

Short-term optimal scheduling of a price-maker hydro producer in a pool-based day-ahead market

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Abstract: This paper proposes a stochastic mixed-integer linear programming approach to maximise the total expected profit of a price-maker hydro generating company. Start-up and shutdown procedures, discrete hydro unit-commitment constraints, ramp rates, minimum and maximum power output constraints, and head effects, are all taken into account in an efficacious way. Uncertainties are modelled considering sets of scenarios to describe the residual demand curves. The proposed approach is successfully applied to a Portuguese hydro system in cascaded configuration. Conclusions are duly drawn.

Key words: Hydro producer; price-maker; mixed-integer linear programming; residual demand curves; optimal bidding

1 Introduction

The deregulated environment of the electricity sector, such as the one in mainland Spain and Portuguese systems, has induced the creation of mechanisms to encourage competition. In this framework, the main goal of deregulation is to assure a clear decoupling between generation, electricity sales and network operations.

Hydro energy is currently one of the most important renewable energies in the Portuguese system [1]. Hydro units are fast units compared to coal-fired units and liquefied natural gas units [2]. Also, hydro units produce less pollution than competing technologies, being considered favourable options for electricity generation at intermediate, peak and base loads [3].

Managing the available water in reservoirs in the best way possible, safeguarding its future use, provides self-scheduling and represents a significant advantage for hydro generating companies in face of competition. Taking into account this self-schedule, a hydro company submits optimal offers to the market. Hence, in a deregulated environment, short-term hydro scheduling represents a crucial tool to support bidding decisions [4].

The market environment typically is composed of a variety of submarkets to facilitate trade between consumers and power producers, such as the pool and the bilateral contracts markets [5]. The pool market is particularly relevant to our problem.

The pool is composed of the day-ahead market, the balancing market and the adjustment market [6]. Most of the energy is negotiated in the day-ahead market, so the other two markets provide the final tuning of the traded energy. The pool is based on purchase bids and sale offers by consumers and generating companies, respectively.

As a consequence of the market power of some producers, two types of generating companies can be determined: price-makers [7, 8] and price-takers [9–11]. If a perfectly competitive market is assumed, the generating companies are considered as price-takers.

Price-takers accept market prices without being able to change them [12]. The bidding process in those markets can be seen in [13, 14]. If an oligopolistic market is assumed, the bidding decisions made by participants with market power influence prices [15]. Bid optimization models for those markets can be seen in [16].

A novel stochastic mixed-integer linear programming (SMILP) approach is proposed in this paper for solving the short-term optimal scheduling and trading in the day-ahead pool market of a price-maker hydro producer. The effect on market prices is modelled by using stochastic residual demand curves. The proposed approach contains continuous and binary variables, and allows making decisions considering parameter uncertainty explicitly. Uncertainties are modelled by considering sets of scenarios to describe the residual demand curves, where optimal bids (quota, price) are selected for each scenario. The proposed approach is optimized for sets of scenarios using the amount of power supplied by each unit facing the 24-hours residual demand curves of every scenario.

The proposed SMILP approach efficiently solves the stochastic optimization problem of a price-maker hydro producer in cascaded configuration including the head change effect. The cascaded configuration, stochasticity and price-maker objective function are not considered in [17]. A price-maker formulation is considered in [18, 19], but power generation is assumed to be linearly dependent on water discharge. Also, a deterministic residual demand curve is considered in [20], while in our paper it is modelled as stochastic.

The new contributions of the paper are threefold:

- (i) to assess the impact of a hydro system on market prices based on sets of scenarios that describe the residual demand curves;
- (ii) to construct the supply curves for the day-ahead market, thus providing a production strategy to maximise profits for a price-maker hydro producer;
- (iii) to analyze a realistic case study, based on a Portuguese hydro system in cascaded configuration.

Nomenclature

Ω, ω	set and index of residual demand curve scenarios
K, k	set and index of hours in the considered time horizon
J, j	set and index of reservoirs/plants owned by the hydro producer
S, s	set and index of steps of the residual demand curve
R, r	set and index of the volume intervals
I, i	set and index of the breakpoints of the unit performance curves
M_j, m	set and index of reservoirs upstream to reservoir j
Π_j	feasible operating region of plant j
τ_{mj}	time required for the water discharged/spilled from reservoir m to reach reservoir j , in hours
ρ_ω	probability of scenario ω
$\lambda_{\omega,k}(q_{\omega,k})$	residual demand curve, expressing the market price as a stepwise monotonically decreasing function, for scenario ω in period k , of the price-maker quota
$q_{\omega,k}$	hydro producer's quota for scenario ω in period k
$\varphi _{\tilde{v}^r}(t_{\omega,j,k})$	piecewise linear approximation of the power generation function for a parametric number of water volumes, \tilde{v}^r
$P_{\omega,j,k}$	power generation of plant j for scenario ω in period k
p_j^{\max}, p_j^{\min}	power generation limits of plant j
$\lambda_{\omega,k,s}$	price matching step s of the residual demand curve for scenario ω in period k
$f_{\omega,k,s}$	fraction of the producer's quota for residual demand curve step s
$f_{\omega,k,s}^{\max}$	maximum producer quota for the selected residual demand curve step s for scenario ω in period k
$u_{\omega,k,s}$	binary variable equal to 1 if step s is the last step required to obtain quota $q_{\omega,k}$ for scenario ω in period k , and equal to 0 otherwise

