

# A practical approach for profit-based unit commitment with emission limitations

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## Abstract

This paper proposes a practical approach for profit-based unit commitment (PBUC) with emission limitations. Under deregulation, unit commitment has evolved from a minimum-cost optimisation problem to a profit-based optimisation problem. However, as a consequence of growing environmental concern, the impact of fossil-fuelled power plants must be considered, giving rise to emission limitations. The simultaneous address of the profit with the emission is taken into account in our practical approach by a multiobjective optimisation (MO) problem. Hence, trade-off curves between profit and emission are obtained for different energy price profiles, in a way to aid decision-makers concerning emission allowance trading. Moreover, a new parameter is presented, ratio of change, and the corresponding gradient angle, enabling the proper selection of a compromise commitment for the units. A case study based on the standard IEEE 30-bus system is presented to illustrate the proficiency of our practical approach for the new competitive and environmentally constrained electricity supply industry.

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*Keywords:* Profit-based unit commitment (PBUC); Electricity market; Emission limitations; Multiobjective optimisation (MO)

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## 1. Introduction

Energy conversion from fossil fuels into electric energy provides the backbone of the electricity supply industry worldwide. Fossil fuels provide a reliable and affordable source of energy. The technology for exploitation of fossil fuels is well developed and available in virtually every country of the world. However, one of the main contributions to the emission of greenhouse gases into the atmosphere, which is thought to be responsible for climate change on our environment, is through the use of fossil-fuelled power plants [1].

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It is now recognised that the greenhouse effect can be slowed down only if the emission of carbon dioxide and other greenhouse gases is reduced drastically. A major step in this direction is the Kyoto Protocol, an international treaty and an agreement under which industrialized countries will reduce their collective emissions of greenhouse gases by 5% over the five-year period of 2008–2012 compared to the year 1990. For the European Union (EU) the Kyoto target is an 8% reduction.

In December 2002, the EU created an emissions trading scheme (ETS) in an effort to meet the Kyoto targets. Quotas were introduced in six key industries: energy, steel, cement, glass, brick making, and paper/cardboard. There are also fines for member nations that fail to meet their obligations, starting at 40 €/ton CO<sub>2</sub> in 2005 and rising to 100 €/ton CO<sub>2</sub> in 2008. Also, the new March 2006 European green paper on a “secure, competitive and sustainable energy policy for Europe” will try to make the case for greater integration and cooperation of EU energy policies [2].

An unprecedented change is bound to occur in the new competitive and environmentally constrained electricity supply industry, where the role of the traditional coal-fired power plant is likely to change. Coal is by far the most abundant and cheapest fossil fuel with sufficient resources to sustain our long run needs for energy during centuries, but the combustion of coal in power plants discharges significant quantities of ash, nitrogen, sulphur oxides, mercury and greenhouse gases into the atmosphere.

In the old carbon unconstrained electricity supply industry, coal-fired power plants achieved a superior merit order, due to lower fuel costs, although posing a higher impact on the environment. In the presence of emission allowances, coal-fired power plants may move down in the merit order, due to higher carbon emission intensity. They will run less than it was normal in the old carbon unconstrained case. Hence, natural gas-fired power plants in combined cycle configuration, or even the new promising technology for coal power plants with zero emissions, will go up in the merit order. Gas-fired power plants will need less emission allowances than coal-fired power plants, resulting in a tendency for a shift in the merit order of the power plants [3].

Market prices to buy more emission allowances will add up a cost to the marginal cost of power generation for fossil-fuelled power plants [4]. Instead, clean and environmentally friendly electric energy options will face a competitive advantage, namely renewable such as hydro power plants [5], since they will not have to buy or own emission allowances.

Environmental issues have become more and more important for fossil-fuelled power generation and they have to be included in the optimisation, giving rise to emission limitations. Fossil-fuelled power plants posing different emission levels should not be considered in the same way in what regards the generation decision. The research work available in the literature, concerning emission limitations, is mainly for the economic dispatch (ED) problem [6,7], deciding only the power contribution of each unit but not its commitment status and availability for generation at each hour.

