on STHS in order to obtain more realistic and feasible results. The head change effect together with the cascaded hydro configuration, implying spatial-temporal coupling among reservoirs, increases the problem complexity.

Dynamic programming (DP) is among the earliest methods applied to the STHS problem [12]. Although, DP can handle the non-concavities and the nonlinear characteristics present in the hydro model, direct application of DP methods for hydro systems with many coupled plants is impractical due to the well-known DP curse of dimensionality, more difficult to avoid in short-term than in long-term optimisation without losing the accuracy needed in the model [13].

Neural networks [14], genetic algorithms [15] and other evolutionary algorithms such as particle swarm optimization [16–18] and differential evolution [19,20], have been recently applied for solving the STHS problem, giving good performance. Nevertheless, these methods typically depend on the expertise of the operator to properly calibrate the parameters.

The network flow technique is especially effective for solving problems associated with the mathematical modelling of hydro resources [21], because of the underlying network structure subjacent in cascaded reservoirs [22,23]. A set of cascaded reservoirs, each one having just one downstream neighbour, can be represented by a tree. Their nodes represent the reservoirs and their arcs represent the water releases. The replication of this tree for each period results in a particular network. The arcs connecting the trees represent the water stored in the reservoirs [24]. For cascaded hydro systems, as there are water linkage and electric connections among plants, the advantages of the network flow technique are salient: for instance, it is more difficult for other approaches to consider the water travel delay between reservoirs in a river effectively, especially when the river is branched [25].

The network flow model is often simplified to a linear or piecewise linear one. Linear programming (LP) is a widely used method for STHS [26]. LP algorithms lead to extremely efficient codes, implementations of which can be found commercially. Also, mixed-integer linear programming (MILP)

is becoming frequently used for STHS [27–31], where binary variables allow modelling of start-up costs and discrete hydro unit-commitment constraints. The number of start-ups of hydro units should be low, since frequent start-ups shorten the lifetime of the units as a result of mechanical stress. The start-up costs are mainly due to wear and tear of the windings and to malfunctions of the control equipment [32].

However, LP algorithms imply that power generation is linearly dependent on water discharge, thus neglecting head dependence to avoid nonlinearities, leading to inaccuracy. The discretisation of the nonlinear dependence between power generation, water discharge and head, used in MILP to model head variations, augment the computational burden required to solve this problem. Furthermore, methods based on successive linearization in an iterative scheme depend on the expertise of the operator to properly calibrate the parameters. For instance, the selection of the best under-relaxation factor in [5,30] is empiric and case-dependent, rendering some ambiguity to these methods.

Hydro scheduling is in nature a nonlinear optimisation problem. A nonlinear programming (NLP) method expresses hydro generation characteristics more accurately and the head change effect can be taken into account. Although there were considerable computational difficulties in the past to directly use NLP methods to this sort of problem [33,34], with the drastic advancement in computing power and the development of more effective nonlinear solvers in recent years this disadvantage is being overcome.

In earlier studies [2,8,35], the use of a NLP method in some case studies leads to a result that exceeds by at least three percent what is obtained by LP, requiring a negligible extra computation time. However, the nonlinear model cannot avoid water discharges at forbidden zones, and ignoring the start/stop of units may give schedules unacceptable from an operation point of view. Moreover, it is important to notice that a minor change in the energy price may give a significant change in the water discharge, and consequently in the power generation of plants. Therefore, ramp rate of water discharge should be included in the constraints to keep a lesser and steady head variation, which is particularly important for reservoirs with a task of navigation. These concerns lead to our novel method, based on mixed-integer nonlinear programming (MINLP), for solving the STHS problem.

The nonlinear dependence between the power generation, the water discharge and the head is taken into account in the novel mixed-integer nonlinear formulation. In our earlier formulation, the hydro unit commitment was not considered. As new contributions, discharge ramping constraints and start/stop of units are also considered in the novel formulation, in order to obtain more realistic and feasible results.

The paper is structured as follows. Section 2 provides the notation used throughout the paper along with the mathematical formulation of the STHS problem. Section 3 develops the proposed method for solving the STHS problem considering head dependence, discharge ramping constraints and start/stop of units. Section 4 presents two case studies, illustrating the numerical simulation results. Section 5 provides conclusions.