$q_{\omega,k,s}^{\min}$	summation of power blocks from step 1 to step $s-1$ of the residual demand curve for scenario ω in period k
$a_{j,k}$	reservoir j inflow during period k
$v_{\omega,j,k}$	reservoir j storage for scenario ω at end of period k
v_j^{\max}, v_j^{\min}	water storage limits of reservoir j
$v_{j,0}$	initial storage of reservoir j
$v_{j,K}$	final storage of reservoir j
$t_{\omega,j,k}$	plant j discharge for scenario ω during period k
t_j^{\max}, t_j^{\min}	limits on water discharge of plant j
$s_{\omega,j,k}$	spillage of water by reservoir j for scenario ω during period k
$w_{\omega,j,k}$	decision to commit plant j for scenario ω during period k
R_j	discharge ramping limit of plant j
SU_j	plant j start-up cost
$y_{\omega,j,k}$	binary variable equal to 1 if plant j is starting-up for scenario ω at beginning of period k
$z_{\omega,j,k}$	binary variable equal to 1 if plant j is shutting-down for scenario ω at beginning of period k
$g_{\omega,\omega',k}$	binary variable linking the offers for scenarios ω and ω' in hour k
$d_{\omega,j,k,r}$	binary variable choosing the right curve according to the reservoir j volume for interval r ($d_{\omega,j,k,r} = 1$ if $H_{j,r-1} \leq v_{\omega,j,k} < H_{j,r}$)
$m_{\omega,j,k,i}$	binary variable assuring the limits of the water discharged of plant j between the breakpoint i and $i+1$ in period k ($m_{\omega,j,k,i} = 1$ if $T_{j,i-1} \leq t_{\omega,j,k} < T_{j,i}$ or $T_{j,i} \leq t_{\omega,j,k} < T_{j,i+1}$)
$\pi_{\omega,j,k,i}$	weight of breakpoint i for plant j for scenario ω during period k
$T_{j,i}$	plant j discharge at breakpoint i

$P_{j,i,r}$	plant j power output at breakpoint i for interval r
$\Delta P_{j,r}$	maximum power difference between intervals R and r
$H_{j,r}$	reservoir j volume for interval r
A	matrix of constraints
x	vector of decision variables
c	vector of coefficients for the linear term
b^{\max}, b^{\min}	superior and inferior limit vectors on constraints
x^{\max}, x^{\min}	superior and inferior limit vectors on decision variables

2 Price-maker hydro scheduling problem

In the majority of electricity markets generating companies make decisions according to the ones made by the other companies. However, there are few generating companies able to exercise market power, meaning that a perfect competition model cannot be achieved [21–23]. According to the rules set forth by the Iberian Electricity Market (MIBEL), the price-maker producer condition is applied when a company has a market quota higher than 10%, measured in terms of electricity generated within the MIBEL [24]. Such producer is able to strategically manage his flexible hydro plants in the short term, in order to manipulate market prices and maximize profit [18]. The flexibility of the hydro plants is a crucial feature because they can be started and stopped, and output levels can be changed, almost instantaneously. Another feature that confers market power for the producer is his relative size, i.e., the hydro producer can exercise market power by manipulating prices through capacity withholding [25]. Besides, in [18] it is stated that when hydro generators have market power they would tend to allocate more hydro production to off-peak hours than to peak hours, in order to provoke price spikes. By doing so, they can exploit competitors' capacity constraints, reducing their own supply and driving up the market price when demand is at the peak [26].

Accordingly, the optimal bidding of a price-maker hydro producer, and, consequently, the market price for a given hour, is determined by the so-called residual demand curve that defines the market price as a monotonically non-increasing function of the producer's quota [27]. This curve is obtained by subtracting the quantity offered by the competitors from the total demand for each hour. Thus, the market price is obtained as a function of the quantity that the price-maker company offers to the day-ahead market.

The price-maker hydro scheduling problem can be formulated as a non-linear optimization problem with linear constraints, because the profit is the result of the market price multiplied by the quota of the price-maker. To overcome the difficulty of having a non-linear optimization problem, several linearizing methods can be adopted to define the residual demand curves, such as: 1) polynomial approximation; 2) piecewise linear approximation; 3) stepwise approximation [27]. The method adopted in this work is characterized by a stepwise approximation since it is the way bids are made in most pool-based electricity markets. Moreover, the stepwise approximation provides a closer agreement between expected and resulting prices, as stated in [27]. According to [21], the number of steps to describe a residual demand curve is small for fairly small changes in the quota, e.g., a variation of 20% in the quota commonly results in no more than 10 steps. This provides a convenient framework for the construction of the mentioned curves.

Fig. 1 shows a typical residual demand curve. This curve is presented as a pair (quota, price) [28], where it is possible to control the total energy in each step through binary variables, $u_{k,s}$.

The problem of developing the optimal offering strategies for a generating company takes place in a day-ahead market, consisting of 24 hourly auctions. Although the 24 hourly auctions are cleared simultaneously, the result of each auction is based only on the energy offers that have been accepted for that hour at the corresponding marginal price [29].

The residual demand curves of a price-maker producer can be determined: 1) by market simulation or 2) employing forecasting techniques [30]. The hourly residual demand curves are considered as known data, as in [28].

Uncertainty is modelled in this paper by a set of scenarios for the hourly residual demand. The number of scenarios affects the shape of the offer curves decided by the proposed approach. A small number of scenarios may not be enough to describe the uncertainty throughout the decision-making horizon. In [22], eleven scenarios were considered since the variation in the objective function is not relevant for a larger number of scenarios. This proves that the accuracy with which the day-ahead market uncertainty can be represented tends to saturate when the number of scenarios considered increases. Hence, in order to provide a good representation of the producer's interaction within the electricity market, in this paper ten demand residual scenarios are considered for each hour. Indeed, considering a large number of residual demand curve scenarios may result in high CPU times or even intractability, due to the overwhelming need of having binary variables for its modeling. Scenario reduction techniques are only advisable for eliminating scenarios with very low probability and bundle scenarios that are very close [31] while keeping, as much as possible, the stochastic properties of the original one [32].

