

# Scheduling of Head-Sensitive Cascaded Hydro Systems: A Nonlinear Approach

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**Abstract**—In this paper, we propose a novel nonlinear approach to solve the short-term hydro scheduling problem under deregulation, considering head-dependency. The actual size of hydro systems, the continuous reservoir dynamics and constraints, the hydraulic coupling of cascaded hydro systems, and the complexity associated with head-sensitive hydroelectric power generation still pose a real challenge to the modelers. These concerns are all accounted for in our approach. Results from a case study based on one of the main Portuguese cascaded hydro systems are presented, showing that the proposed nonlinear approach is proficient.

**Index Terms**—Hydroelectric power generation, nonlinear programming, power generation scheduling.

## NOTATION

The notation used throughout the paper is stated as follows.

$I, i$	Set and index of reservoirs.
$K, k$	Set and index of hours in the time horizon.
$\lambda_k$	Forecasted energy price in hour $k$ .
$p_{ik}$	Power generation of plant $i$ in hour $k$ .
$\Psi_i$	Future value of the water stored in reservoir $i$ .
$v_{ik}$	Water storage of reservoir $i$ at end of hour $k$ .
$a_{ik}$	Inflow to reservoir $i$ in hour $k$ .
$M_i$	Set of upstream reservoirs to reservoir $i$ .
$q_{ik}$	Water discharge by reservoir $i$ in hour $k$ .
$s_{ik}$	Water spillage by reservoir $i$ in hour $k$ .
$h_{ik}$	Head of plant $i$ in hour $k$ .
$l_{ik}$	Water level in reservoir $i$ in hour $k$ .
$\eta_{ik}$	Power efficiency of plant $i$ in hour $k$ .
$\bar{v}_i, \underline{v}_i$	Water storage limits of reservoir $i$ .
$\bar{q}_{ik}$	Maximum water discharge by reservoir $i$ in hour $k$ .
$\underline{q}_i$	Minimum water discharge by reservoir $i$ .

$v_{i0}$	Initial water storage of reservoir $i$ .
$F(\cdot)$	Nonlinear function of decision variables.
$A$	Constraint matrix.
$\bar{b}, \underline{b}$	Upper and lower bound vectors on constraints.
$x$	Vector of decision variables.
$\bar{x}, \underline{x}$	Upper and lower bound vectors on decision variables.
$\bar{\eta}_i, \underline{\eta}_i$	Power efficiency limits of plant $i$ .
$\bar{h}_i, \underline{h}_i$	Head limits of plant $i$ .
$\bar{l}_i, \underline{l}_i$	Water level limits of reservoir $i$ .
$\bar{q}_i^{\text{hx}}$	Maximum water discharge by reservoir $i$ at $\bar{h}_i$ .
$\underline{q}_i^{\text{hn}}$	Maximum water discharge by reservoir $i$ at $\underline{h}_i$ .
$\bar{p}_i^{\text{hx}}$	Maximum power generation of plant $i$ at $\bar{h}_i$ .
$\underline{p}_i^{\text{hn}}$	Maximum power generation of plant $i$ at $\underline{h}_i$ .
$H$	Hessian matrix.
$f$	Vector of coefficients for the linear term.
$L$	Lagrange function.

## I. INTRODUCTION

IN this paper, the short-term hydro scheduling (STHS) problem of a head-sensitive cascaded hydro system is considered. In hydro plants with a large storage capacity available, as it is the case in the Brazilian system for instance, head variation has negligible influence on operating efficiency in the short-term [1]. In hydro plants with a small storage capacity available, also known as run-of-the-river hydro plants, operating efficiency is sensitive to the head—head change effect [2]. For instance, in the Portuguese system there are several cascaded hydro systems formed by many but small reservoirs. Hence, it is necessary to consider head-dependency on STHS. In a cascaded hydraulic configuration, where hydro plants can be connected in both series and parallel, the release of an upstream plant contributes to the inflow of the next downstream plants, implying spatial-temporal coupling among reservoirs.

Hydro plants particularly run-of-the-river hydro plants are considered to provide an environmentally friendly energy option, while fossil-fuelled plants are considered to provide an environmentally aggressive energy option, but nevertheless still in nowadays a necessary option [3]. However, the rising demand for electric energy, likely increases in fossil-fuel prices, and the

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need for clean emission-free generation sources, are trends in favor of increasing generation from renewable sources.

The Portuguese fossil fuels energy dependence is among the highest in the European Union. Portugal does not have endogenous thermal resources, which has a negative influence on Portuguese economy. Moreover, the Portuguese greenhouse emissions are already out of Kyoto target and must be reduced in the near future. Hence, promoting efficiency improvements in the exploitation of the Portuguese hydro resources reduces the reliance on fossil fuels and decreases greenhouse emissions.

In the STHS problem a time horizon of one to seven days is considered, usually divided into hourly intervals. Hence, the STHS problem is treated as a deterministic one. Where the problem includes stochastic quantities, such as inflows to reservoirs or energy prices, the corresponding forecasts are used [4].

