Stochastic Modeling of Lead-Acid Battery Parameters

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***Abstract* – Renewable energies are in constant growth and evolution, being a clean way to provide the energy required for the sustainable development of human society. In this context, energy storage systems are a key factor in the integration of renewable generation, because through them, the flexibility of the power system can be increased. Lead-acid batteries have been extensively used to provide electricity in isolated and rural locations, and could be integrated to the smart grid in order to improve its performance. However, this is a complex element due to its working principle, specifically during charging periods. In this paper, a general purpose model is formulated from a probabilistic point-of-view in order to determine the range of possible values of state-of-charge due to the uncertainty and to estimate the battery efficiency. A case study is analyzed and the results are compared with Monte Carlo Simulation approach in order to evaluate the proposed model.**

***Index Terms* – Battery energy storage systems; Battery efficiency; Lead-acid batteries; Smart grid.**

I. Introduction

Renewable energy is one of the most promising power sources to ensure the industrial and economic development of the human society in a clean and sustainable way. This fact combined with a moderate growing in energy demand, the increasing use of natural gas, and the well-known behaviour of conventional power plants [1] have motivated researchers worldwide to solve technical problems related with the integration of wind and solar photovoltaic (PV) generation, mainly due to the variability and natural uncertainty of these energy sources.

Energy storage systems can mitigate the effects of renewable integration by increasing the flexibility of the power system. Moreover, with the modernization of power system in order to provide it with smart capabilities battery energy storage systems (BESSs) have gain importance. Within BESSs technologies, lead-acid batteries have been extensively analysed in the literature for rural or isolated electrification [2], load levelling of power systems [3], and controlling of smart grids [4].

Optimal design of hybrid energy systems consists on finding a rated power of PV generator and wind turbine, besides of rated capacity of BESSs in order to minimize the net present cost (NPC) or levelized cost of energy. This task is carried out by using an optimization technique such as genetic algorithms, which require a detail mathematical model of the energy system [5]. Under this context, the accuracy of the model used is crucial to successfully design and control the energy system and take advantage of the energetic resources available.

Coulombic efficiency (CE) is a critical parameter that can influence the estimation of state-of-charge (SOC) at a determined instant in a significant way, specifically during charging process. According to the experimental analysis carried out by [6], from SOC between its minimum value and about 79%, charging efficiency is about 80%. Then, for higher values of SOC, specifically 84% CE reduces to 54%. This operative condition in a typical PV system could influence the estimation on available energy on BESS, due that available resources are used to supply BESS losses during charging process. The aforementioned topic has been carefully analysed in the literature.

Regarding modelling and design of wind/battery systems, if CE charge controller operation and ambient temperature are not considered, an increment of 25% would be required in order to reach the reliability level desired [7]. Moreover, when a combined system (integration between PV, wind, conventional, and BESS) is analysed, a variation of approximately 33% is observed in the number of hours of operation of diesel generators, 31% in the fuel consumption, and 31% in the estimation of. However, the aforementioned variations depend on the hybrid system configuration and the BESS size [8].

Several models of lead-acid batteries have been proposed in the literature specially to analyse the dynamic behaviour of PV systems. For instance, in [9], a general purposes model was developed based on the assumption that the product of internal resistance and the battery capacity remains constant for two different types of lead-acid batteries. Hence, generalized expressions for discharge and charge voltage, variation of the battery capacity with the current, charge efficiency, and SOC were estimated.

In [10], a model was able to include the ageing processes by means of the application of weighted factors Ampère-hour (Ah) throughput. In more detail, the behaviour of the battery depends on its depth-of-discharge (DOD), battery current, acid stratification, and time since its last full charging. By this way, control strategy and sizing of battery bank under different operative conditions can be carried out by taking into account the end-of-charge voltage, frequency of full charging, gassings periods, and maximum DOD.

In [11], a comparative analysis between lead-acid battery models considering the effects of charge controller operation on battery lifetime was presented. Weighted Ah throughput model was implemented and evaluated, observing an estimation error between 6% and 14% depending on the application.