The hydro producer must settle on the hourly offer curves that should be submitted to the day-ahead market to maximise profit [23]. Deciding the location of the intersection points between residual demand curve scenarios and the offer curve allows selecting the appropriate bids, which should be increasing both in quota and in price. All scenarios of a particular hour are connected by a set of increasing constraints. Fig. 2 shows three residual demand curve scenarios and the corresponding supply curve built through $(q_{\omega,k}, \lambda_{\omega,k,s})$ pairs. According to Fig. 2, each residual demand curve scenario, ω , must have one, and only one, corresponding pair $(q_{\omega,k}, \lambda_{\omega,k,s})$ that must be located in the residual demand curve.

3. SMILP formulation of a price-maker hydro producer

The problem can be stated as to find out the optimal price-quota combination that maximises the total profits of the price-maker hydro producer in the day-ahead market.

3.1 General SMILP approach

The general formulation for a SMILP problem can be defined as:

$$\max F(x) = \sum_{\omega=1}^{\Omega} \rho_{\omega} c^T x \quad (1)$$

subject to:

$$x^{\min} \leq x \leq x^{\max} \quad (2)$$

$$b^{\min} \leq Ax \leq b^{\max} \quad (3)$$

$$x_j \text{ integer } \forall j \in J \quad (4)$$

The market clearing price is considered, in our paper, as a linear function of the producer's quota. Each power production function is approximated through three preset values, $H_{j,r}$, of the storage. Also, for each water volume, the $p_{\omega,j,k} - q_{\omega,j,k}$ relation is characterized by a piecewise linear approximation with four breakpoints, as will be detailed later on.

3.2 SMILP non-linear problem formulation

The concept of residual demand curve, that defines market clearing price as a monotonically non-increasing function of the producer's quota, can be used. Hence, the operation of a hydro producer acting as a price-maker can be mathematically modelled following the formulation of section 3.2 as:

$$\text{Maximize } \sum_{\omega=1}^{\Omega} \rho_{\omega} \sum_{k=1}^K \left[\lambda_{\omega,k}(q_{\omega,k}) q_{\omega,k} - \sum_{j=1}^J SU_j y_{\omega,j,k} \right] \quad (5)$$

subject to:

$$p_{\omega,j,k} \in \Pi_j, \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (6)$$

$$q_{\omega,k} = \sum_{j=1}^J p_{\omega,j,k} \quad \forall \omega \in \Omega, \quad \forall k \in K \quad (7)$$

The first term in (5) is related to the profit of the price-maker hydro producer, while the second term is related to the start-up costs associated with each plant j . This first term is non-linear, since the profit results from multiplying the price by the quota. The set of constraints (6) allows modelling the features of the hydro units, such as, the start-up and shutdown procedures, discrete hydro unit-commitment constraints, ramp rates, minimum and maximum power output constraints, and also the head effects, using an improved linearization method as in [17]. The set of constraints (7) expresses the price-maker quota as the total power production of its units for each hour.

3.3 SMILP linear problem formulation

The previous optimization problem cannot be solved directly using standard software, since the problem is non-linear, discontinuous and large-scale.

Therefore, it is assumed that the residual demand curves can be expressed as stepwise curves, whose step size is connected to the size of the energy block at given prices. An equivalent formulation of the problem based on SMILP is presented in this paper, using continuous and binary variables.

3.3.1 Objective function

The problem can be defined as:

$$\text{Maximize} \quad \sum_{\omega=1}^{\Omega} \rho_{\omega} \sum_{k=1}^K \left[\sum_{s=1}^{n_k} \lambda_{\omega,k,s} (f_{\omega,k,s} + u_{\omega,k,s} q_{\omega,k,s}^{\min}) - \sum_{j=1}^J SU_j y_{\omega,k,j} \right] \quad (8)$$

The objective function expresses the price-maker hydro producer profit in the day-ahead market. The producer's revenues, for hour k , are approximated by a stepwise function using binary variables, as shown in Fig. 1. The binary variables are employed to define each step s of the stepwise function corresponding to the residual demand curve.

3.3.2 Price-maker constraints

These constraints are defined as in [28], given by:

$$q_{\omega,k} = \sum_{j=1}^J p_{\omega,j,k} \quad \forall \omega \in \Omega, \quad \forall k \in K \quad (9)$$

$$q_{\omega,k} = \sum_{s=1}^S (f_{\omega,k,s} + u_{\omega,k,s} q_{\omega,k,s}^{\min}) \quad \forall \omega \in \Omega, \quad \forall k \in K \quad (10)$$

The set of constraints (9) are identical to (7). The set of constraints (10) determines the value of the hydro producer's quota, $q_{\omega,k}$, in every hour, depending on the variables $f_{\omega,k,s}$ and $u_{\omega,k,s}$. The minimum quota for step s is defined by the parameter $q_{\omega,k,s}^{\min}$ (note that $q_{\omega,k,1}^{\min} = 0, \forall k$), whereas the nonnegative continuous variables $f_{\omega,k,s}$ express the additional fraction of step s that is filled. Step s can only be used if step $s - 1$ is fulfilled.

$$0 \leq f_{\omega,k,s} \leq u_{\omega,k,s} f_{\omega,k,s}^{\max} \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (11)$$

In (11) the lower bound assumes nonnegative values for $f_{\omega,k,s}$, whereas the upper bound does not exceed the maximum producer quota for the selected residual demand curve step s for period k ; if step s is not selected $u_{\omega,k,s} = 0$, then the upper bound of $f_{\omega,k,s}$ is zero and $f_{\omega,k,s} = 0$.

$$\sum_{s=1}^S u_{\omega,k,s} = 1 \quad \forall \omega \in \Omega, \quad \forall k \in K, \quad \forall s \in S \quad (12)$$

In (12) the summation of the binary variables that represents each step s of the residual demand curve for an hour k is equal to one. This means that one variable $u_{\omega,k,s}$ only is active in each hour, selecting the optimal step that is added to the producer's quota. If $u_{\omega,k,s} = 1$ then only step s is selected for hour k .