Traditionally, in a regulated environment, the goal of the STHS problem is the minimization of expected costs, while maintaining an adequate security of supply [5]. This problem could be a part of the traditional hydrothermal coordination problem, typically solved with methods based on decomposition approaches. The issue of considering constraints imposed by the electric network within the hydrothermal coordination problem usually implies adding a set of constraints that are equivalent to a power flow for each period. Thus, the electric network may be represented by dc [6]–[9] or ac [10] power flow models. The electric network constraints may imply a reduction in the total power generation. For instance, a reduction of about 25% has been reported [8] while accounting for transmission capacity limits, relatively to the transmission-unconstrained case. However, the issue of considering electric network constraints is mainly relevant for Latin American countries that feature extensive and weakly-meshed networks, highly-loaded power lines, and generation plants located far from the load [10]; otherwise, the implications of neglecting electric network constraints are lesser.

In a deregulated environment, such as the Norwegian case [11] or concerning Portugal and Spain given the Iberian Electricity Market, a hydro generating company (H-GENCO) is usually an entity owning generation resources and participating in the electricity market with the ultimate goal of maximizing profits, without concern of the system, unless there is an incentive for it [5]. The system-wide balance of supply and demand is assumed to be managed by an independent system operator (ISO), which maintains the system security and reliability. Nevertheless, an appropriate representation when transmission security is considered in the STHS problem can be seen for instance in [12].

The optimal management of the water available in the reservoirs for power generation, regarding future operation use, delivers a self-schedule and represents a major advantage for the H-GENCO to face competitiveness given the economic stakes involved [13]. Based on the self-schedule, the H-GENCO is able to submit bids with rational support to the electricity market. Thus, for deregulation applications, STHS solution is very important as a decision support for developing bidding strategies in the market [14], guided by the forecasted energy prices, and a more realistic modeling is crucial for surviving nowadays com-

petitive framework. The development of bidding strategies is outside the scope of this paper, but can be seen for instance in [15] and [16].

Dynamic programming (DP) is among the earliest methods applied to the STHS problem [17], [18]. Although DP can handle the nonconvex, nonlinear characteristics present in the hydro model, direct application of DP methods for cascaded hydro systems is impractical due to the well-known DP curse of dimensionality, more difficult to avoid in short-term than in long-term optimization without losing the accuracy needed in the model [19].

Artificial intelligence techniques have also been applied to the STHS problem [20]–[23]. However, a significant computational effort is necessary to solve the problem for cascaded hydro systems, particularly, with a time horizon of 168 hourly intervals. Also, due to the heuristics used in the search process only sub-optimal solutions can be reached.

A natural approach to STHS is to model the system as a network flow model, because of the underlying network structure subjacent in cascaded hydro systems [7], [24]. For cascaded hydro systems, as there are water linkage and electric connections among plants, the advantages of the network flow technique are salient.

Hydroelectric power generation characteristics are often assumed as linear or piecewise linear in hydro scheduling models [25]–[27]. Accordingly, the solution procedures are based on linear programming (LP) or mixed-integer linear programming (MILP). LP is a well-known optimization method and standard software can be found commercially. MILP is very powerful for mathematical modeling and is applied successfully to solve large-size scheduling problem in power systems. Hence, MILP is becoming often used for STHS [28]–[34], where integer variables allow modeling of start-up costs and discrete hydro unit-commitment constraints.

However, LP typically considers that hydroelectric power generation is linearly dependent on water discharge, thus ignoring head-dependency to avoid nonlinearities. This is nowadays not appropriated for a realistic modeling of run-of-the-river hydro plants. The discretization of the nonlinear dependence between power generation, water discharge and head, used in MILP to model head variations, augments the computational burden required to solve the STHS problem. For instance, the optimal solution reported in [31] required 22 min of CPU time, on a 400-MHz-based processor with 500 MB of RAM. Furthermore, methods based on successive linearization in an iterative scheme depend on the calibration of parameters that behave like magic numbers. For instance, the selection of the best under-relaxation factor in [32] and [33] is empiric and case-dependent, rendering some ambiguity to these methods.

Hydro scheduling is in nature a nonlinear optimization problem. A nonlinear model has advantages compared with a linear one. A nonlinear model expresses hydroelectric power generation characteristics more accurately and head-dependency on STHS can be taken into account. In the past, there were considerable computational difficulties to directly use nonlinear programming (NLP) methods to this sort of problem [35]–[37]. The cascaded hydraulic configuration coupled with the head change effect augments the problem dimension and the

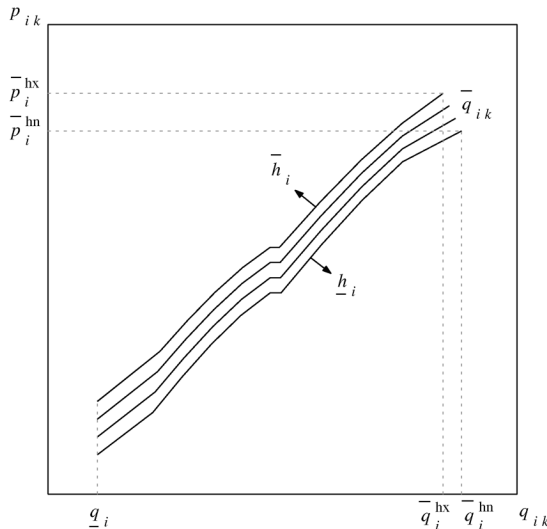


Fig. 1. Unit performance curves.

complexity. As a result of the nonlinear nature of the problem, computational limitations prevented a direct optimization or simplifications of the model were imposed. However, with the advancement in computing power and the development of more effective nonlinear solvers in recent years, this disadvantage has much less influence. We show as a new contribution that this disadvantage is mitigated by applying the proposed nonlinear approach to a realistically-sized hydro system with seven cascaded reservoirs, which was not possible with earlier approaches and computational resources.