The unit commitment (UC) problem comprises both deciding the commitment status, a discrete value, and the power contribution, a continuous value. The economic consequences of UC are recognised as very important; savings of a small percent value represent a significant reduction in the fuel consumption. In the UC problem a time horizon of one day to one week is considered, usually divided in hourly intervals. Hence, the UC problem is treated as a deterministic one due to the short-term time horizon. Where stochastic quantities are included, such as energy prices, the corresponding forecasts are used [8].

For many years, central planning was the dominant approach in the electricity supply industry. Utilities had an obligation to serve their customers. This was translated into a demand constraint that ensured all demand would be met. Hence, the main goal of the UC problem was the minimisation of the total fuel cost throughout the time horizon considered, satisfying the demand of electrical energy and all physical and operational constraints. In this context, a large number of optimisation techniques for solving the UC problem have been used by different researchers. Some of the techniques that have been used are integer programming [9], Lagrangian relaxation [10,11], and heuristic methods [12,13].

Nowadays, the electric utility deregulation process has introduced competition through bidding to win the best profit in the electricity market, as well as the possibility of the consumer to choose which supplier he or she wants. Under deregulation, UC has evolved from a minimum-cost policy to a profit-based policy, giving rise to the new profit-based unit commitment (PBUC) problem [14].

The account of emission limitations in the UC problem, as in [15,16], did not receive lately as much attention as in the ED problem. The recent advent of the ETS in the EU has renewed interest in the environmentally constrained UC problem [3,17]. Still, the environmental issues have been included only in the minimum-cost optimisation problem, but not in the profit-based optimisation problem with different energy price profiles, which represents the new contribution of this paper.

Emission allowances are yearly allocated. Hence, the environmentally constrained UC problem requires a medium-term scheduling. An estimation of the daily or weekly allowances of each unit is obtained by means of annual allowances [4].

A practical approach based on multiobjective optimisation (MO) is proposed in this paper to solve the PBUC problem with emission limitations. The proposed practical approach is applied on a case study based on the standard IEEE 30-bus system. Trade-off curves between profit and emission are presented for different energy price profiles, graphically illustrating the non-dominated or Pareto-optimal solution set. Moreover, a new parameter is presented, ratio of change, and the corresponding gradient angle, enabling the proper selection of a compromise commitment for the units.

The paper is structured as follows. Section 2 provides the notation used throughout the paper along with the mathematical formulation of the PBUC problem. Section 3 develops the proposed practical approach for solving the PBUC problem with emission limitations. Section 4 presents a case study, illustrating the numerical simulation results. Section 5 provides conclusions.

## 2. Problem formulation

### Notation

$I$	total number of thermal units.
$K$	total number of hours in the scheduling time horizon.
$\pi_k$	forecasted energy price during period $k$ .
$C_{ik}$	total fuel cost incurred by thermal unit $i$ during period $k$ .
$x_{ik}$	state of thermal unit $i$ during period $k$ .
$u_{ik}$	commitment decision (on-line or shutdown) of thermal unit $i$ during period $k$ .
$p_{ik}$	power generation of thermal unit $i$ during period $k$ .
$p_i^{\max}$	maximum power generation of thermal unit $i$ .
$p_i^{\min}$	minimum power generation of thermal unit $i$ .
$D_k$	demand of electrical energy during period $k$ .
$B_n$	set of thermal units on the $n$ th cumulative constraint.
$H_{ni}$	function which describes a contribution of thermal unit $i$ to $n$ th cumulative constraint.
$H_n^{\text{req}}$	upper bound on $n$ th cumulative constraint.
$N$	set of cumulative constraints.
$A_{ik}$	state function of thermal unit $i$ during period $k$ .
$P_{ik}$	dispatch function of thermal unit $i$ during period $k$ .

$U_{ik}$	set of admissible decisions for thermal unit $i$ during period $k$ .
$X_i^0$	set of initial states for thermal unit $i$ .
$X_i^f$	set of final states for thermal unit $i$ .
$E_{ik}$	total emission caused by thermal unit $i$ during period $k$ .
$w$	weighting factor.
$\xi$	scaling factor.
$M$	set of Pareto-optimal solutions.
$\varepsilon$	allowable level.
$\mathbf{x}$	vector of all state variables.
$\mathbf{u}$	vector of all commitment decision variables.
$\mathbf{p}$	vector of all power generation variables.

The traditional UC problem is defined as the task of establishing the minimum total fuel cost for the hourly generation schedule of the thermal units during a time horizon of one day up to one week, satisfying the demand of electrical energy and all physical and operational constraints.