3.3.3 Market offer constraints

In order to develop offering strategies as non-decreasing curves it is needed that all the residual demand curve scenarios are considered for each hour. This implies that each offer

curve is completely defined by a set of pairs of scenarios ω and ω' , with $\omega \neq \omega'$, establishing a link between different scenarios. The difference between ω and ω' is explained as the relative position of two possible residual demand curve scenarios, ω and ω' , where all possible combinations among them is given by $\Omega(\Omega - 1)/2$.

The following conditions are imposed, for each pair of offers (ω, ω') , to ensure that an increasing bid is submitted to the market:

$$q_{\omega,k} - q_{\omega',k} \geq -M^q g_{\omega, \omega',k} \quad \forall \omega, \omega' \in \Omega, \quad \forall k \in K \quad (13)$$

$$q_{\omega',k} - q_{\omega,k} \geq -(1 - g_{\omega, \omega',k}) M^q \quad \forall \omega, \omega' \in \Omega, \quad \forall k \in K \quad (14)$$

$$\sum_{s=1}^S \lambda_{\omega,k,s} u_{\omega,k,s} - \sum_{s=1}^S \lambda_{\omega',k,s} u_{\omega',k,s} \geq -M^p g_{\omega, \omega',k} \quad \forall \omega, \omega' \in \Omega, \quad \forall k \in K, \quad \forall s \in S \quad (15)$$

$$\sum_{s=1}^S \lambda_{\omega',k,s} u_{\omega',k,s} - \sum_{s=1}^S \lambda_{\omega,k,s} u_{\omega,k,s} \geq -(1 - g_{\omega, \omega',k}) M^p \quad \forall \omega, \omega' \in \Omega, \quad \forall k \in K, \quad \forall s \in S \quad (16)$$

Constraints (13)–(16) are compatible with the case of residual demand curves that intersect, such as the ones depicted in Fig. 2. These constraints have been adopted from [7, 23]. According to Fig. 2, each residual demand curve scenario, ω , must have one (and only one) corresponding pair $(q_{\omega,k}, \lambda_{\omega,k,s})$ that must be located in the residual demand curve.

The variable $g_{\omega, \omega',k}$ is a binary variable, M^q and M^p are a large quota and a large price, respectively. If $g_{\omega, \omega',k} = 0$, constraints (13) and (15) are enforced, whereas if $g_{\omega, \omega',k} = 1$, constraints (14) and (16) are active.

3.3.4 Hydro constraints

Hydro constraints are presented hereafter:

$$p_{\omega,j,k} - \varphi|_{\bar{v}^r}(t_{\omega,j,k}) = 0 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (17)$$

In (17), power generation, $p_{\omega,j,k}$, is related to the water discharge and the characteristics of the reservoir. To avoid the non-linearities a piecewise linear approximation of the power generation function is desirable. An enhanced linearization method is considered taking into

account the head change effect. This method corresponds to: 1) an extension of [9] in order to slightly generalize its approach to a parametric number of water volumes, and 2) a more accurate assessment of the power production upper bound through a convex combination method considering both volumes and discharges [17]. Three fixed values $H_{j,r}$ of the storage are implemented. The $p_{\omega,j,k} - q_{\omega,j,k}$ relation is denoted by a linear piecewise approximation with four breakpoints for each water volume.

$$v_{\omega,j,k} = v_{\omega,j,k-1} + a_{j,k} + \sum_{m \in M_j} (t_{\omega,m,k-\tau_{mj}} + s_{\omega,m,k-\tau_{mj}}) - t_{\omega,j,k} - s_{\omega,j,k} \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (18)$$

$$v_j^{\min} \leq v_{\omega,j,k} \leq v_j^{\max} \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (19)$$

$$w_{\omega,j,k} t_j^{\min} \leq t_{\omega,j,k} \leq w_{\omega,j,k} t_j^{\max} \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (20)$$

$$t_{\omega,j,k} - R_j \leq t_{\omega,j,k+1} \leq t_{\omega,j,k} + R_j \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (21)$$

Eq. (18) is related to the water balance equation for each reservoir. In (19), the inferior and superior water storage bounds are defined. In (20), the same happens for water discharge. The binary variable, $w_{\omega,j,k}$, is equal to 1 if plant j is on-line in hour k , and 0 otherwise. Also, constraints on discharge ramp rates are implemented in (21), which might be a consequence of environment or navigational impositions.

$$w_{\omega,j,k} p_j^{\min} \leq p_{\omega,j,k} \leq w_{\omega,j,k} p_j^{\max} \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (22)$$

$$s_{\omega,j,k} \geq 0 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (23)$$

$$y_{\omega,j,k} - z_{\omega,j,k} = w_{\omega,j,k} - w_{\omega,j,k-1} \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (24)$$

$$y_{\omega,j,k} + z_{\omega,j,k} \leq 1 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (25)$$

In (22), the inferior and superior bounds on power generation are defined. Eq. (23) defines a non-negative value for water spillage. Eqs. (24) and (25) model the starting-up and shutting-down of hydro plants.

$$t_{\omega,j,k} - \sum_{i \in Z} T_{j,i} \pi_{\omega,j,k,i} = 0 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (26)$$

$$\sum_{i \in Z} \pi_{\omega,j,k,i} - w_{\omega,j,k} = 0 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (27)$$

$$\pi_{\omega,j,k,i} - m_{\omega,j,k,i} \leq 0 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (28)$$

$$m_{\omega,j,k,i} + m_{\omega,j,k,l} \leq 1 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K, \quad \forall l \in Z: i < l - 1 \quad (29)$$

$$\sum_{r \in R} d_{\omega,j,k,r} = 1 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (30)$$

$$p_{\omega,j,k} - \sum_{i \in Z} P_{j,i,r} \pi_{\omega,j,k,i} - \Delta P_{j,r} (1 - d_{\omega,j,k,r}) \leq 0 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (31)$$

$$v_{\omega,j,k} - \sum_{r \in R} H_{j,r-1} d_{\omega,j,k,r} \geq 0 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (32)$$

$$v_{\omega,j,k} - \sum_{r \in R} H_{j,r} d_{\omega,j,k,r} \leq 0 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (33)$$