In this paper, we propose a novel nonlinear approach to solve the STHS problem under deregulation, considering head-dependency. The actual size of hydro systems, the continuous reservoir dynamics and constraints, the hydraulic coupling of cascaded hydro systems, and the complexity associated with head-sensitive hydroelectric power generation still pose a real challenge to the modelers. These concerns are all accounted for in our approach. Results from a case study based on one of the main Portuguese cascaded hydro systems are presented, showing that the proposed nonlinear approach is proficient.

This paper is organized as follows. Section II provides the mathematical formulation of the STHS problem. Section III presents the nonlinear approach to solve the STHS problem. Section IV provides the results from a case study based on one of the main Portuguese cascaded hydro systems. Finally, concluding remarks are given in Section V.

## II. PROBLEM FORMULATION

The STHS problem can be stated as to find out the water discharges, the water storages, and the water spillages, for each reservoir  $i$  at all scheduling time periods  $k$  that maximizes (or minimizes) a performance criterion subject to all hydraulic constraints.

### A. Objective Function

Depending on the system characteristics and operational requirements, the objective function chosen can be in many forms

[30]. In this paper, the objective function to be maximized is expressed as

$$\sum_{i=1}^I \sum_{k=1}^K \lambda_k p_{ik} + \sum_{i=1}^I \Psi_i(v_{iK}). \quad (1)$$

In (1), the first term is related to the revenues of each plant  $i$  in the hydro system during the short-term time horizon and the last term expresses the future value of the water stored in the reservoirs in the last period  $K$ . Water value is a function of water stored in the reservoirs at the last period. An appropriate representation when this term is explicitly taken into account can be seen for instance in [38]. The storage targets for the short-term time horizon can be established by medium-term planning studies.

### B. Hydro Constraints

The hydro constraints are of two kinds: equality constraints and inequality constraints or simple bounds on the decision variables.

1) *Water Balance*: The water balance equation for each reservoir is formulated as

$$v_{ik} = v_{i,k-1} + a_{ik} + \sum_{m \in M_i} (q_{mk} + s_{mk}) - q_{ik} - s_{ik} \quad \forall i \in I, \quad \forall k \in K \quad (2)$$

assuming that the time required for water to travel from a reservoir to a reservoir directly downstream is less than the 1-h period.

2) *Head*: The head is considered a function of the water levels in the upstream reservoir, denoted by  $f(i)$  in subscript, and downstream reservoir, denoted by  $t(i)$  in subscript, depending, respectively, on the water storages in the reservoirs

$$h_{ik} = l_{f(i)k}(v_{f(i)k}) - l_{t(i)k}(v_{t(i)k}), \quad \forall i \in I, \quad \forall k \in K. \quad (3)$$

Typically for a powerhouse with a reaction turbine, where the tail water elevation is not constant, the head is modeled as in (3), and for a powerhouse with an impulse turbine, where the tail water elevation remains constant, the head depends only on the upstream reservoir water level.

3) *Power Generation*: Power generation is considered a function of water discharge and hydro power efficiency

$$p_{ik} = q_{ik} \eta_{ik}(h_{ik}), \quad \forall i \in I, \quad \forall k \in K. \quad (4)$$

Hydro power efficiency is expressed as the output-input ratio, depending on the head. The hydroelectric power generation characteristics can be graphically represented by a family of nonlinear curves, also known as unit performance curves, each curve for a specific value of the head (see Fig. 1). These curves are to be linearized in this paper.

4) *Water Storage*: Water storage has lower and upper bounds

$$\underline{v}_i \leq v_{ik} \leq \bar{v}_i, \quad \forall i \in I, \quad \forall k \in K. \quad (5)$$

5) *Water Discharge*: Water discharge has lower and upper bounds

$$\underline{q}_i \leq q_{ik} \leq \bar{q}_{ik}(h_{ik}), \quad \forall i \in I, \quad \forall k \in K. \quad (6)$$

The minimum water discharge is considered null in our case study, but may be considered nonzero due to navigation, recreational or ecological reasons. The maximum water discharge is considered a function of the head. Hence, the maximum water discharge may be different for each period  $k$  according to the value of the head, which represents a real feature that is required in our case study in order to achieve better exploitation efficiency.

6) *Water Spillage*: We consider a null lower bound for water spillage

$$s_{ik} \geq 0, \quad \forall i \in I, \quad \forall k \in K. \quad (7)$$

Water spillage can occur when without it the water storage exceeds its upper bound, so spilling is necessary due to safety considerations. The initial water storages and inflows to reservoirs are assumed as known input data.

The H-GENCO analyzed in this paper is considered to be a price-taker, i.e., it does not have market power. Therefore, energy prices  $\lambda_k$  in (1) are assumed known, as in [31] and [34]. A discussion about the most appropriate techniques available in the literature to forecast these prices is presented in Section IV.

