Optimal Operation of Energy Hubs Considering Uncertainties and Different Time Resolutions

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*Abstract***--This paper presents a robust chance-constrained optimization framework for the optimal operation management of an energy hub in the presence of electrical, heating, and cooling demands and renewable power generation. The proposed strategy can be used for optimal decision making of operators of energy hubs (EHs) or energy providers. The electrical energy storage device in the studied energy hub can handle the fluctuations in operating points raised by such uncertainties. In order to model the hourly demands and renewable power generation uncertainties, a robust chanceconstrained close-to-real-time model is adopted in this paper. The considered energy hub in this study follows a centralized framework and the energy hub operator is responsible for the optimal operation of the hub assets based on the day-ahead scheduling. A thorough analysis of energy flows with different carriers is presented. In addition, a numerical stability test regarding the selection of the time step size is performed to guarantee the solution's time resolution independence, occurring in previous studies.**

*Index Terms***--Electrical energy storage, energy hub, stochastic programming, optimal operation**

NOMENCLATURE

Parameters

Sets

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Variables

Symbols and Abbreviations

H2G	Hub to Grid transactions
G2H	Grid to Hub transactions
CHP	Combined Heat and Power
EES	Electrical Energy Storage
COP	Coefficient of Performance
$Ch.$, Dis.	Charge and discharge
Max, Min	Maximum and minimum

I. INTRODUCTION

A. Background and Motivation

More than twelve years ago, the vision of Energy Hub (EH) was firstly proposed by *Giedl et al.* [1], [2], as a framework for optimal simultaneous operation of a system with multiple energy carriers. Being an ambitiously futuristic proposal at the time, not much dedication was given to the EH research in the subsequent few years.

Integrating backup systems and energy conservation measures, all of which rely on the high penetration of local generation and storage resources would help achieve a smart grid (SG) [3], [4]. Aiming at using the maximum potential of local resources, an increased integration of different energy carriers has to be resorted. Often being inside a single distributed generation (DG) or storage unit, this intertwining of different energy carriers gave rise to "ubiquitous energy" paradigms, being generic systems with various intermittent distributed energy resources (DERs), relying on a wide spectrum of carriers for energy conversion or storage [5]. As SG moved further from being purely electrical, particularly in the demand side, challenges in economic management and operation became more pronounced [6], [7]. This unavoidable and increased intertwining of different energy carriers revived interest in the concept EHs; due to them being an ideal modelling and optimization approach to ensure suitable techno-economic and environment-friendly operation of hybrid SGs [8], [9].

B. Review of Recent Scientific Literature

In [10], an EH-based model for microgrids was introduced containing thermal and gas-based subnetworks. The proposed centralized optimal operation framework aimed at minimizing the operating costs. Stochastic programming was used for uncertainties of loads and renewable energy sources (RESs) in a mixed-integer linear programming (MILP) model. A graph theory approach was used in [11] to formulate a standard matrix-based EH model based on a conversion matrix corresponding to different energy carriers. The EH was treated as a "black box", providing a direct conversion between input and output energies by different carriers. Conversion efficiencies inside the EH were assumed to be constant without accounting for inherent uncertainties. While the proposed generalized coupling matrix was nonlinear, it was shown how to decrease the nonlinearity of an EH optimization model.