In a competitive environment, a generating company (GENCO) has the goal to produce electricity and sell it with maximum profit. The system-wide balance of supply and demand is assumed to be managed by an independent system operator, which maintains the system security and reliability. Hence, the GENCO can consider a generation schedule that produce demand less than the forecasted level if it creates more profit. Indeed, according to the price profile, the total generation may change.

Redefining the UC problem for the competitive environment involves changing the demand constraint from an equality to less than or equal constraint, and changing the objective function from cost minimisation to profit maximisation. Moreover, in the new competitive and environmentally constrained electricity supply industry, a GENCO with thermoelectric facilities faces the optimal trade-off problem of how to achieve the maximum profit by the management of the energy available in fossil fuels for power generation minimising the environmental impact.

In the PBUC problem under consideration, the objective function is a measure of the profit attained by the conversion of the energy available in fossil fuels into electric energy. Thus, the objective function to be minimised can be expressed as:

$$g(\mathbf{x}, \mathbf{u}, \mathbf{p}) = \sum_{i=1}^I \sum_{k=1}^K C_{ik}(x_{i,k-1}, u_{ik}, p_{ik}) - \pi_k p_{ik} \quad (1)$$

The objective function in (1) is composed of two parts. The first part represents the total fuel cost incurred on the commitment of the units. The second part expresses the revenues of each unit in the thermal system during the short-term time horizon, where  $\pi_k$  is the forecasted energy price during period  $k$  and  $p_{ik}$  is the power generation of thermal unit  $i$  during period  $k$ .

The commitment decision  $u_{ik}$  identifies if the unit is on-line or shutdown. The unit's state depends not only on the commitment decision, but also on the start-up and shutdown constraints. Once started or shutdown, a unit must remain committed or uncommitted for minimum durations: min up and min down times. In addition to constraints on start-up and shutdown, a unit may have ramp-rate constraints: some generation levels cannot be reached from one period to the next [8].

The total fuel cost incurred by thermal unit  $i$  during period  $k$  is given by the sum of the start-up cost with the operation cost. The start-up cost is given as a constant, and the operation cost is mathematically modelled as a convex function.

The operation cost is assumed to be computed by a quadratic function of power generation as [18]:

$$C_{ik}^{\text{op}}(u_{ik}, p_{ik}) = u_{ik} (a_i + b_i p_{ik} + c_i p_{ik}^2); \quad \forall i \in I, \quad \forall k \in K \quad (2)$$

where  $a_i$ ,  $b_i$  and  $c_i$  are the cost coefficients for thermal unit  $i$ .

Alternatively, the objective function to be minimised can be the total emission, expressed as:

$$h(\mathbf{x}, \mathbf{u}, \mathbf{p}) = \sum_{i=1}^I \sum_{k=1}^K E_{ik}(x_{i,k-1}, u_{ik}, p_{ik}) \quad (3)$$

The emission is assumed to be computed by the sum of quadratic and exponential functions of power generation as [18]:

$$E_{ik}^{\text{em}}(u_{ik}, p_{ik}) = u_{ik} [10^{-2} (\alpha_i + \beta_i p_{ik} + \gamma_i p_{ik}^2) + \zeta_i \exp(\lambda_i p_{ik})]; \quad \forall i \in I, \quad \forall k \in K \quad (4)$$

where  $\alpha_i$ ,  $\beta_i$ ,  $\gamma_i$ ,  $\zeta_i$  and  $\lambda_i$  are the emission coefficients for thermal unit  $i$ . The emission coefficients in (4) are computed by the given data for the type of pollutant.

The optimal value of the objective function is determined subject to constraints: global constraints and local constraints.

The following equations represent the set of global constraints.

1) *Hourly Generation Constraints:*

$$\sum_{i=1}^I p_{ik} \leq D_k; \quad \forall k \in K \quad (5)$$

2) *Cumulative Constraints:*

$$\sum_{i=1}^{B_n} \sum_{k=1}^K H_{ni}(x_{i,k-1}, u_{ik}, p_{ik}) \leq H_n^{\text{req.}}, \quad \forall n \in N \quad (6)$$

In (5) the power generated by the thermal units is less than or equal to the demand of electrical energy  $D_k$  during period  $k$ . An example of the cumulative constraints, given in (6), would be the limitation on emission by a group of units over the scheduling time horizon [15,16], where  $B_n$  is the set of thermal units on the  $n$ th cumulative constraint,  $H_{ni}$  is the function which describes a contribution of thermal unit  $i$  to  $n$ th cumulative constraint,  $H_n^{\text{req.}}$  is the upper bound on  $n$ th cumulative constraint and  $N$  is the set of cumulative constraints.