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In [12], the operative categories were defined taking into account some stress factors such as charge factor, Ah throughput, highest discharge rate, time between full charge, time with low SOC, partial cycling, low environmental temperature, and temperature acceleration factor, making possible a development of several recommendations and categorical analysis for isolated renewable energy systems.

In this work, experimental information reported in [6] is combined with the model presented in [9] using a probabilistic perspective. In specific, the information shown in both references are used to build the probability distribution of uncertain parameters related to the estimation of charging efficiency. Hence, the corresponding distribution is estimated. In order to corroborate the obtained results, a comparative analysis with Monte Carlo simulation (MCS) approach is presented. The paper is organized as follow: Section II describes the probabilistic model of lead-acid batteries including deterministic model, the analysis of uncertain parameters, and the probabilistic SOC estimation. In Section III is proposed the modelling method through the analysis of charging process of a single cell presented and finally, conclusions are presented in Section IV.

II. PROBABILISTIC MODEL OF LEAD-ACID BATTERIES

Frequently, the main variable considered for controlling and optimizing energy systems provided with BESSs is SOC. This variable depends on the specific characteristics of battery under analysis and the interaction between the system components, available renewable resources, among other factors. It will present a deterministic general purposes model in Subsection II-A, while in Subsection II-B and Subsection II-C present the analysis of uncertain parameters related to charge efficiency, and the method to estimate PDF of SOC, respectively, and finally the model of probabilistic SOC used in Subsection II-D.

A. General Purposes Lead-Acid Battery Model

Deterministic model used in this work is previously proposed by [9], where maximum storage capacity available (C_T) is estimated according to (1), C_{10} is the capacity in 10h, the capacity (C) available at a determined charge current (I) is calculated by using (2), while charge efficiency (η_c) is estimated through (3), and actual SOC by (4):

$$\boldsymbol{C_T = 1.67C_{10}(1 + 0.005\Delta T)} \quad (1)$$

$$\boldsymbol{C = \frac{C_T}{1 + 0.67\left(\frac{I}{I_{10}}\right)^{0.9}}} \quad (2)$$

$$\boldsymbol{\eta_c = 1 + exp\left[\frac{a}{I/I_{10} + b}(SOC_t - 1)\right]} \quad (3)$$

$$\boldsymbol{SOC_t = \left(\frac{Q}{C}\right)\eta_c} \quad (4)$$

where $Q = Tt$ are the Ah delivered to the battery during charge process, $\Delta T = T - 25$ is the variation of ambient temperature from the reference value of 25°C, I_{10} is battery current at 10h, while a and b are coefficient to be experimentally determined.

B. Analysis of Uncertain Parameters

Equations (1) and (2) have general character and these could be easily estimated from data typically provided by the manufacturers, if a specific battery needs to be simulated. Moreover, the parameters a and b depends on the constructive characteristics of the battery under analysis, e.g., frequently values used for these coefficients are $a = 20.73$ and $b = 0.55$ fitted by using batteries with tubular positive plates and low-antimony alloys [9].

Fig. 1 shows the behavior of charging efficiency presented in [6] (measurement in Fig. 1) and the corresponding curve described by (3) (fitting in Fig. 1), where the values obtained were $a = 2.53$ and $b = 0.48$, respectively. Fig. 2 presents the behavior of both curves for SOC between 0.3 and 1. It is possible to note the important difference between both models. However, the information provided by them could be integrated in a probabilistic model of BESS to estimate SOC values. Then, this work aims to estimate the probability distribution function (PDF) of coefficients a and b using the experimental data available in [6] and [9], respectively.

C. Probabilistic State-of-Charge Estimation

Fig. 3 shows the scheme of the proposed probabilistic model. Besides of coefficients a and b in the model, charge current and SOC at previous time step $(t-1)$ have been considered as probabilistic variables. Assuming that the behavior of forecasting error could be reasonable modeled by a Beta PDF [13], all probabilistic variables have been represented by using this distribution. PDF of coefficients a and b, charge current (I), and SOC at time $t-1$ (SOC_{t-1}) are incorporated into the model through a discretization process. The method used in this work was originally developed in [14]. This method discretizes Beta PDF in the compact interval $\{0, 1\}$. Discretized variable is represented by the set (K) according to (5):