Constraints (26)–(33) complete the model (8)–(25) by approximating the function of power production (17) with a parametric number of water volumes. Constraints (26)–(29) express the plant j discharge during period k . According to (28), $\pi_{\omega,j,k,i}$ can only be nonzero if the related binary variable, $m_{\omega,j,k,i}$, is equal to one. Note that $0 \leq \pi_{\omega,j,k,i} \leq 1$. Eq. (30) establishes the logical value of the d variables responsible for determining volumes intervals. Eq. (31) expresses the power generation $p_{\omega,j,k}$ for volume interval r . Constraints (32) and (33) define, for every hour k , the two opposite water volumes of the interval where the calculated volume v_k lies.

$$v_{\omega,j,0} = v_j^{Initial} \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (34)$$

$$v_{\omega,j,K} = v_j^{Final} \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (35)$$

In (34) and (35), the initial, $v_j^{Initial}$, and final, v_j^{Final} , reservoir levels are imposed.

4 Case study

The SMILP approach has been tested on a Portuguese hydro system in cascaded configuration with three reservoirs. Table 1 shows the hydro data. The modelling was carried out in MATLAB environment and solved using CPLEX 12.1, considering a 3.47-GHz dual processor with 48 GB RAM.

Table 1: Hydro data

#	v_j^{\min} (hm ³)	v_j^{\max} (hm ³)	$v_{j,0}$ (hm ³)	p_j^{\min} (MW)	p_j^{\max} (MW)	t_j^{\min} (m ³ /s)	t_j^{\max} (m ³ /s)
1	0	9.90	7.92	0	174.00	111.67	335.00
2	0	13.50	10.80	0	191.00	110.00	330.00
3	0	26.40	21.12	0	240.00	140.00	408.89

Final storage in the reservoirs is assumed identical to the initial value. The targets on storage can be determined by medium-term horizons [33]. The start-up costs of the hydro units are assumed to be given by $SU_j = p_j^{\max} \times 2.5$, and forbidden zones are taken into account using (20). The time horizon considered is one day, since we model a day-ahead market. The residual demand curves are expressed as stepwise functions with 5 steps. The total number of scenarios for the residual demand curves is 10.

Fig. 3 shows the residual demand curves scenarios at hours 1, 5, 6, 12, 16 and 23. The optimal solution corresponds to the specific points that define the optimal offering strategies to submit in the day-ahead market for each residual demand curve.

In order to prove the efficiency of the SMILP approach, a comparison with another approach (called AP.1) is provided. The AP.1 approach also considers water discharges in forbidden zones, discharge ramping constraints and units start/stops, but the power generation is considered to be depending linearly on water discharge, thus disregarding head dependency.

Table 2 summarizes the dimensions of the two optimization problems.

Table 2: Optimizing characteristics of each approach

#	AP.1	SMILP
Constraints	11760	24720
Continuous variables	3600	7200
Binary variables	4440	9480

The optimal storages of the reservoirs are presented in Fig. 4 for scenario 1. The optimal discharges of the plants are presented in Fig. 5. A different behaviour is likely to be verified in Fig. 4 for the first and second reservoirs, as the AP.1 approach ignores the head change effect. Indeed, the water storage in the first reservoir is higher for the proposed approach, increasing the head variation between consecutive reservoirs.

The comparison of SMILP with AP.1 approach results, shown in Fig. 4, reveals the influence of considering the head change effect in the behavior of the reservoirs. The upstream reservoir should operate at a suitable high storage level in order to benefit the power generation efficiency of its associated plant, due to the head change effect. Hence, the storage trajectory of the upstream reservoir is pulled up using the SMILP approach. Instead, the storage trajectory of the last downstream reservoir is pulled down using the SMILP approach, thereby improving the head for the immediately upstream reservoirs.

The results in Fig. 5 are consistent with those in Fig. 4. Hence, even if the average water discharge is equal for both approaches, the average storage and the average quota are higher with the SMILP approach, as shown in Table 3.

Table 3: Computational results for both approaches

Approach	Average Discharge (%)	Average Storage (%)	Average Market Clearing Price (€/MWh)	Average Quota (MW)
AP.1	65.25	69.94	28.67	398.37
SMILP	65.25	70.11	24.38	485.72

Table 4 provides the maximum achievable profit for each approach.

Table 4: Expected total profits for both approaches

Approach	Expected Total Profit (€)	CPU Time (s)	MIP Gap (%)
AP.1	246946	384	0.99
SMILP	261129	542	0.99

Higher quota and reservoir storage (Table 3) reveal that head effect modeling provides better results than linear power generation functions. Optimal profit results in Table 4 do indicate that head effect modeling leads to higher profits. However, please note that this might not be valid in some cases. Particularly, a price-maker hydro producer might seek to maximise profit by withholding production (and thus reducing quota) and/or spilling water (thus reducing storage) in order to provoke price spikes, according to [34].

The expected total profit for the SMILP approach is 5.74% higher than the one obtained with the AP.1 approach, while the CPU time is still acceptable as well as the MIP Gap.

The CPLEX 12.1 solver used on a large scale problem provides the MIP Gap parameter to improve its performance. The MIP Gap is a good parameter of optimality used as convergence criteria. In [35], the MIP Gap is computed by:

$$\text{MIP Gap} = \frac{|\text{best node} - \text{best integer}|}{10^{-10} + |\text{best integer}|} \quad (36)$$

In (36), a small enough positive constant is included to ensure that this criterion can be used for the case where the best integer objective takes null value.