The following equations represent the set of local constraints.

1) *State Equations for the Thermal Units:*

$$(x_{ik}, p_{ik}) = A_{ik}(x_{i,k-1}, u_{ik}); \quad \forall i \in I, \quad \forall k \in K \quad (7)$$

2) *Power Generation Admissible Range:*

$$p_{ik} = P_{ik}(x_{ik}, u_{ik}) \quad \forall i \in I, \quad \forall k \in K \quad (8)$$

3) *Decision, Initial State and Final State:*

$$u_{ik} \in U_{ik}, \quad x_{i0} \in X_i^0, \quad x_{if} \in X_i^f; \quad \forall i \in I, \quad \forall k \in K \quad (9)$$

Eq. (7) provides the state and power generation of thermal unit  $i$  during period  $k$  for the state during period  $k-1$  and the commitment decision during period  $k$ . The time dependence of the state function  $A_{ik}$  is needed to account for the user-specified time-varying state constraints [8].

In (8), a dispatch function  $P_{ik}$  maps the decision  $u_{ik}$  and the resulting state  $x_{ik}$  into the power generation admissible range. In (9) the decision  $u_{ik}$  belongs to the set of admissible decisions  $U_{ik}$ , which is state dependent; and the initial state  $x_{i0}$  and final state  $x_{if}$  belong to the initial state set  $X_i^0$  and the final state set  $X_i^f$ , respectively.

Constraints (5) to (9) define the set of feasible variables.

### 3. The proposed practical approach

The PBUC problem with emission limitations is formulated as the following MO problem:

$$\text{Min } \{ g(\mathbf{x}, \mathbf{u}, \mathbf{p}), h(\mathbf{x}, \mathbf{u}, \mathbf{p}) \} \quad (10)$$

Subject to

$$(\mathbf{x}, \mathbf{u}, \mathbf{p}) \in F \quad (11)$$

The first application of MO with power systems has been addressed in [19]. MO with conflicting objective functions gives rise to a set of optimal solutions, instead of one optimal solution. The reason for the optimality of many solutions is that no one can be considered to be better than any other with respect to all objective functions. These optimal solutions are known as non-dominated or Pareto-optimal solutions [18]. The trade-off curve represents the image of the Pareto-optimal set into the space of objectives.

If the problem had been reduced to a single objective problem by treating the emission as a constraint, it would be difficult to obtain the trade-off relations. This is an advantage of using the multiobjective criteria instead of a single objective regarding the profit maximisation. The availability of the trade-off curve between profit and emission will give a quantitative base to decision-makers for readjusting the scheduling according to emission allowance trading.

The most widely used method for generating non-dominated solutions and trade-off curve is the weighted sum method, especially when the MO problem has only two objectives. Adopting the weighted sum method, a non-dominated solution to the MO problem can be determined by a convex combination of the objective functions:

$$o(\mathbf{x}, \mathbf{u}, \mathbf{p}) = w g(\mathbf{x}, \mathbf{u}, \mathbf{p}) + (1 - w) \xi h(\mathbf{x}, \mathbf{u}, \mathbf{p}) \quad (12)$$

where  $w$  is the weighting factor and  $\xi$  is the scaling factor, given for instance by the emission market price, which is assumed constant over the scheduling time horizon.



The trade-off curve can be found by parametrically varying the weighting factor  $w$  between 0 and 1, thus solving single objective optimisation problems. The best emission commitment (BEC) corresponds to  $w = 0$ , while the best profit commitment (BPC) corresponds to  $w = 1$ .

Our practical approach may merge the weighted sum method with the  $\varepsilon$ -constraining method into a hybrid method, which constraints the objective functions by some allowable levels  $\varepsilon$ :

$$\sum_{i=1}^I \sum_{k=1}^K C_{ik} - \pi_k p_{ik} \leq \varepsilon_C^{\text{req}} \quad (13)$$

or

$$\sum_{i=1}^I \sum_{k=1}^K E_{ik} \leq \varepsilon_E^{\text{req}} \quad (14)$$

in order to overcome the difficulty on finding the non-convex Pareto-optimal set for the MO problem.