$$K = \{k_q, P_r\{k_q\}; q = 0,1, \dots, Q\} \quad (5)$$

where q is the discretization state, k_q is the value of corresponding state q, while $P_r\{k_q\}$ is the probability associated with state. Values k_q are determined by using (6):

$$\boldsymbol{k_q = \begin{cases} max\left[\left\{\frac{q}{Q} - \frac{\lambda}{Q}, 0\right\}; \frac{q}{Q} - \frac{\lambda}{Q} + \frac{1}{Q}\right); q = 0, \dots, Q-1 \\ \left[\frac{q}{Q} - \frac{\lambda}{Q}, 1\right]; q = Q \end{cases}} \quad (6)$$

where Q is the total amount of discrete states and λ is a coefficient to be adjusted by user, typically equal to 1/2. Then, the probability of each discrete state is calculated through (7):

$$\boldsymbol{P_r\{k_q\} = \frac{(1+q)^{\alpha-1}(Q+1-q)^{\beta-1}}{\sum_{m=0}^{Q}(1+m)^{\alpha-1}(Q+1-m)^{\beta-1}}, \; q = 0, 1, \dots, Q} \quad (7)$$

where α and β are the parameters of Beta PDF. Fig. 4 shows each of parameters involved in discretization process previously described.

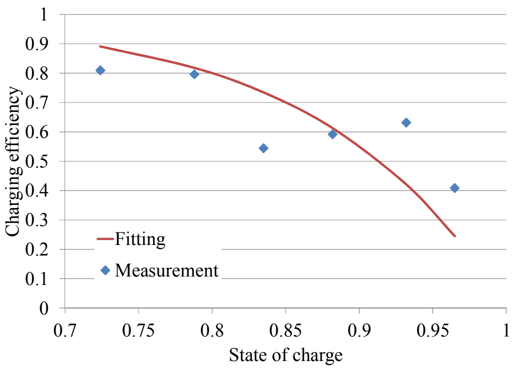

Fig. 1. Fitting of efficiency model using data measured in [6].

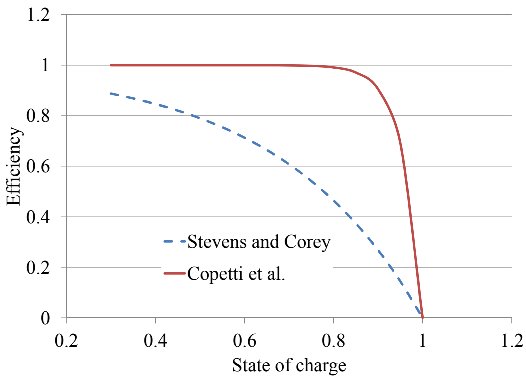

Fig. 2. Comparison of efficiency model from [6], and [9] data.

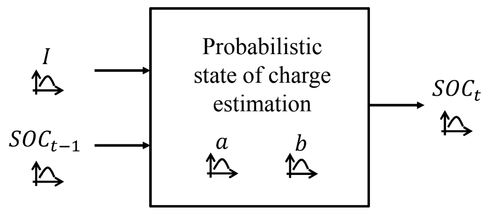

Fig. 3. Probabilistic lead-acid battery model.

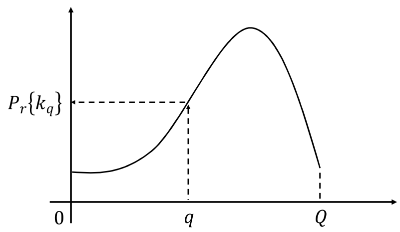

Fig. 4. Discretized Beta PDF.

Finally, all values in the compact interval are displaced in order to represent the corresponding variable of interest (F). In this step a new discrete state defined as $h = q + 1$ is introduced. For example, if the coefficient a is discretized, in the formulation of (8), it is assumed that $a = F$, f_{max} would be equal to the maximum value of a, and f_{min} would be equal to the minimum value of a.