To consider uncertainty on energy prices requires a stochastic programming approach. A scenario tree should be adequately constructed and trimmed, which is outside the scope of this paper since we consider the STHS problem as a deterministic one. Nevertheless, an appropriate representation when market uncertainty is explicitly taken into account via price scenarios can be seen for instance in [15], [16], [33], and [39].

### III. NONLINEAR APPROACH

The NLP problem can be stated as to maximize

$$F(x) \quad (8)$$

subject to

$$\underline{b} \leq Ax \leq \bar{b} \quad (9)$$

$$\underline{x} \leq x \leq \bar{x}. \quad (10)$$

In (8), the function  $F(\cdot)$  is a nonlinear function of the vector  $x$  of decision variables: water discharges, water storages, and water spillages. Equation (9) corresponds to the equality constraints in (2), with  $\underline{b} = \bar{b}$ . Equation (10) corresponds to the inequality constraints or simple bounds on the decision variables in (5)–(7). Also, the upper bound for water discharge implies a new inequality constraint that will be rewritten into (9).

As expressed in (3) and (4), water level and hydro power efficiency depend, respectively, on water storage and head.

We consider a linearization of hydro power efficiency of plants, expressed as the output-input ratio, Fig. 1. Hence, we consider the hydro power efficiency given by

$$\eta_{ik} = \alpha_i h_{ik} + \eta_i^0, \quad \forall i \in I, \quad \forall k \in K \quad (11)$$

where the parameters  $\alpha_i$  and  $\eta_i^0$  are given by

$$\alpha_i = (\bar{\eta}_i - \underline{\eta}_i) / (\bar{h}_i - \underline{h}_i), \quad \forall i \in I \quad (12)$$

$$\eta_i^0 = \bar{\eta}_i - \alpha_i \bar{h}_i, \quad \forall i \in I. \quad (13)$$

Also, we consider a linearization of the water level function given by

$$l_{ik} = \beta_i v_{ik} + l_i^0, \quad \forall i \in I, \quad \forall k \in K \quad (14)$$

where the parameters  $\beta_i$  and  $l_i^0$  are given by

$$\beta_i = (\bar{l}_i - \underline{l}_i) / (\bar{v}_i - \underline{v}_i), \quad \forall i \in I \quad (15)$$

$$l_i^0 = \bar{l}_i - \beta_i \bar{v}_i, \quad \forall i \in I. \quad (16)$$

Substituting (11) into (4), we have

$$p_{ik} = q_{ik} (\alpha_i h_{ik} + \eta_i^0), \quad \forall i \in I, \quad \forall k \in K. \quad (17)$$

Therefore, substituting (3) and (14) into (17), power generation becomes a nonlinear function of water discharge and water storage, given by

$$p_{ik} = \alpha_i \beta_{f(i)} q_{ik} v_{f(i)k} - \alpha_i \beta_{t(i)} q_{ik} v_{t(i)k} + \chi_i q_{ik} \quad \forall i \in I, \quad \forall k \in K \quad (18)$$

with

$$\chi_i = \alpha_i (l_{f(i)}^0 - l_{t(i)}^0) + \eta_i^0, \quad \forall i \in I. \quad (19)$$

Hence, a major advantage of our nonlinear approach is to consider the head change effect in a single function (18) of water discharge and water storage that can be used in a straightforward way, instead of deriving several curves for different heads.

The parameters given by the product of  $\alpha$ 's by  $\beta$ 's are of crucial importance for the behavior of head-sensitive reservoirs in a hydro system, setting optimal reservoirs storage trajectories in accordance to their relative position in the cascade. It should be noted that these parameters are not related to the solution procedure. Instead, they are determined only by physical data defining the hydro system. Alternative physical data resulting in different values for these parameters were considered in our previous study [2], with no model consideration for head-sensitive maximum water discharge. However, in this paper, we only use real data from one of the main Portuguese cascaded hydro systems.

In our model, the maximum water discharge, thus giving the maximum power generation, is considered head-sensitive, given by

$$\bar{q}_{ik} = -\delta_i h_{ik} + q_i^0, \quad \forall i \in I, \quad \forall k \in K \quad (20)$$

where the parameters  $\delta_i$  and  $q_i^0$  are given by

$$\delta_i = (\bar{q}_i^{\text{hn}} - \bar{q}_i^{\text{hx}}) / (\bar{h}_i - \underline{h}_i), \quad \forall i \in I \quad (21)$$

$$q_i^0 = \bar{q}_i^{\text{hn}} + \delta_i \underline{h}_i, \quad \forall i \in I. \quad (22)$$

Substituting (3) and (14) into (20), the maximum water discharge becomes a function of water storage, given by

$$\bar{q}_{ik} = -\delta_i [\beta_{f(i)} v_{f(i)k} - \beta_{t(i)} v_{t(i)k}] + \gamma_i, \quad \forall i \in I, \quad \forall k \in K \quad (23)$$

with

$$\gamma_i = q_i^0 - \delta_i (l_{f(i)}^0 - l_{t(i)}^0), \quad \forall i \in I. \quad (24)$$

Hence, the new inequality constraint to be rewritten into (9) is given by

$$q_{ik} + \delta_i [\beta_{f(i)} v_{f(i)k} - \beta_{t(i)} v_{t(i)k}] \leq \gamma_i, \quad \forall i \in I. \quad (25)$$

Our STHS problem can be formulated as a quadratic programming (QP) problem with a quadratic objective function and a linear set of equality and inequality constraints. Primarily, QP with linear constraints can be viewed as a generalization of the LP problem with a quadratic objective function [40].