A non-dominated solution  $m$  in the Pareto-optimal set, representing a 168 hours generation schedule, is characterized by a total profit and a total emission in the space of objectives.

Upon having the Pareto-optimal set and trade-off curve, the proposed practical approach extracts one solution to the decision-maker as the best compromise solution. This compromise solution denotes the amount of percentage decrease in total profit that the decision-maker is willing to accept in exchange for a certain amount of percentage decrease in total emission [20].

The ratio of change is obtained for each non-dominated solution  $m$  with respect to the previous non-dominated solution  $m-1$ , comparatively to the maximum ratio of change, given by:

$$\mu^m = \frac{h_{\%}(\mathbf{x}^m, \mathbf{u}^m, \mathbf{p}^m) - h_{\%}(\mathbf{x}^{m-1}, \mathbf{u}^{m-1}, \mathbf{p}^{m-1})}{g_{\%}(\mathbf{x}^m, \mathbf{u}^m, \mathbf{p}^m) - g_{\%}(\mathbf{x}^{m-1}, \mathbf{u}^{m-1}, \mathbf{p}^{m-1})} \times \frac{g_{\%}^{\text{max}}}{h_{\%}^{\text{max}}} \quad (15)$$

The corresponding gradient angle is also obtained, given by:

$$\theta^m = \tan^{-1}(\mu^m) \quad (16)$$

The new parameter, ratio of change, and the corresponding gradient angle, enable the selection of the best compromise commitment (BCC) for the units. On the one hand, if the gradient angle assumes small values, the percentage decrease in total emission would be small for a significant percentage decrease in total profit. On the other hand, if the gradient angle assumes large values, the decision-maker may decide in favour of a further percentage decrease in total emission at the expense of some percentage decrease in total profit.

In our approach, the BCC is selected for a ratio of change equal to 1, corresponding to a gradient angle of  $45^\circ$ , since a ratio of change less than 1 means that the percentage decrease in total emission is less than the corresponding percentage decrease in total profit.

#### 4. Case study

The proposed practical approach has been applied on a case study based on the standard IEEE 30-bus system. The fuel cost and emission coefficients, unit's characteristics and constraints on start-up and shutdown are shown in Table 1.

Table 1  
Fuel cost and emission coefficients, unit's characteristics and constraints on start-up and shutdown

	1	2	3	4	5	6
$a$	10	10	20	10	20	10
$b$	200	150	180	100	180	150
$c$	100	120	40	60	40	100
$\alpha$	4.091	2.543	4.258	5.426	4.258	6.131
$\beta$	-5.554	-6.047	-5.094	-3.550	-5.094	-5.555
$\gamma$	6.490	5.638	4.586	3.380	4.586	5.151
$\zeta$	2.0E-4	5.0E-4	1.0E-6	2.0E-3	1.0E-6	1.0E-5
$\lambda$	2.857	3.333	8.000	2.000	8.000	6.667
$p_i^{\max}$ (MW)	50	60	100	120	100	60
$p_i^{\min}$ (MW)	5	5	5	5	5	5
Start-up (\$)	20	20	40	20	40	20
Min up (h)	2	2	2	2	2	2
Min down (h)	2	2	2	2	2	2

Our practical approach was developed and implemented on a 2.8-GHz-based processor with 512 MB of RAM using Fortran language. The scheduling time horizon chosen is one week divided into 168 hourly periods.

In restructured markets, price forecasting has become an increasingly important activity for both electricity producers and large consumers [21]. An accurate forecast of energy prices is a very important tool for a GENCO to develop an appropriate bidding strategy in the market and to optimally schedule its thermal resources.

Several methodologies have been tried out for energy prices forecasting [22], mainly based on time series models, or on artificial intelligence techniques [23–25].

The result of the optimisation is dependent on the energy price data. Indeed, minor changes in the energy price may give a significant change in the power generation of thermal units. Hence, the influence of price forecasting on profit-based unit commitment with emission limitations is analysed in this paper considering different energy price profiles.