Fig. 6 shows the profit results versus CPU time for the SMILP approach. The proposed approach finds a near-optimal solution relatively fast and spends the remaining time verifying the solution. A feasible solution is found when the gap among the best integer objective and the objective of the best node remaining is within a predefined tolerance, considered to be 1% in this paper.

The SMILP approach terminates with a solution of 261129 €, which has a MIP Gap of 0.99%. A MIP gap whose value is less than 1% can be considered acceptable for a large-scale optimization problem, as stated in [36]. If a MIP Gap of 0.6% were considered, for instance,

the SMILP approach would take 1924 seconds to obtain a solution of 261983 €. This translates into a profit increase of only 0.327%, for a considerable CPU time increase of 255%. Hence, a further decrease in the MIP Gap parameter pushes the solver to find a better solution, but much longer CPU times only yield slightly higher benefits.

To assess the effectiveness of the SMILP approach over a deterministic approach, the value of the stochastic solution (VSS) has been determined. VSS is given by: $VSS = z_{SP} - z_{DP}$, where z_{SP} is the expected profit of the stochastic problem and z_{DP} is the expected profit obtained from the problem where decisions variables are fixed to those resulting from the associated deterministic problem, i.e., from the problem in which stochastic processes are changed by their respective expected values [37].

Aiming for a fair and unambiguous comparison, constraints (13)–(16) are not considered in this comparison. In other words, we disregard the construction of robust supply curves that take into account all residual demand curve scenarios, i.e., the absence of these constraints only considers the optimal point associated to each scenario ω . Thus, each optimal point corresponds to a possible producer's offering strategy. Besides, if two scenarios belong to the same bundle at a time, i.e., are identical, the corresponding operating decisions should be identical too, ensuring the principle of non-anticipativity. Therefore, the VSS is given by: $VSS = 282377 - 267180 = 15197$ €, i.e., $VSS (\%) = 5.69\%$.

Note that 267180 € is the average profit achieved for the optimizer of the deterministic problem. Thus, the solution achieved using the SMILP approach is noticeably better than the one achieved by a deterministic approach.

The average hourly production is shown in Fig. 7, and the respective average market-clearing prices are shown in Fig. 8. The average production is equal to the sum of the products of quota and probability in each scenario ω . According to Fig. 7 and Fig. 8, it is possible to verify that the market-clearing prices do not generally follow the variation pattern of the quota of the price-maker producer, meaning that market power is being implemented. This variation

pattern can be employed as a monitoring variable to measure market power.

Fig. 9 shows the offer curves developed for the day-ahead market, with and without the head change effect. The SMILP approach implies higher quotas for the same price values. Hence, the SMILP approach provides substantially better solutions, within a reasonable CPU time, for head-sensitive price-maker hydro producers with cascaded configurations.

5 Conclusions

A generation scheduling problem for a price-maker hydro producer has been studied to find out the optimal combination of price and quantity bids that maximises the producer's total profits in the day-ahead market. The model is thoroughly tested on a case assuming a realistic hydro system in cascaded configuration in Portugal. The potential market power of a price-maker hydro producer has been analysed. Main results include short-term offering strategies and the resulting market clearing prices, as well as the optimal reservoir storage and plant discharge trajectories, using a novel stochastic mixed-integer linear programming approach that considers sets of scenarios to describe the residual demand curves. The proposed approach assures significantly better results, for head-sensitive price-maker hydro producers with cascaded configurations, guaranteeing also an acceptable CPU time.

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7 References

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Figure captions

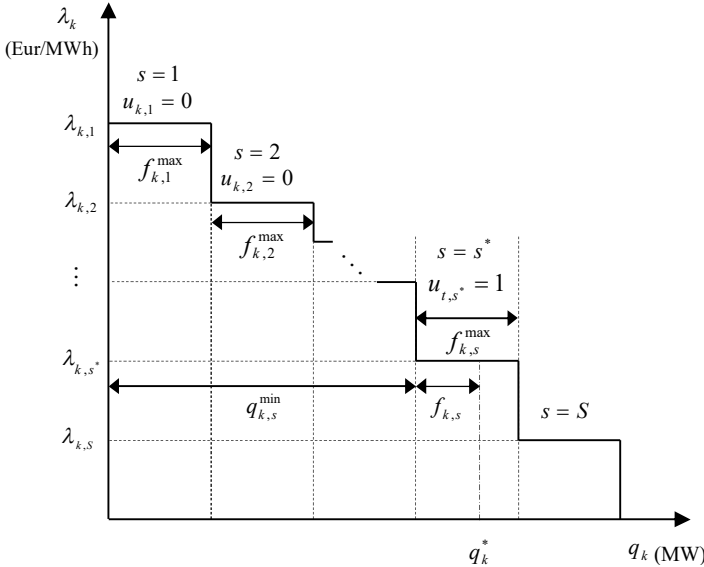


Fig. 1. Typical residual demand curve [25].

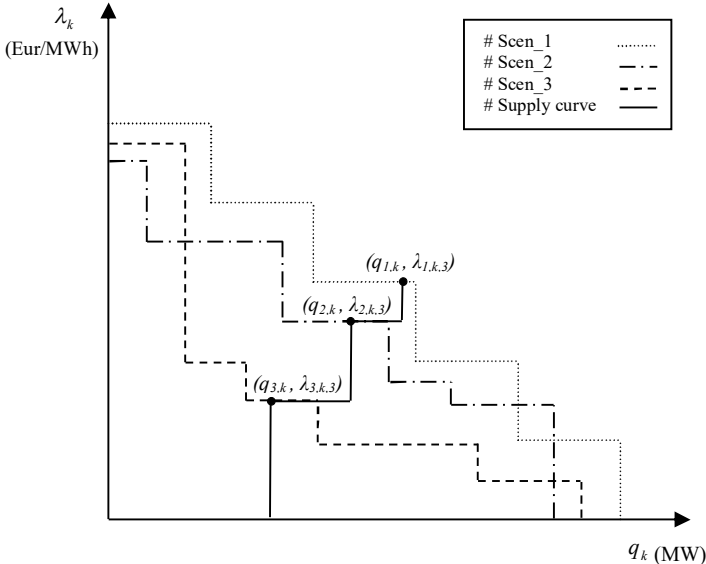


Fig. 2. Supply curve built through $(q_{\omega,k}, \lambda_{\omega,k,s})$ pairs.