This maximum and minimum values are obtained from the experimental data. A similar procedure should be followed to discretize the other variables of the probabilistic model.

$$\boldsymbol{F = \{f_h = (f_{max} - f_{min})k_{h-1} + f_{min};\ h = 1, \dots, H\}} \qquad \textbf{(8)}$$

As the values reported in [9], which were determined by using more batteries than in [6], more probability has been assigned to those values. Considering $a \in \{2, 21\}$ and $b \in [0.48, 0.56]$. Beta PDF was fitted, obtaining for parameter a: $\alpha = 0.76$ and $\beta = 0.27$, and for parameter b: $\alpha = 21.3$ and $\beta = 1.1$. Fig 5 and Fig. 6 show the discretized variables a and b considering 9 intervals ($H = 9$). These variables are then used to determine PDF of SOC at current time instant by means of the model presented in Subsection II-A.

D. *Model of Probabilistic State-of-Charge*

Once parameters a and b have been probabilistically modeled, these parameters are incorporated on the battery model in order to estimate PDF of SOC. The general procedure is shown in Fig. 7.

Let f_h^a, f_h^b, $f_h^{SOC_{t-1}}$, and f_h^I be the value of discretized state (h) of the corresponding probabilistic variable (a, b, SOC_{t-1}, I) of (8).

Then, the value of SOC under this specific conditions $\left(SOC_t^{a,b,SOC,I}\right)$ can be found by evaluating the model presented in (1)-(4), and finally, using the corresponding probabilities, PDF of SOC_t can be built.

Once the of $\left(SOC_t^{a,b,SOC,I}\right)$ has been calculated, this is used to complete PDF of SOC_t. To carry out this task, a determined number of intervals (N) to represent PDF are assigned, and the discrete step ($\Delta dSOC$) is calculated according to (9):

$$\boldsymbol{\Delta dSOC = \frac{(1 - SOC_{min})}{(N-1)}} \qquad \textbf{(9)}$$

where SOC_{min} is the minimum SOC. The value ($dSOC_n; n = 1,2, \dots, N$]) of each discrete state (n) is estimated by means of (10):

$$\boldsymbol{dSOC_n = SOC_{min}, SOC_{min} + \Delta dSOC, \dots, 1} \qquad \textbf{(10)}$$

Then, for each value $\left(SOC_t^{a,b,SOC,I}\right)$ in the algorithm of Fig. 7, the corresponding probability value is assigned by following the algorithm presented in Fig. 8, where the probabilities $P_r\left\{SOC_t = SOC_t^{a,b,SOC,t}\right\}$ can be saved in a vector previously initialized to zero.

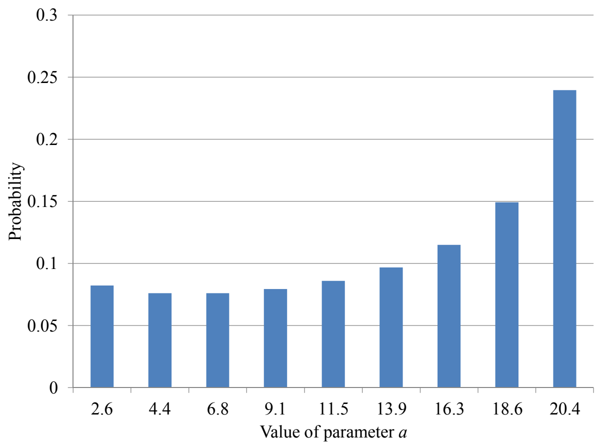

Fig. 5. Probabilistic representation of parameter a.

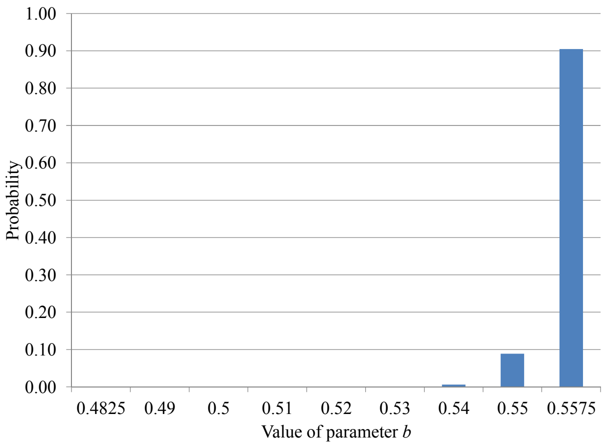

Fig. 6. Probabilistic representation of parameter b.

for f_h^a; $h = 1, \dots, H$
  for f_h^b; $h = 1, \dots, H$
    for $f_h^{SOC_{t-1}}$; $h = 1, \dots, H$
      for f_h^I; $h = 1, \dots, H$
        • Estimate $SOC_t = SOC_t^{a,b,SOC,I}$
        • Complete PDF of SOC_t
      end
    end
  end
end

Fig. 7. Algorithm to build PDF of SOC.