## 2. Problem formulation

### Notation

$K$	total number of hours in the scheduling time horizon.
$J$	total number of hydro resources.
$l_{kj}$	water level in reservoir $j$ during period $k$ .
$l_j^{\max}$	maximum water level in reservoir $j$ .
$l_j^{\min}$	minimum water level in reservoir $j$ .
$h_{kj}$	head of plant $j$ during period $k$ .
$h_j^{\max}$	maximum head of plant $j$ .
$h_j^{\min}$	minimum head of plant $j$ .
$v_{kj}$	water storage of reservoir $j$ at end of period $k$ .
$v_j^{\max}$	maximum storage of reservoir $j$ .
$v_j^{\min}$	minimum storage of reservoir $j$ .
$v_{0j}$	initial water storage of reservoir $j$ .
$v_{Kj}$	final water storage of reservoir $j$ .
$q_{kj}$	water discharge of plant $j$ during period $k$ .
$q_j^{\max}$	maximum water discharge of plant $j$ .
$q_j^{\min}$	minimum water discharge of plant $j$ .
$s_{kj}$	water spillage by reservoir $j$ during period $k$ .
$a_{kj}$	natural inflow to reservoir $j$ during period $k$ .
$p_{kj}$	power generation of plant $j$ during period $k$ .
$u_{kj}$	Commitment decision of plant $j$ during period $k$ .
$R_j$	discharge ramping limit of plant $j$ .
$\eta_{kj}$	efficiency of plant $j$ during period $k$ .
$\eta_j^{\max}$	maximum efficiency of plant $j$ .
$\eta_j^{\min}$	minimum efficiency of plant $j$ .
$\tau_{mj}$	water travel delay between reservoirs $m$ and $j$ .
$\lambda_k$	forecasted energy price during period $k$ .
$\Psi_j$	future value of the water stored in reservoir $j$ .
$M_j$	set of upstream reservoirs to reservoir $j$ .
$SU_j$	start-up cost of plant $j$ .

- $y_{kj}$  binary variable which is equal to 1 if plant  $j$  is started-up at beginning of period  $k$ .  
 $z_{kj}$  binary variable which is equal to 1 if plant  $j$  is shut-down at beginning of period  $k$ .  
 $F$  nonlinear function of variables.  
 $A$  constraint matrix.  
 $\mathbf{b}^{\max}$  upper bound vector on constraints.  
 $\mathbf{b}^{\min}$  lower bound vector on constraints.  
 $\mathbf{x}$  vector of decision variables.  
 $\mathbf{x}^{\max}$  upper bound vector on variables.  
 $\mathbf{x}^{\min}$  lower bound vector on variables.

The STHS problem is formulated as a MINLP problem. The objective function to be maximised can be expressed as:

$$F = \sum_{k=1}^K \sum_{j=1}^J (\lambda_k p_{kj} - SU_j y_{kj}) + \sum_{j=1}^J \Psi_j (v_{Kj}) \quad (1)$$

The first term in (1) represents the profit with the hydro system during the short-term time horizon, where  $\lambda_k$  is the forecasted energy price during period  $k$  and  $p_{kj}$  is the power generation of plant  $j$  during period  $k$ . The second term represents the start-up costs,  $SU_j$ , which is a new contribution to earlier studies [2,8,35]. The last term expresses the water value,  $\Psi_j$ , for the future use of the water stored in the reservoirs at the last period,  $v_{Kj}$ . A representation when this term is explicitly taken into account can be seen in [36]. The storage targets for the short-term time horizon, which are established by medium-term planning studies, may be represented either by a penalty on water storage or by a previously determined ‘future cost function’.

The optimal value of the objective function is determined subject to constraints: equality constraints and inequality constraints or simple bounds on the variables. The following equations represent the set of constraints for the plants over the short-term time horizon.