The QP problem can be stated in general form as to maximize

$$Q(x) = 1/2x^T Hx + f^T x \quad (26)$$

subject to

$$x \in F_{\leq} \cap F_{=} \quad (27)$$

with

$$F_{\leq} = \{x : Cx \leq \bar{c}\} \quad (28)$$

$$F_{=} = \{x : Dx = d\}. \quad (29)$$

Equation (18) can be easily converted in the format of (26), with the parameters  $\alpha_i \beta_{f(i)}$  and  $\alpha_i \beta_{t(i)}$  multiplied by the forecasted energy price  $\lambda_k$  appearing in the Hessian matrix  $H$ , and the parameter  $\chi_i$  also multiplied by the forecasted energy price  $\lambda_k$  appearing in the vector  $f$  of coefficients for the linear term.

The matrix  $H$  is a symmetric matrix, thus all its eigenvalues are real numbers. If the matrix  $H$  is negative semidefinite, i.e., all its eigenvalues are nonpositive, then the problem formulated as in (26) and (27) becomes a concave QP problem. In such case, any local optimum is a global optimum and the problem is solvable in polynomial time. If the matrix  $H$  is indefinite, i.e., has both positive and negative eigenvalues, then the problem formulated as in (26) and (27) becomes an indefinite QP problem, which is NP-hard. The application of local optimization procedures for this problem can no longer guarantee the identification of the global optimum [40]. Our STHS problem is an indefinite QP problem.

The Lagrange function for the problem formulated as in (26) and (27) is given by

$$L = 1/2x^T Hx + f^T x + \varepsilon^T (\bar{c} - Cx) + \mu^T (Dx - d) \quad (30)$$

where  $\varepsilon$  and  $\mu$  are the Lagrange multipliers for the inequality and equality constraints, respectively.

The first-order Karush–Kuhn–Tucker (KKT) conditions, for  $x^*$  to be an optimal solution of this problem, are given by

$$Cx^* \leq \bar{c} \quad (31)$$

$$Dx^* = d \quad (32)$$

$$\varepsilon^T (\bar{c} - Cx^*) = 0 \quad \varepsilon \geq 0 \quad (33)$$

$$f = C^T \varepsilon - D^T \mu - Hx^* \quad (34)$$

where (31) and (32) ensure feasibility of the solution, (33) is the complementary slackness condition, and (34) is the stationarity condition.

The KKT conditions are necessary for all QP problems, whether concave or not, but sufficient only for concave QP problems. Thus, an additional second-order optimality condition is necessary, which is given by

$$y^T H y \leq 0 \quad \forall y \in T(x^*) \quad (35)$$

where  $T(x^*)$  is the tangent space at  $x^*$ , given by

$$T(x^*) = \{y : \forall r \in R^a(x^*), C(r)y = 0, Dy = 0\}. \quad (36)$$

$R^a(x^*)$  is the set of active constraints at  $x^*$  and  $C(r)$  is the vector of line  $r$  of matrix  $C$ .

Indefinite QP is still a research topic among specialists, being less general and taking advantage of the special mathematical structure exhibited by the model.

In order to present the special mathematical structure exhibited by our model for the STHS problem, we assert for the problem formulated as in (26) and (27) three valid observations.

1) *First Observation:* There is always one feasible solution and the objective function is bounded on the feasible region. This is easily proved by the following argument: the objective function is bounded from below by a null profit, and from above by the maximum profit possible with the finite water available in the reservoirs for power generation during the short-term time horizon.

For instance, one way of finding this upper bound is by solving the STHS problem relaxing the equality constraints (29), i.e., to maximize

$$Q(x) = 1/2x^T Hx + f^T x \quad (37)$$

subject to

$$x \in F_{\leq}. \quad (38)$$

2) *Second Observation:* The objective function is a concave function over the feasible set  $F$ , if the global maximizing optimal point is not a boundary point of the set  $F_{\leq}$ . To show this let the inequality constraints (28) be non-active at a local maximizing optimal point  $x^*$ ,  $R^a(x^*) = \phi$ , implying by KKT condition (33) that

$$\varepsilon = 0. \quad (39)$$

For instance, consider by hypothesis that the local maximizing optimal point  $x^*$  is not a global maximizing optimal point, then there exists a feasible  $x$  with a better objective function value for the problem, that is

$$x^T Hx - x^{*T} Hx^* + 2f^T(x - x^*) > 0 \quad (40)$$

thus by (29), (32) and (36), we have

$$x - x^* \in T(x^*) \quad (41)$$

and substituting (34) into (40), we have

$$\exists x - x^* \in T(x^*) : (x - x^*)^T H(x - x^*) > 0 \quad (42)$$

resulting in contradiction with the second-order optimality condition (35).

Hence, if the inequality constraints are non-active, the local maximizing optimal point  $x^*$  is a global maximizing optimal point. This is known as an uniextremality property.