The three energy price profiles considered over the time horizon are shown in Fig. 1 (where \$ is a symbolic economic quantity). Profile 1 is a high-price profile and has a peak value of 434.8 \$/MWh. Profile 2 has a peak value of 347.9 \$/MWh. Profile 3 is a low-price profile and has a peak value of 278.3 \$/MWh.

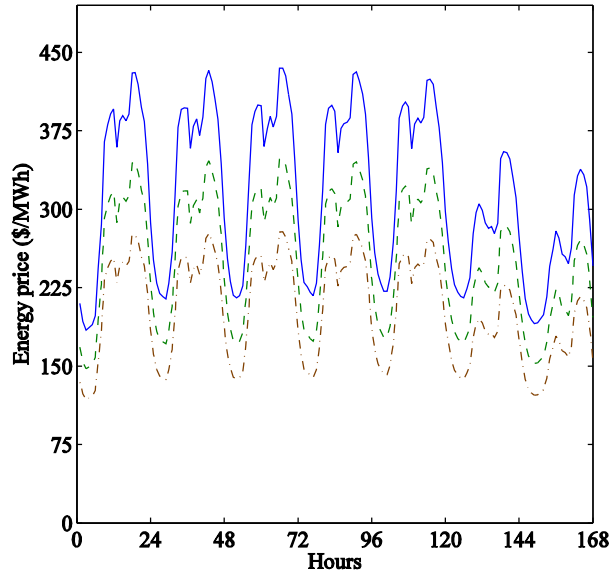


Fig. 1. Energy price profiles considered. The solid line denotes profile 1, the dashed line denotes profile 2 and the dash-dot line denotes profile 3.

The following computation strategy is carried out: at first, profit and emission are independently optimised to determine the anchor points of the trade-off curves: BPC and BEC; then, profit and emission are merged according to the weighted sum method mentioned in our practical approach.

The computed hourly total generation for profile 1, 2 and 3 are shown respectively in Figs. 2, 3 and 4.

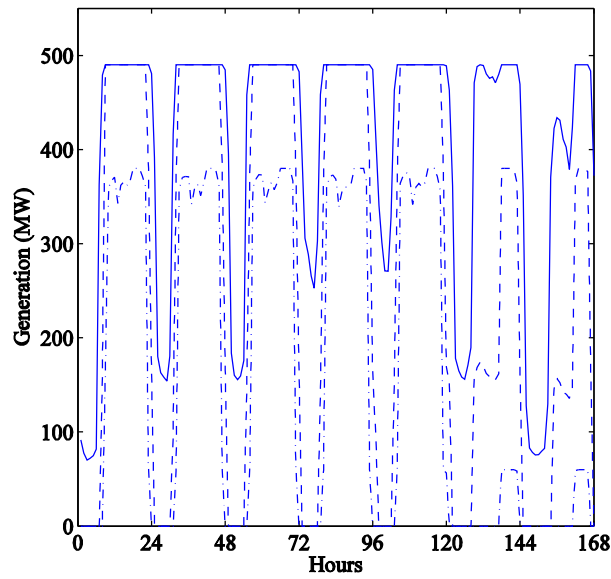


Fig. 2. Hourly total generation for profile 1. The solid line denotes BPC results,  $w = 1$ , while the dashed and dash-dot lines denote compromise commitment results for  $w = 0.6$  and  $w = 0.4$ , respectively.

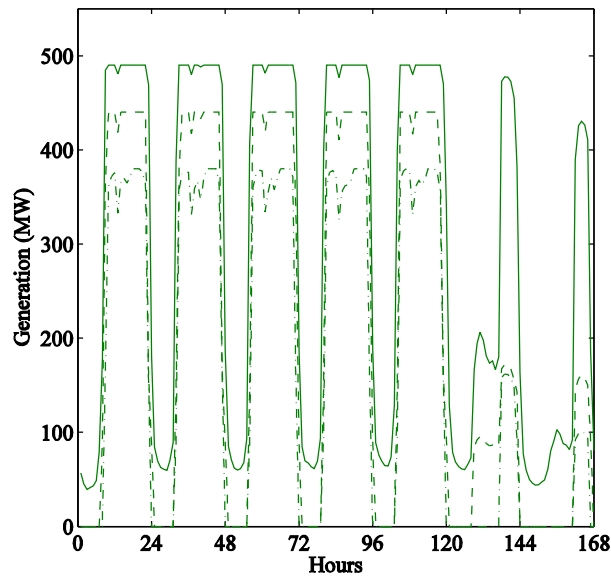


Fig. 3. Hourly total generation for profile 2. The solid line denotes BPC results,  $w = 1$ , while the dashed and dash-dot lines denote compromise commitment results for  $w = 0.6$  and  $w = 0.5$ , respectively.