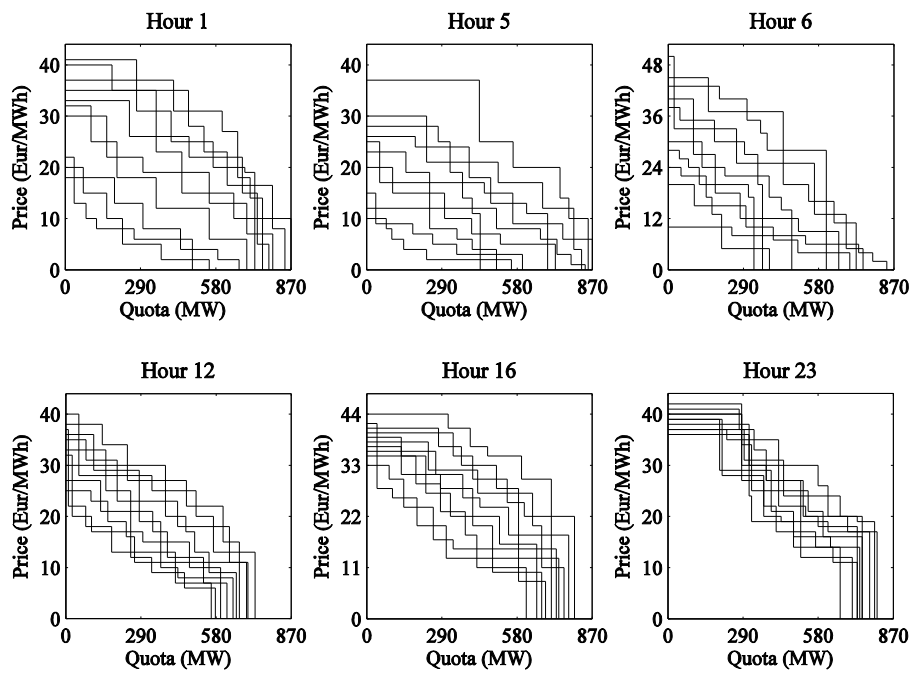


Fig. 3. Residual demand curves scenarios at hours 1, 5, 6, 12, 16 and 23.

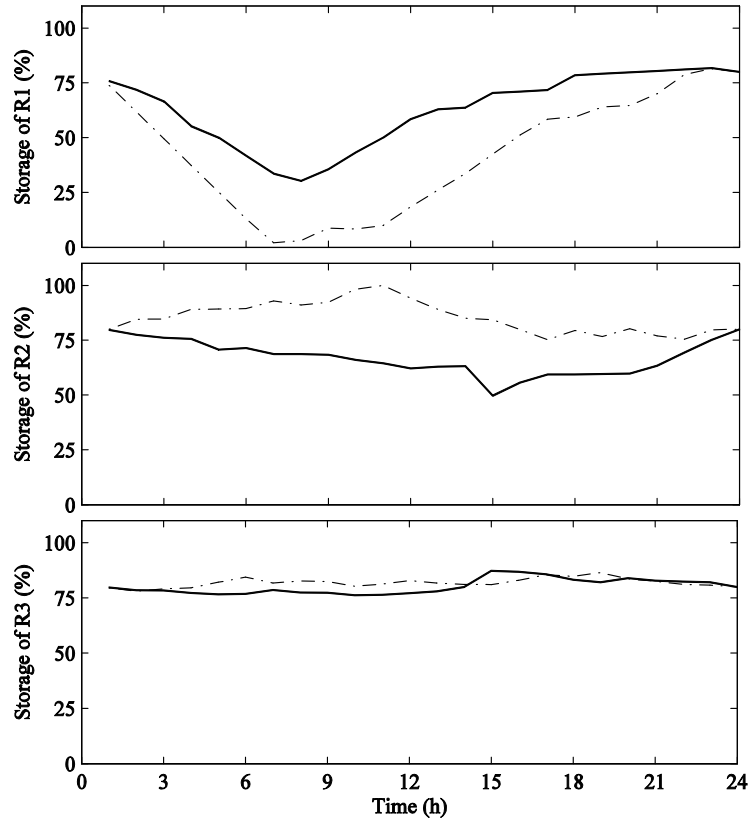


Fig. 4. Optimal reservoir storages for scenario 1. The results of the proposed approach are denoted by a solid line, whereas the AP.1 approach results are denoted by a dashed line.