3) *Third Observation:* Consider the problem formulated as in (37) and (38), i.e., relaxing equality constraints (29), then the global maximizing optimal point  $x^*$  is a boundary point of the set  $F_{\leq}$ . To show this let  $x \in F_{\leq}$ , thus becoming valid the inequality given by

$$Q(x^*) - Q(x) \geq 0 \quad (43)$$

which is equivalent to

$$x^{*T} Hx^* - x^T Hx - 2f^T(x - x^*) \geq 0. \quad (44)$$

The first-order KKT conditions, for  $x^*$  to be an optimal solution of this problem, are given by

$$Cx^* \leq \bar{c} \quad (45)$$

$$\varepsilon^T(\bar{c} - Cx^*) = 0 \quad \varepsilon \geq 0 \quad (46)$$

$$f = C^T \varepsilon - Hx^*. \quad (47)$$

Substituting (47) into (44), we have

$$(x - x^*)^T H(x - x^*) \leq -2\varepsilon^T C(x - x^*) \quad (48)$$

and considering (46), it is easy to conclude that

$$-2\varepsilon^T C(x - x^*) \geq 0. \quad (49)$$

If the inequality constraints (28) are non-active at the local maximizing optimal point  $x^*$ ,  $R^a(x^*) = \phi$ , then by (46), we have  $\varepsilon = 0$ , and by (48), matrix  $H$  is not an indefinite matrix, which is a contradiction. For the local maximizing optimal point  $x^*$  of the problem formulated as in (37) and (38), at least one constraint is active.

Consequently, for indefinite QP problems where the objective function is bounded over the feasible set, there exists a possible optimal solution at a boundary point of this feasible set, not necessarily attained at a vertex as it happens if the problem is transformed into an LP problem. However, it is a good initial guess to start by such a vertex and consider the neighborhood around the vertex a good basis for achieving an enhanced objective function value.

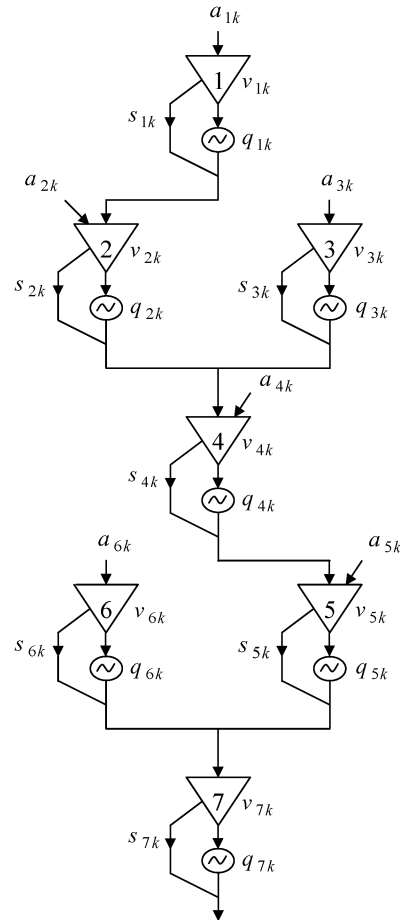


Fig. 2. Hydro system with seven cascaded reservoirs.

We consider a starting point given by a linear approach, and afterwards we check for an enhanced objective function value using the proposed nonlinear approach. In our case study we always arrive at convergence to a better solution.

#### IV. CASE STUDY

The proposed nonlinear approach has been applied on one of the main Portuguese cascaded hydro systems. Our model has been developed and implemented in MATLAB and solved using the optimization solver package Xpress-MP. The numerical testing has been performed on a 600-MHz-based processor with 256 MB of RAM.

The default algorithm of Xpress-MP is Newton Barrier. However, our Hessian matrix is not semidefinite, but rather indefinite. Newton Barrier would normally fail on indefinite QP problems. Hence, the default algorithm of Xpress-MP was changed to Dual Simplex, which worked fine.

##### A. Input Data

The realistically-sized hydro system has seven cascaded reservoirs and is shown in Fig. 2. Table I shows the data of these plants.

The hydro plants numbered in Fig. 2 as 1, 2, 4, 5, and 7 are run-of-the-river hydro plants. The hydro plants numbered as 3 and 6 are storage hydro plants. Hence, for the storage hydro plants head-dependency may be neglected, due to the small head

TABLE I  
HYDRO DATA

#	$\underline{v}_i$ (hm <sup>3</sup> )	$\bar{v}_i$ (hm <sup>3</sup> )	$v_{i0}$ (hm <sup>3</sup> )	$\bar{p}_i^{\text{hx}}$ (MW)	$\bar{p}_i^{\text{hn}}$ (MW)	$\bar{q}_i^{\text{hx}}$ (m <sup>3</sup> /s)	$\bar{q}_i^{\text{hn}}$ (m <sup>3</sup> /s)
1	5.18	12.94	10.35	188.08	157.30	1018.89	1144.50
2	5.32	13.30	10.64	237.14	136.54	823.47	1080.00
3	39.00	97.50	78.00	60.00	56.56	16.36	16.40
4	4.80	12.00	9.60	185.99	155.69	767.13	900.00
5	4.40	11.00	8.80	201.02	196.37	622.23	881.31
6	36.89	58.38	46.70	134.02	133.91	310.76	326.34
7	8.60	21.50	17.20	117.01	108.37	1077.03	1356.51

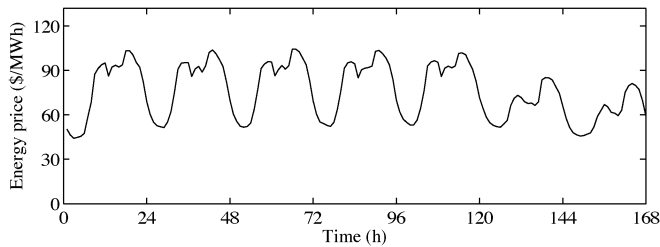


Fig. 3. Energy price profile.

variation during the short-term time horizon. Inflow is considered only on reservoirs 1 to 6. The final water storage in the reservoirs is constrained to be equal to the initial water storage.