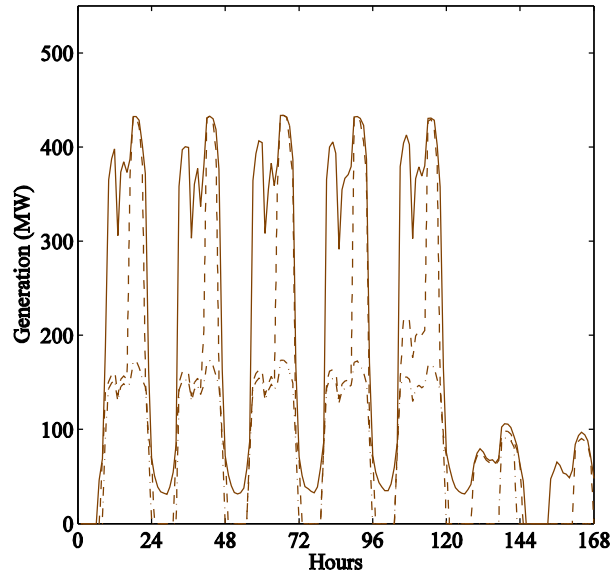


Fig. 4. Hourly total generation for profile 3. The solid line denotes BPC results,  $w = 1$ , while the dashed and dash-dot lines denote compromise commitment results for  $w = 0.7$  and  $w = 0.5$ , respectively.

In the BPC results,  $w = 1$ , the units are committed in order to achieve maximum profit, regardless of emission. The generation profile tends to follow the shape of the energy price profile. In the compromise commitment results, the maximum power generation is reduced as the weighting factor  $w$  decreases, in order to attain an adequate emission level, thus implying a lower total profit. In the BEC results,  $w = 0$ , all units are uncommitted in order to achieve minimum emission, since no must-run units were considered in this case study. Also, if necessary, a non-null profit or emission can be considered as a minimum value to avoid total shutdown.

Fig. 5 shows the trade-off curves in three dimensions, depicting the selected best compromise solutions, for each profile considered. The Pareto-optimal set has 201 non-dominated solutions. Trade-off characteristics give the percentage decrease in total emission against percentage decrease in total profit.