1) *Water Balance Equation:*

$$v_{kj} = v_{k-1,j} + a_{kj} + \sum_{m \in M_j} (q_{k-\tau_{m,j},m} + s_{k-\tau_{m,j},m}) - q_{kj} - s_{kj}; \quad \forall k \in K, \quad \forall j \in J \quad (2)$$

2) *Power Generation Equation:*

$$p_{kj} = q_{kj} \eta_{kj} (h_{kj}); \quad \forall k \in K, \quad \forall j \in J \quad (3)$$

3) *Head Equation:*

$$h_{kj} = l_{kf(j)}(v_{kf(j)}) - l_{kt(j)}(v_{kt(j)}); \quad \forall k \in K, \quad \forall j \in J \quad (4)$$

4) *Water Storage Constraints:*

$$v_j^{\min} \leq v_{kj} \leq v_j^{\max}; \quad \forall k \in K, \quad \forall j \in J \quad (5)$$

5) *Water Discharge Constraints:*

$$u_{kj} q_j^{\min} \leq q_{kj} \leq u_{kj} q_j^{\max}; \quad \forall k \in K, \quad \forall j \in J \quad (6)$$

6) *Discharge Ramping:*

$$q_{kj} - R_j \leq q_{k+1,j} \leq q_{kj} + R_j; \quad \forall k \in K, \quad \forall j \in J \quad (7)$$

7) *Water Spillage Constraints:*

$$s_{kj} \geq 0; \quad \forall k \in K, \quad \forall j \in J \quad (8)$$

8) *Logical Status of Commitment:*

$$y_{kj} - z_{kj} = u_{kj} - u_{k-1,j}; \quad \forall k \in K, \quad \forall j \in J \quad (9)$$

Eq. (2) corresponds to the water conservation equation, where  $v_{kj}$  is the water storage of reservoir  $j$  at end of period  $k$ ,  $a_{kj}$  is the natural inflow to reservoir  $j$  during period  $k$ ,  $q_{kj}$  is the water discharge of plant  $j$  during period  $k$ ,  $s_{kj}$  is the water spillage by reservoir  $j$  during period  $k$ ,  $\tau_{mj}$  is the water travel delay between reservoirs  $m$  and  $j$ ,  $K$  is the total number of hours in the scheduling time horizon,  $J$  is the total number of hydro resources and  $M_j$  is the set of upstream reservoirs to reservoir  $j$ . The travel time between reservoirs must be taken into account if the transportation delays are not negligible. Time-delay is a difficult issue, depending on the distance between the reservoirs and on the water discharge, deserving particular attention and research. Time-delay can be accounted for by considering a different model structure for different flow levels in an iterative procedure, which is outside the scope of this paper. Also, in our case studies, the time required for water to travel from a reservoir to a reservoir directly downstream is considered less than the one hour period, independently of water discharge, due to the

small distance between consecutive reservoirs. In (3) power generation,  $p_{kj}$ , is considered a function of water discharge,  $q_{kj}$ , and of efficiency,  $\eta_{kj}$ , expressed as the output-input ratio, which in turn depends on the head,  $h_{kj}$ . Hence, the power output of a hydro plant depends on the water discharge, the head, and the efficiency. The operating points are restricted by minimal and maximal water discharges [37]. In (4) the head,  $h_{kj}$ , is considered a function of the water level in the upstream reservoir  $f(j)$ ,  $l_{kf(j)}$ , and of the water level in the downstream reservoir  $t(j)$ ,  $l_{kt(j)}$ , both levels depending on the water storages in the respectively reservoirs. Typically for a powerhouse with a reaction turbine, where the tail water elevation is not constant, the head is modelled as in (4), and for a powerhouse with an impulse turbine, where the tail water elevation remains constant, the head depends only on the upstream reservoir water level, as in [30]. Hence, tailrace effects can be considered by including a correction in the data regarding reservoir water levels. In (5) water storage has lower and upper bounds. Here, for each reservoir  $j$ ,  $v_j^{\min}$  is the minimum storage capacity and  $v_j^{\max}$  is the maximum storage capacity. In (6) water discharge has lower and upper bounds. Here, for each plant  $j$ ,  $q_j^{\min}$  is the minimum water discharge and  $q_j^{\max}$  is the maximum water discharge. The maximum water discharge may be considered a function of the head, as in [2]. As a new contribution to earlier studies [2,8,35], the commitment decision of each hydro plant is ascertained. Hence, the binary variable,  $u_{kj}$ , is equal to 1 if plant  $j$  is on-line in period  $k$ , otherwise is equal to 0. In (7) water discharge has ramping limit, which is also a new contribution to earlier studies [2,8,35]. Discharge ramping constraints may be imposed due to navigation, recreational or ecological requirements [31]. In (8) a null lower bound is considered for water spillage. Water spillage by the reservoirs can occur only in normal schedule situations when without it the water storage exceeds its upper bound, so spilling is necessary due to safety considerations. The spillage effects were considered in [38]. The initial water storages,  $v_{0j}$ , and the inflows to reservoirs,  $a_{kj}$ , are assumed as known input data. Eq. (9) is necessary to model the start-up and shut-down status of the plants. Although variables  $z_{kj}$  may seem superfluous since they only appear in (9), extensive numerical simulations have proven their ability in considerably reducing computation time [28].