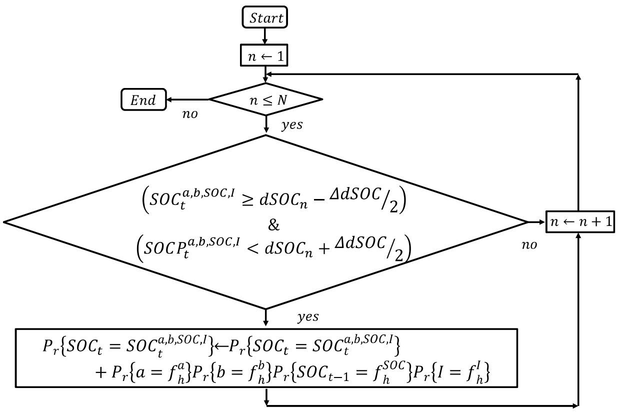

Fig. 8. Algorithm to complete PDF of SOC.

III. ENERGY SYSTEM MODELLING

The probabilistic methodology proposed in this work is illustrated by analyzing the dynamic behavior of a typical lead-acid battery with capacity in 10h equal to 180Ah ($C_{10} =$ 180Ah) and minimum SOC of 30% ($SOC_{min} = 0.3$). SOC at previous time step $t-1$ was modeled as a Beta PDF with $\alpha = 5$ and $\beta = 2.5$ in interval $\{SOC_{min}, 1\}$, while discretization process was carried out by considering 9 intervals ($H = 9$) and $\lambda = 0.5$.

The model of this variable is shown in Fig. 9, where SOC has been expressed in Ah. In a similar way, charge current was modeled by considering a Beta PDF with $\alpha = 5$ and $\beta =$ 2.5 in the interval $\{0, I_{10}\}$discretized with 9 intervals, and the obtained model is presented in Fig. 10.

Regarding the representation of SOC at current time (t), this was built by considering 150 intervals ($N = 150$). In order to verify the results obtained from the probabilistic approach, it was compared with MCS approach with 5,000 trials. Comparative results are presented in Fig. 11, where important differences can be observed, mainly related to the reduced amount of intervals ($H = 9$) used to model each probabilistic variable.

In other words, from results obtained in Fig. 11, as the number of intervals is increased, results obtained from MCS and probabilistic approach converges to the same behavior, and observed in Fig. 12, where $H = 17$ has been used, however, the computational burden increases considerably.

Table I presents maximum, expected and minimum values considering a significance level of 1%. As can be observed, the results obtained from the proposed model can reasonable estimate the extreme and average values without losing too much the information and with a reduced computational time, which makes it suitable for simulations in yearly basis, considering the typical application of this type of simulation models.

The proposed model presented in this work was implemented in MATLAB programming language; using a computer provided with a i7-3630QM CPU at 2.40GHz, 8GB of RAM and 64-bit operating system.

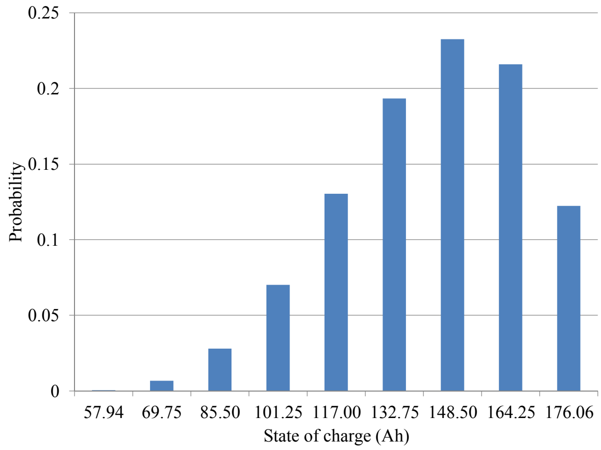

Fig. 9. Probabilistic representation of SOC at previous time step.