The number of the decision variables for the problem can be determined as three times the number of reservoirs multiplied by the number of scheduling time periods [41]. The number of constraints can be determined as two times the number of decision variables plus two times the number of reservoirs multiplied by the number of scheduling time periods. Hence, for our case study the number of variables is 3528 and the number of constraints is 9408.

The time horizon is of 168 hourly intervals, starting on Monday and finishing on Sunday. The energy price profile considered over the short-term time horizon is shown in Fig. 3 (\$ is a symbolic economic quantity).

The competitive environment coming from the deregulation of the electricity markets brings energy prices uncertainty, placing higher requirements on forecasting. A good price forecasting tool reduces the risk of under/over estimating the profit of the H-GENCO and provides better risk management. In the short-term, a generating company needs to forecast energy prices to derive its bidding strategy in the market and to optimally schedule its energy resources [42].

Price forecasting has become in recent years an important research area in electrical engineering, and several techniques have been tried out in this task.

In general, hard and soft computing techniques could be used to predict energy prices. The hard computing techniques include time series [43], auto regressive (AR) [11], and auto regressive integrated moving average (ARIMA) [44] models. An input/output hidden Markov model (IOHMM) [45], wavelet-ARIMA [46], and weighted nearest neighbors (WNN) techniques [47] have also been proposed. The soft computing techniques include neural networks [48], [49] and neuro-fuzzy approaches [50]. These energy prices are considered as deterministic input data for our STHS problem.

## B. Results Analysis

The benefits of considering head-dependency are shown by providing a linear approach that does not consider the impact of variable head. Hence, a comparison of NLP with LP results is presented thereafter. This comparison occurs while satisfying the same hydro constraints, for the sake of a fair comparison.

The storage trajectories of the run-of-the-river reservoirs are shown in Fig. 4. The solid lines denote NLP results while the dashed lines denote LP results.

The comparison of NLP with LP results, shown in Fig. 4, reveals the influence of considering the head change effect in the behavior of the reservoirs. The upstream reservoir should operate at a suitable high storage level in order to benefit the power generation efficiency of its associated plant, due to the head change effect. Hence, the storage trajectory of the upstream reservoir is pulled up using the NLP approach. Instead, the storage trajectory of the last downstream reservoir is pulled down using the NLP approach, thereby improving the head for the immediately upstream reservoirs. Hence, a higher efficiency of the last downstream plant is not important for the overall profit in this hydro system.

The discharge profiles for the run-of-the-river reservoirs are shown in Fig. 5. Again, the solid lines denote NLP results while the dashed lines denote LP results.

The comparison of NLP with LP results, shown in Fig. 5, reveals that the water discharge changes more quickly from the minimum value to the upper value in the LP results than in the NLP results, due to the head change effect.

As a new contribution to earlier studies, some shape adaptation is imposed due to the consideration of the maximum power generation as head-sensitive. This implies that there is a slope shape at the most favorable price hours of each day, instead of the normal flat shape when the maximum water discharge was considered constant.

The main numerical results for the hydro system are summarized in Table II.

Although the average water discharge is as expected the same for both optimization methods, the average storage is superior with the nonlinear approach, due to the consideration of the head change effect.

Thus, regardless of the price scenario considered, with the proposed nonlinear approach we have a higher total profit for the H-GENCO, about 4%. Moreover, the additional CPU time required is negligible, converging rapidly to the optimal solution. Hence, the proposed nonlinear approach provides better results for head-sensitive cascaded hydro systems.