### 3. The proposed method

In order to solve the STHS problem, it is essential to use appropriate models, considering power generation as a function of water discharge and also of the head for run-of-the-river hydro plants. The main contribution of this paper is that discharge ramping constraints and start/stop of units are also considered, in order to obtain more realistic and feasible results.

The STHS problem can be formulated as the following mixed-integer nonlinear optimisation problem:

$$\text{Max } F(\mathbf{x}) \quad (10)$$

Subject to:

$$\mathbf{b}^{\min} \leq \mathbf{A} \mathbf{x} \leq \mathbf{b}^{\max} \quad (11)$$

$$\mathbf{x}^{\min} \leq \mathbf{x} \leq \mathbf{x}^{\max} \quad (12)$$

$$\mathbf{x}_i \text{ integer}; \quad i \in I \quad (13)$$

where  $F(\cdot)$  is a nonlinear function of the vector  $\mathbf{x}$  of decision variables,  $\mathbf{A}$  is the constraint matrix,  $\mathbf{b}^{\max}$  is the upper bound vector on constraints,  $\mathbf{b}^{\min}$  is the lower bound vector on constraints,  $\mathbf{x}^{\max}$  is the upper bound vector on variables and  $\mathbf{x}^{\min}$  is the lower bound vector on variables. Equality constraints are defined by setting the lower bound equal to the upper bound, i.e.  $\mathbf{b}^{\min} = \mathbf{b}^{\max}$ . The variables  $\mathbf{x}_i$  are restricted to be integers. The lower and upper bounds for water discharge imply new inequality constraints that will be rewritten into (11).

In (3) the efficiency depends on the head. We consider it given by:

$$\eta_{kj} = \eta_j^0 + \alpha_j h_{kj}; \quad \forall k \in K, \quad \forall j \in J \quad (14)$$

where the parameters  $\eta_j^0$  and  $\alpha_j$  are respectively the offset and the slope given by:

$$\eta_j^0 = \eta_j^{\max} - \alpha_j h_j^{\max}; \quad \forall j \in J \quad (15)$$

$$\alpha_j = (\eta_j^{\max} - \eta_j^{\min}) / (h_j^{\max} - h_j^{\min}); \quad \forall j \in J \quad (16)$$

In (16) parameter  $\alpha_j$  depends on the extreme values for efficiency and head, where  $\eta_j^{\max}$  is the maximum efficiency,  $\eta_j^{\min}$  is the minimum efficiency,  $h_j^{\max}$  is the maximum head and  $h_j^{\min}$  is the minimum head.

In (4) the water level depends on the water storage. We assume it given by:

$$l_{kj} = l_j^0 + \beta_j v_{kj}; \quad \forall k \in K, \quad \forall j \in J \quad (17)$$

where the parameters  $l_j^0$  and  $\beta_j$  are respectively the offset and the slope given by:

$$l_j^0 = l_j^{\max} - \beta_j v_j^{\max}; \quad \forall j \in J \quad (18)$$

$$\beta_j = (l_j^{\max} - l_j^{\min}) / (v_j^{\max} - v_j^{\min}); \quad \forall j \in J \quad (19)$$

this assumption implies reservoirs with vertical walls, which is a good approximation for run-of-the-river reservoirs, due to its small storage capacity, as our data have shown for the case studies.

In (19) parameter  $\beta_j$  depends on the extreme values for water level and storage, where  $l_j^{\max}$  is the maximum water level,  $l_j^{\min}$  is the minimum water level,  $v_j^{\max}$  is the maximum storage and  $v_j^{\min}$  is the minimum storage.