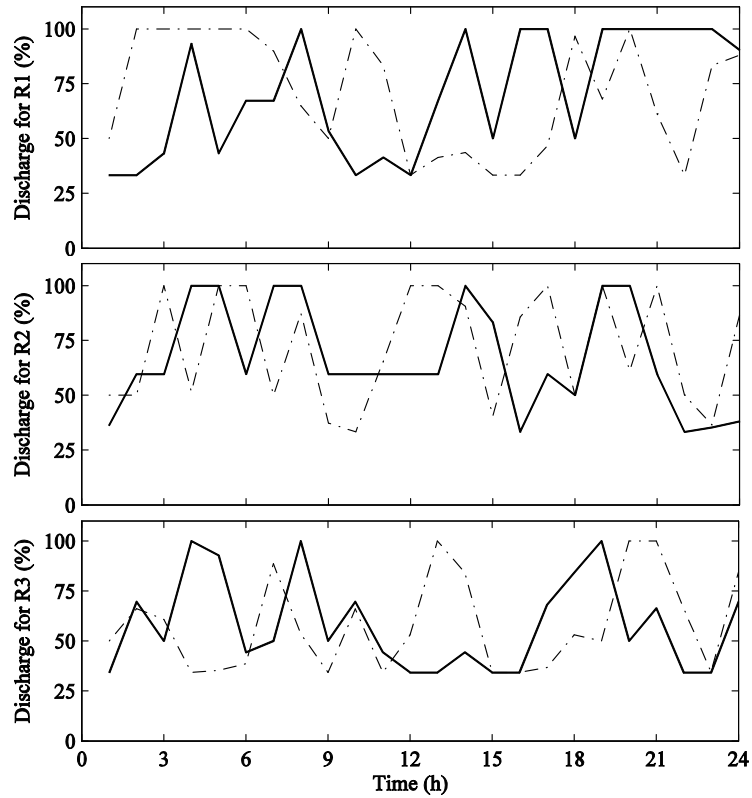


Fig. 5. Optimal plant discharges for scenario 1. The results of the proposed approach are denoted by a solid line, whereas the AP.1 approach results are denoted by a dashed line.

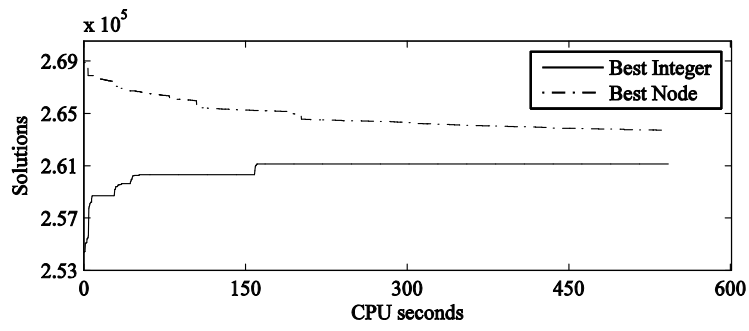


Fig. 6. Solutions versus CPU time.

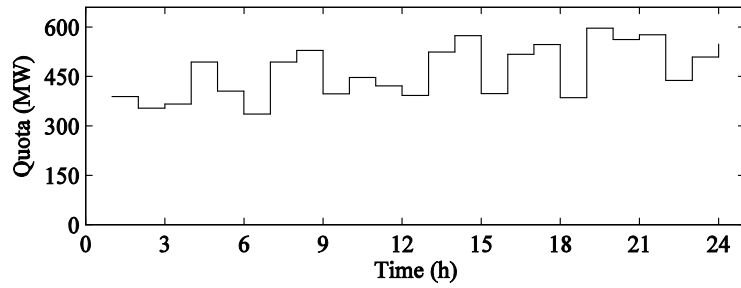


Fig. 7. Average hourly production.

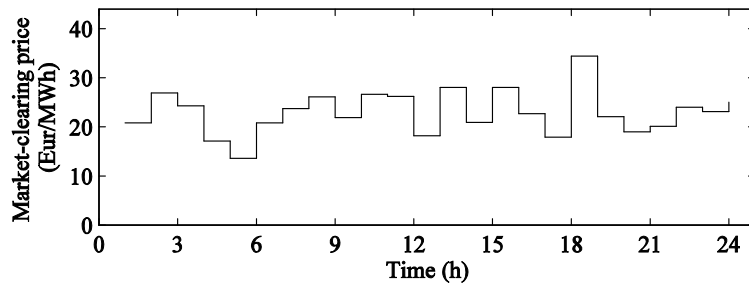


Fig. 8. Average market-clearing prices.

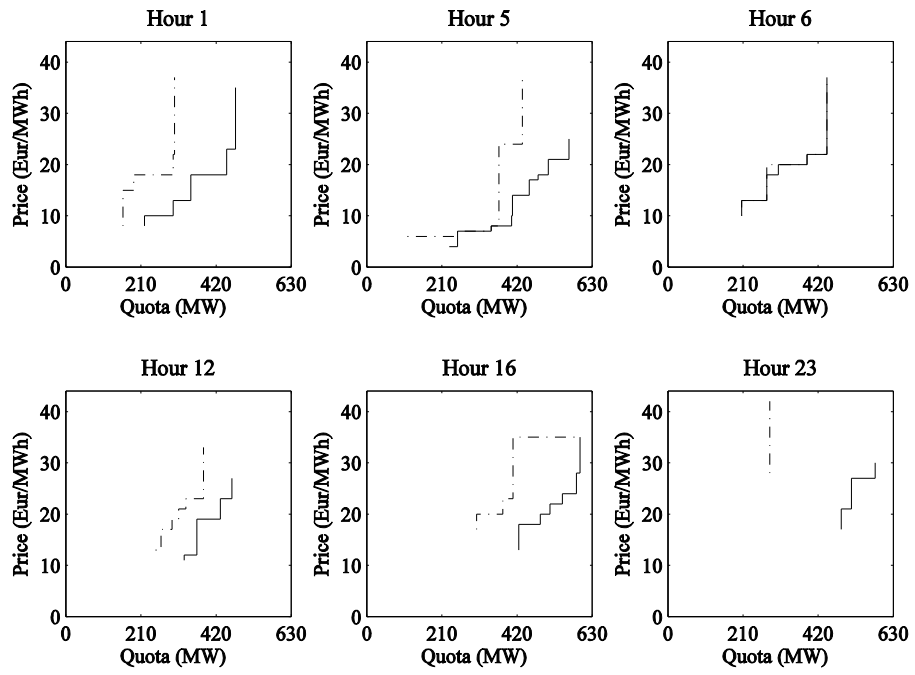


Fig. 9. Offering strategies for the day-ahead market, with and without head change effect. The results with the proposed approach are denoted by a solid line, whereas the AP.1 approach results are denoted by a dashed line.