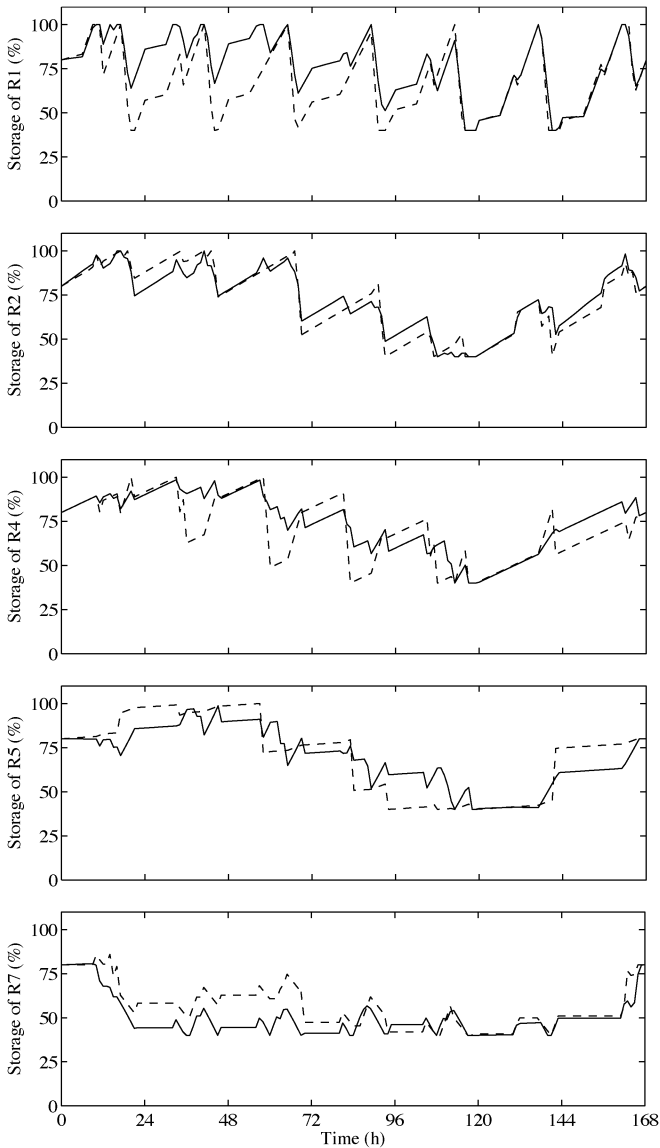


Fig. 4. Storage trajectories of the reservoirs 1, 2, 4, 5, and 7.

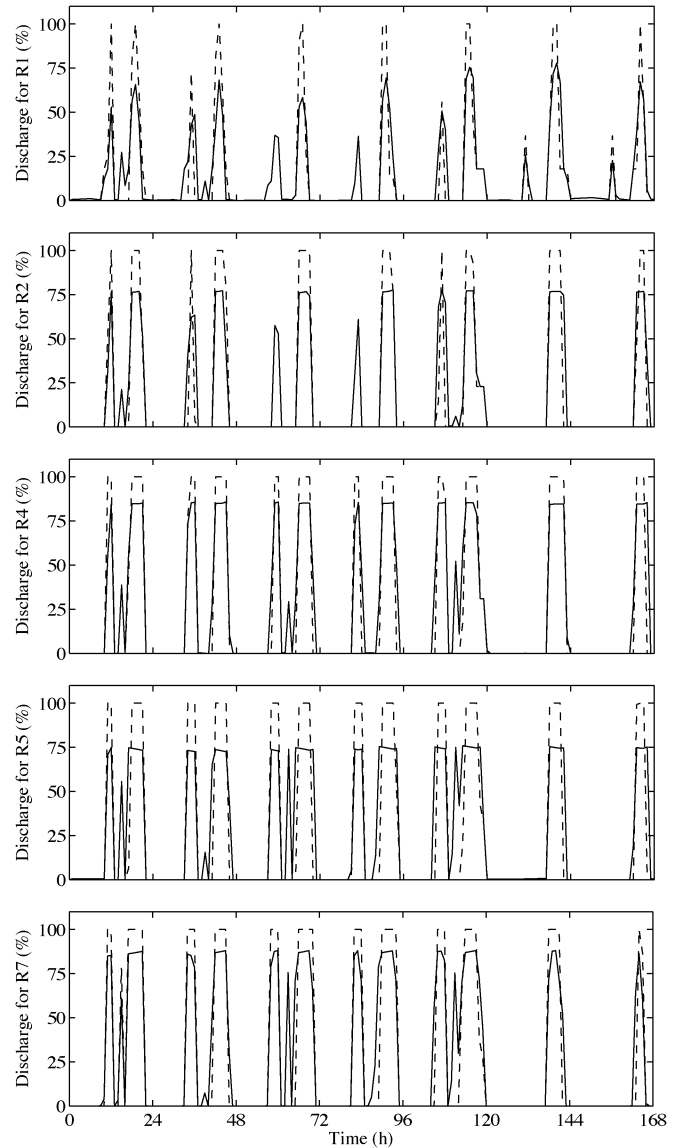


Fig. 5. Discharge profiles for the reservoirs 1, 2, 4, 5, and 7.

### V. CONCLUSION

A novel nonlinear approach is proposed for the STHS problem, considering head-dependency. A nonlinear model has advantages compared with a linear one: hydroelectric power generation characteristics are expressed more accurately and head-dependency on STHS can be taken into account. Our approach considers not only the nonlinear dependence between the power generation, the water discharge and the head, but also that the maximum water discharge, giving the maximum power generation, is a function of the head. A major advantage of our nonlinear approach is to consider the head change effect in a single function of water discharge and water storage that can be used in a straightforward way, instead of deriving several curves for different heads. Due to the more realistic modeling presented in this paper, an enhanced STHS is provided in comparison with a linear approach, assuring simultaneously a negligible computation time. The case study is illustrative of the advantages of our nonlinear model in terms of bene-

TABLE II  
COMPARISON OF NLP WITH LP RESULTS

Method	Average Discharge (%)	Average Storage (%)	Total Profit (\$ $\times 10^3$ )	CPU (s)
LP	23.73	70.32	4567.32	2.25
NLP	23.73	71.97	4753.50	5.90

fits. Hence, the proposed nonlinear approach is both accurate and computationally acceptable, providing better results for head-sensitive cascaded hydro systems.

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