Substituting (14) into (3) we have:

$$p_{kj} = q_{kj} (\eta_j^0 + \alpha_j h_{kj}); \quad \forall k \in K, \quad \forall j \in J \quad (20)$$

By substituting (4) and (17) into (20) power generation becomes a nonlinear function of water discharge and water storage, given by:

$$p_{kj} = \eta_j^0 q_{kj} + \alpha_j l_{f(j)}^0 q_{kj} - \alpha_j l_{t(j)}^0 q_{kj} + \alpha_j \beta_{f(j)} q_{kj} v_{kf(j)} - \alpha_j \beta_{t(j)} q_{kj} v_{kt(j)}; \quad \forall k \in K, \quad \forall j \in J \quad (21)$$

The parameters given by the product of  $\alpha$ 's by  $\beta$ 's are of crucial importance for the behaviour of head-dependent reservoirs in a cascaded hydro system, setting optimal reservoirs storage trajectories in accordance to their relative position in the cascade. It should be noted that these parameters are not related to the solution procedure. Instead, they are determined only by physical data defining the hydro system [8].

A major advantage of our method is to consider the head change effect in a single function of water discharge and water storage, (21), which can be used in a straightforward way, instead of deriving several curves for different heads.

As new contributions to earlier studies [2,8,35], we report the consideration of discharge ramping constraints and start/stop of units. Therefore, more realistic and feasible results are attainable using the proposed method, based on MINLP. We consider for the optimisation procedure a starting point given by the solution of an MILP problem and using the proposed method in our case studies we always arrive at convergence to a superior solution.

#### 4. Case studies

The proposed method has been initially applied on a case study consisting of three head-dependent cascaded reservoirs, based on a Portuguese cascaded hydro system. The spatial coupling among reservoirs is shown in Fig. 1.

"See Fig. 1 at the end of the manuscript".

Only the first reservoir has inflow. This inflow is due to an upstream watershed belonging to a different company and is shown in Fig. 2.

"See Fig. 2 at the end of the manuscript".

Our model was implemented on a 600-MHz-based processor with 256 MB of RAM using the optimisation solver package Xpress-MP under MATLAB. The scheduling time horizon chosen is one week divided into 168 hourly periods.

The energy price profile over the time horizon is shown in Fig. 3 (where \$ is a symbolic economic quantity).

"See Fig. 3 at the end of the manuscript".

In restructured markets, price forecasting is extremely important for all market players [39,40]. An accurate forecast of energy prices has become a very important tool for a GENCO to develop an appropriate bidding strategy in the market and to optimally schedule its hydro resources. Several techniques have been tried out for energy prices forecasting [41], mainly based on time series models [42,43], or on artificial intelligence techniques [44–46]. These energy prices are considered as deterministic input data for the STHS problem.

The final water storage in reservoirs is constrained to be equal to the value at the beginning of the scheduling time horizon. Hence, the future value of the water stored in reservoirs is not considered. The cost of a hydro unit start-up has been estimated as \$2.5 per MW unit nominal output, as in [30]. Also, we consider forbidden zones for the hydro units. These zones result from mechanical vibrations, cavitation, and low efficiency level [47].

A comparison of MINLP with MILP results is presented thereafter. The computed 168-hours optimal reservoir storage trajectories are shown in Fig. 4.

"See Fig. 4 at the end of the manuscript".

Considering the head change effect, the reservoirs should operate at an appropriated high storage level in order to achieve the most benefiting point of the overall efficiency for the conversion of potential energy of the water into electric energy. The storage trajectories of the first and second reservoirs are pulled up, opposing to the change in the storage trajectory of the third reservoir. Nevertheless, due to the constraint on final water storage, the storage trajectory of the third reservoir is pulled up near the final hours of the time horizon, implying a decrease on the storage trajectory of the second reservoir. This behaviour is in favour of the overall power generation efficiency thereby yielding an increase on total profit for the GENCO.

In Fig. 5 the computed 168-hours optimal plant discharge trajectories are shown.

"See Fig. 5 at the end of the manuscript".

The water discharge and consequently the hydro production tend to follow the shape of the price profile in Fig. 3. As new contributions to earlier studies [2,8,35], we report the consideration of discharge ramping constraints and start/stop of units. Including start-up costs in the objective function implies a different behaviour of the reservoirs: once a hydro unit is committed, it tends to remain on-line during more hours, avoiding frequent start-ups. This behaviour is possible to be observed for the first reservoir, at a neighbourhood of 132h, and for the second reservoir, at a neighbourhood of 156h. Also, ramp rate of water discharge is included in the constraints. Therefore, more realistic and feasible results are attainable using the proposed method, based on MINLP.