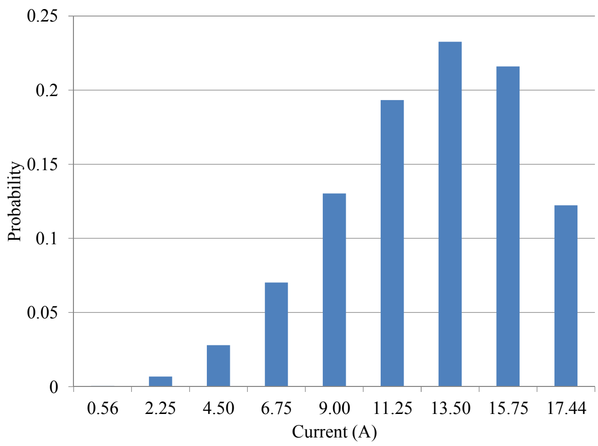

Fig. 10. Probabilistic representation of charge current.

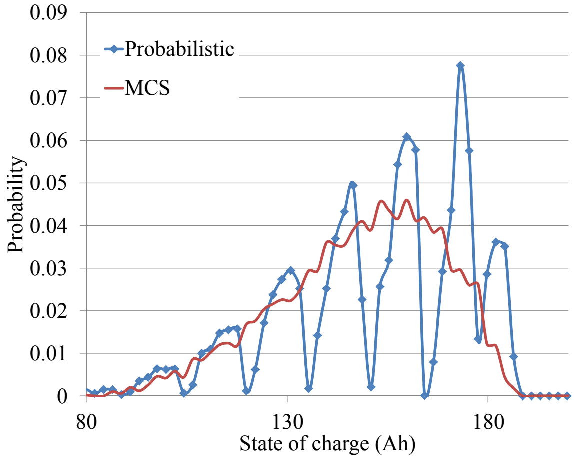

Fig. 11. Comparison of SOC PDF at actual time step ($H = 9$).

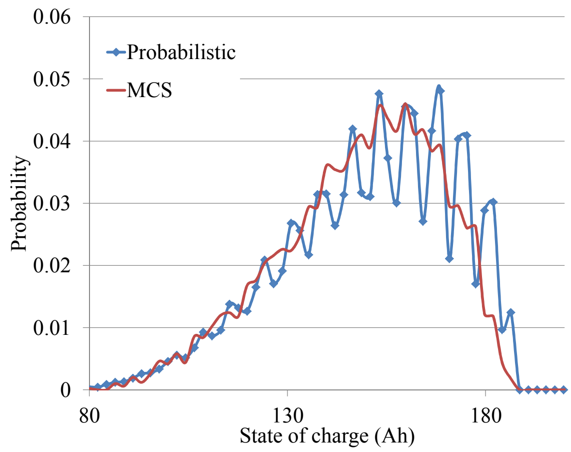

Fig. 12. Comparison of SOC PDF at actual time step ($H = 17$).

TABLE I
COMPARATIVE RESULTS

$SOC_t(Ah)$	MCS (5,000 trials)	Proposed (*H*=9)	Proposed (*H*=17)
Minimum	96.486723	91.910538	93.516201
Expected	148.307890	152.179101	150.367768
Maximum	181.259304	184.142344	184.627168
Time (s)	52.893	4.922	57.268

IV. CONCLUSIONS

BESSs are in continuous technological development because they are a key element in the integration and increment of penetration level of renewable power sources at residential level, specifically under smart grid environments. This storage technology has a complex behavior, difficult to represent in general perspective, and for this reason in this work a probabilistic treatment of lead-acid batteries has been proposed and illustrated, specifically able to model the imprecision related to the efficiency during charge process. From the analysis of a single cell, it was possible to conclude that the proposed model can reasonably estimate extreme and average conditions without losing too much information and with a reduced computational time.

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