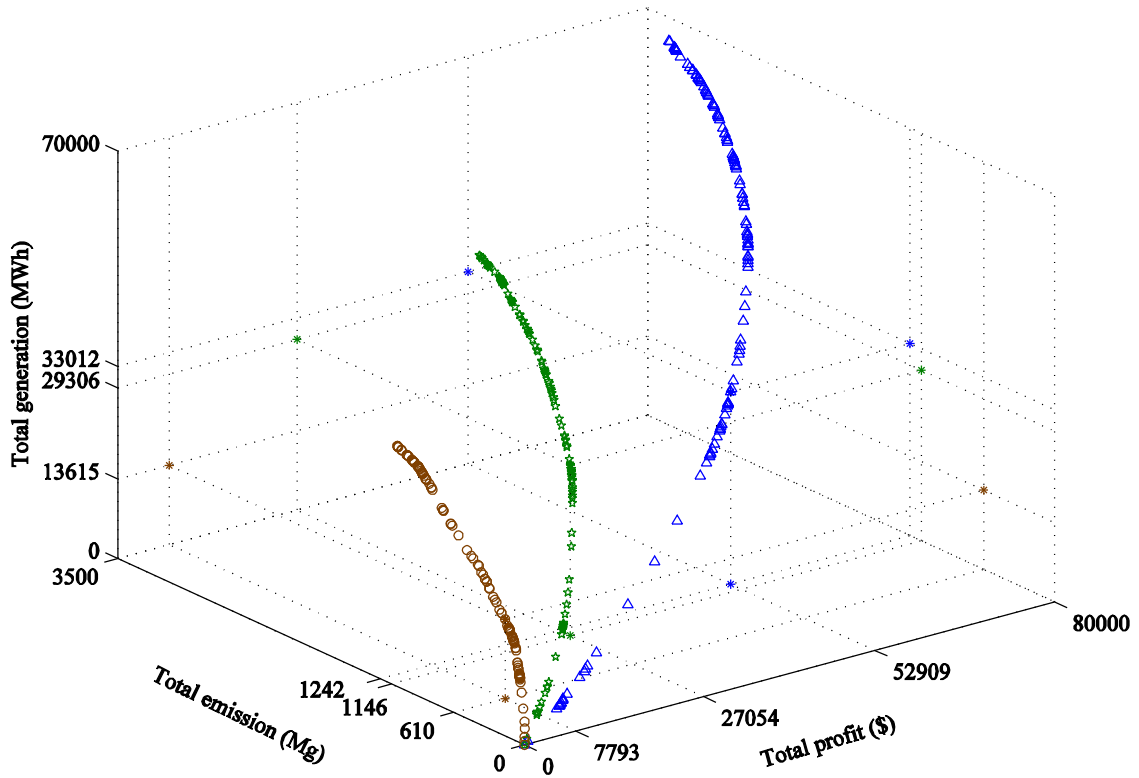


Fig. 5. Trade-off curves in three dimensions, depicting the selected best compromise solutions, for each profile considered.

The trade-off curves have a sharp slope at the BPC neighbourhood. At the beginning of the curves, a significant percentage decrease in total emission is obtained with a small percentage decrease in total profit. For instance, a 16.3% reduction in total emission can be achieved by only a 1.9% decrease in total profit for profile 1. It should be noted that at the end of the curves the opposite occurs.

The new parameter, ratio of change, and the corresponding gradient angle, enable the selection of the BCC for the units, between the BPC and the BEC. Table 2 shows the computational results for the proposed practical approach.

Table 2

Computational results for the proposed practical approach

			Total profit (\$)	Total generation (MWh)	Total emission (Mg)
Profile 1	BPC	$w = 1$	77926	67668	3199
	CC	$w = 0.800$	76594	59857	2701
	CC	$w = 0.600$	69206	46409	2022
	BCC	$w = 0.500$	52909	33012	1242
	CC	$w = 0.400$	45400	26747	992
	CC	$w = 0.200$	0	0	0
	BEC	$w = 0$	0	0	0
Profile 2	BPC	$w = 1$	36203	50564	2451
	CC	$w = 0.800$	35583	45903	2142
	CC	$w = 0.600$	31561	35412	1545
	BCC	$w = 0.545$	27054	29306	1146
	CC	$w = 0.400$	13965	12004	461
	CC	$w = 0.200$	0	0	0
	BEC	$w = 0$	0	0	0
Profile 3	BPC	$w = 1$	10780	32451	1707
	CC	$w = 0.800$	10625	29557	1470
	BCC	$w = 0.610$	7793	13615	610
	CC	$w = 0.600$	7544	12619	567
	CC	$w = 0.400$	5772	7038	373
	CC	$w = 0.200$	0	0	0
	BEC	$w = 0$	0	0	0

The computation time for a trade-off curve was about 10.98 s, with an average 0.05 s for each non-dominated solution representing a 168 hours generation schedule. Hence, the proposed practical approach is computationally acceptable. The proposed practical approach could be applied on larger problems, since the computation time scales up linearly with number of hours and units.

## 5. Conclusion

The new competitive and environmentally constrained electricity supply industry requires new computing tools to ensure both competitiveness to generating companies in the electricity market and environmental protection by limiting the emission of greenhouse gases into the atmosphere. This paper

proposes a practical approach based on multiobjective optimisation to solve the profit-based unit commitment problem with emission limitations. Instead of directly solving the problem by assuming certain weighting factor, trade-off curves between profit and emission are developed for different energy price profiles. The paper also proposed two indices to select a solution of the Pareto's set. Numerical testing results, based on the standard IEEE 30-bus system, show that the proposed practical approach is efficient for obtaining the generation schedule and the trade-off curve, allowing the selection of the best compromise solution by the decision-maker with an acceptable computation time requirement. Hence, the proposed practical approach enables the user to obtain an extra value and cope easier with the demands of energy economics.

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