Fig. 6 shows the power generation per water discharge at each plant.

"See Fig. 6 at the end of the manuscript".

The comparison of MINLP with MILP results is in favour of the proposed method, achieving a higher total profit with an increase of about 4% as shown in the Table 1.

"See Table 1 at the end of the manuscript".

The computation time for this first case study was about 107 s. Although the computation time may not be considered negligible, it is perfectly acceptable for the scheduling time horizon assumed. Without considering discharge ramping constraints and start/stop of units, the profit would be slightly higher, about 5,491 k\$. Moreover, the computation time would be significantly lower, about 4 s. However, these results would be less realistic.

Also, it is important to notice that if head variations were modelled in MILP the computational burden would increase significantly. For instance, the optimal solution reported in [28] required 22 minutes of computation time, on a 400-MHz-based processor with 500 MB of RAM. A major advantage of our method is to consider the head change effect in a single function of water discharge and water storage, which can be used in a straightforward way, instead of deriving several curves for different heads. Hence, the proposed method is not only more accurate but also computationally acceptable, considering head dependence, discharge ramping constraints and start/stop of units.

The proposed method has also been applied on a case study consisting of seven cascaded reservoirs, based on one of the main Portuguese cascaded hydro systems. The spatial coupling among reservoirs and the hydro data is given in [35]. The scheduling time horizon chosen is one day divided into 24 hourly periods, corresponding to a day-ahead electricity market.

The comparison of MINLP with MILP results is again in favour of the proposed method, achieving a higher total profit with an increase of about 4% as shown in the Table 2.

"See Table 2 at the end of the manuscript".

The computation time for this second case study was about 9 s. Without considering discharge ramping constraints and start/stop of units, the profit would be slightly higher, about 751 k\$. Moreover, the computation time would be significantly lower, about 3 s. However, these results would be again less realistic.

Hence, the proposed method based on MINLP has been successfully tested on two case studies based on Portuguese cascaded hydro systems, providing a higher profit in comparison with classical optimisation methods based on MILP. Moreover, the computation time is perfectly acceptable.

## 5. Conclusion

The new environment of competitive electricity markets for energy requires new computing tools to allow generating companies to achieve a better short-term hydro schedule, which is crucial to face competitiveness. A generating company should not ignore the head change effect for head-dependent cascaded reservoirs in order to improve power generation efficiency. Also, ignoring discharge ramping constraints may allow sudden head changes, and ignoring the start/stop of units may give schedules unacceptable from an operation point of view. This paper proposes a novel method, based on mixed-integer nonlinear programming, for scheduling head-dependent cascaded reservoirs. As new contributions to earlier studies, we report the consideration of discharge ramping constraints and start/stop of units. The proposed method has been successfully tested on two case studies based on Portuguese cascaded hydro systems, providing a higher profit in comparison with classical optimisation methods based on mixed-integer linear programming. Although the computation time may not be considered negligible, it is perfectly acceptable. The increased modelling accuracy is very significant, using the proposed method, which implies more realistic and feasible results in comparison with earlier methods. Hence, the proposed method is not only more accurate but also computationally acceptable, providing a novel and better approach to optimise power generation efficiency for head-dependent cascaded reservoirs.

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## Figure captions

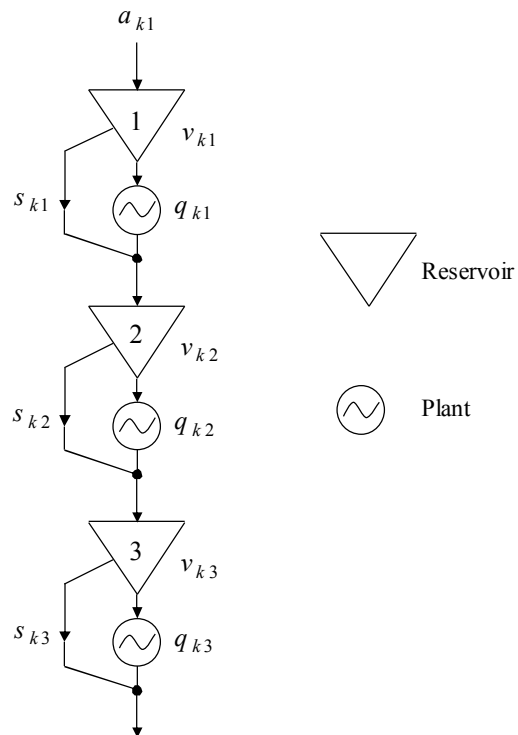


Fig. 1. Hydro system with three cascaded reservoirs.

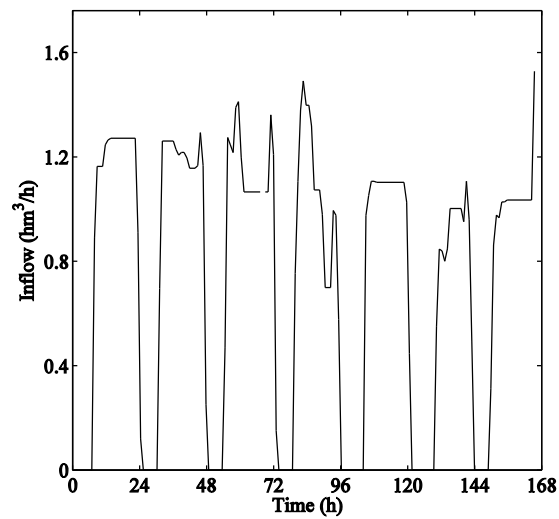


Fig. 2. Inflow on the first reservoir.

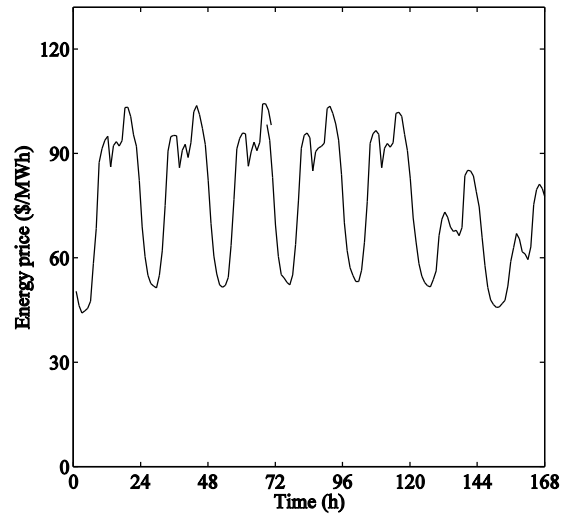


Fig. 3. Energy price profile considered.

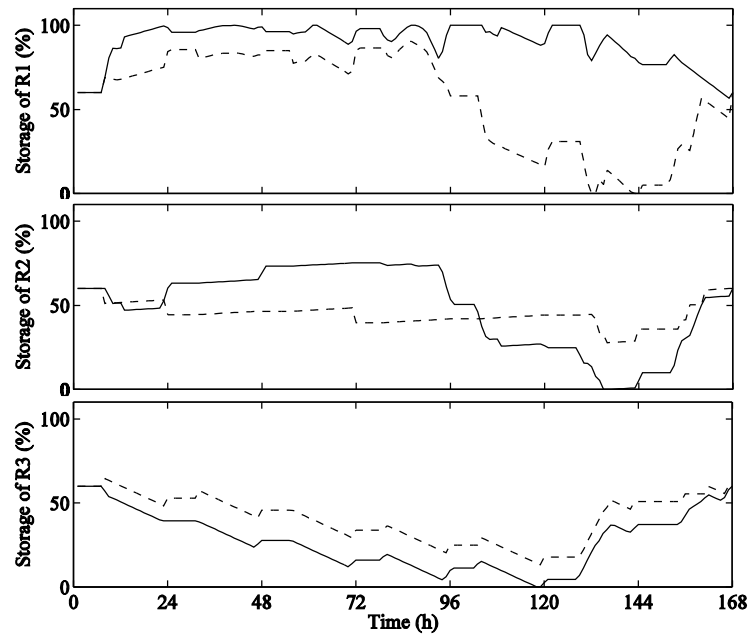


Fig. 4. Optimal reservoir storage trajectories. The solid lines denote MINLP results, while the dashed lines denote MILP results.

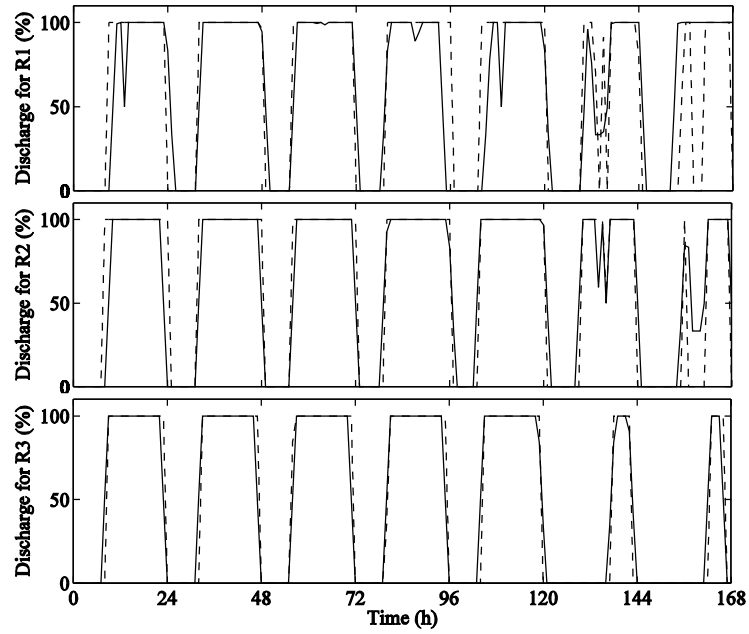


Fig. 5. Optimal plant discharge trajectories. The solid lines denote MINLP results, while the dashed lines denote MILP results.

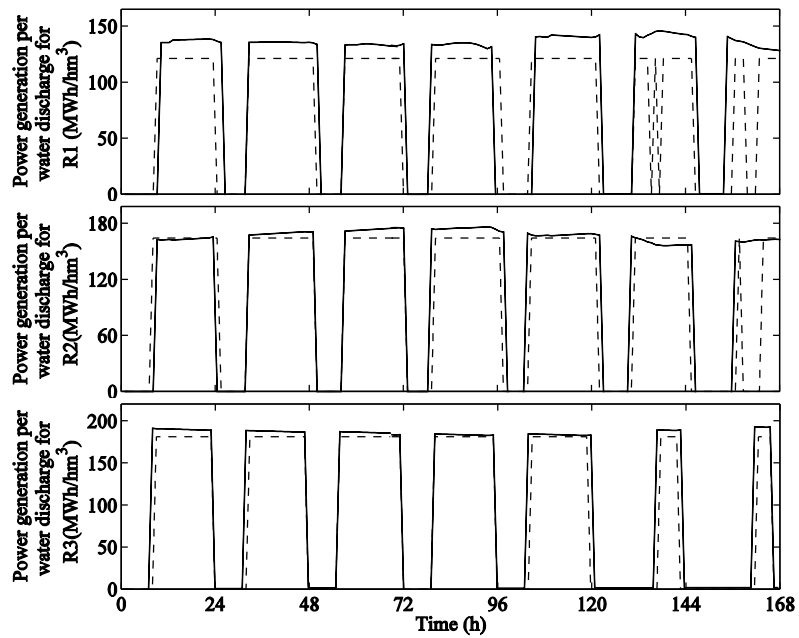


Fig. 6. Power generation per water discharge at each plant. The solid lines denote MINLP results, while the dashed lines denote MILP results.

**Tables**

Table 1

Comparison of MILP with MINLP considering three cascaded reservoirs and 168 h time horizon

Method	Average Discharge (%)	Average Storage (%)	Profit (k\$)	% Increase	CPU time (s)
MILP	58.47	47.34	5,258	-	2.16
MINLP	58.47	55.47	5,466	3.96	106.62

Table 2

Comparison of MILP with MINLP considering seven cascaded reservoirs and 24 h time horizon

Method	Average Discharge (%)	Average Storage (%)	Profit (k\$)	% Increase	CPU time (s)
MILP	25.00	83.08	714	-	1.75
MINLP	25.00	83.86	745	4.34	9.26