A centralized control algorithm for voltage regulation of a ring DC microgrid connected to different EHs was proposed in [12], based on coordination between the voltage source converters and the EHs. To this end, an energy management system (EMS) for each EH is used to control its local resources: a battery tank, an Electrolyzer, a Proton Exchange Membrane Fuel Cell (PEMFC), a chiller, a compressor, and a hydrogen conversion/storage unit. The proposed algorithm was capable of successfully improving the voltage profile in the electrical network. A risk-constrained dynamic stochastic optimization model has been developed in Ref. [13] for the day-ahead operation of an EH. A distributionally robust optimization technique has been suggested in Ref. [14] to optimally schedule an EH, equipped with an energy storage system in the presence of multimodal forecast errors of the photovoltaic (PV) panel's power output. The problem has been formulated as a two-stage programming model, where the first stage is implemented to mitigate the cost and the second stage includes a real-time dispatch with determined PV power output forecast. In [15], 100% RES-based EHs were considered with local wind and solar DGs, in addition to a biomass digester. A mixed-integer programming (MIP) model was proposed in [16] for planning of EHs, aimed at obtaining the optimal configuration of EHs, taking into consideration a large set of candidate components of various systems (RESs, storage systems, and conventional generation units). While conversion efficiencies were considered constant, a scenario-based approach was used to account for RES uncertainty. A receding-horizon scheme based framework has been proposed in [17], where diverse generation and energy storage technologies have been taken into account to optimally control the flow of active and reactive power in a microgrid. Furthermore, an energy management system has been developed in [3] for a microgrid at The University of Genoa in Italy. This microgrid includes different technologies for generating power, such as trigeneration units, renewable energy based technologies, local heating, besides thermal and electrical storage technologies. Ref. [18] proposed a cooperative scheduling framework for an EH community, where EHs are connected to the same electricity and gas utilities, and communicating in a bargaining game framework to obtain the global optimal schedule for all players. Local EMSs at each EH utilized exchanged information to obtain the optimal scheduling of local resources, including combined heat and power (CHP) plants, gas furnaces, storage devices, and local RES-based DGs. However, the load demand and RES uncertainties were not applied to the model. Distributed optimization was performed by decomposing the global objective function for all EHs, using the alternating direction method of multipliers (ADMM). The results indicated that the cooperative scheduling outperforms the non-cooperative one from an economic perspective. Another cooperative framework, presented in [19] was based on an event-triggered framework for the day-ahead and real-time operation of EHs. In [20], a deterministic MILP framework has been developed for profitseeking EHs, participating in electricity and heat markets with distribution companies. Ref. [21] also developed a framework for EHs, participating in a deregulated market, albeit using a decentralized EMS. Both algorithms showed significant technical and economic benefits of employing the EMSs considering market participation. A hybrid artificial neuro-fuzzy inference system (ANFIS) genetic algorithm (GA) based framework was developed in [22] for the multiobjective optimization of EHs. While the considered EH was minimal, with no local RES or storage units, the proposed model was applicable to any generic case due to the machine

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learning (ML) approach employed. Similarly, while uncertainties were not explicitly incorporated, their effects were captured by the proposed ML algorithm. In [23], a twostage Bender's decomposition model (LP and MILP) and robust optimization were proposed for the resilience enhancement in regional-district EHs. The two-stage Bender's decomposition based framework has also been utilized in Ref. [24] for the integrated planning and operation of an EH. Recently, the chance constrained method has been widely applied to power system problems, among which a two-stage chance constrained stochastic MILP framework has been used in Ref. [25] for a hybrid ac/dc microgrid integrated EH platform with multiple EHs. A chance-constrained optimization has been presented in Ref. [26] and used to tackle the optimal energy flow problem in an interconnected system, comprised of three EHs while it is aimed at minimizing the operating costs. RESs, storage systems, and conventional generation units were included in the EHs, interconnected through the electricity and gas distribution networks while supplying electrical and thermal loads. The study showed the success of the proposed approach in enhancing the resilience of the systems under study. In [27], the ADMM was used to develop a game-theoretic auctioning mechanism for a building EH with multiple users and an EH manager. The algorithm's objective was to achieve a globally optimal (Nash equilibrium) scheduling of EH resources to satisfy all users based on their submitted demand bids.

C. Novel Contributions

 This paper proposes a robust chance-constrained framework for the optimal energy hub management in the presence of uncertainties. In this respect, the proposed robust chance-constrained technique would guarantee that the probability of satisfying a certain constraint is above a certain level. This method is relatively straightforward to implement and it does not make the original problem more complicated. Besides, the loadability of the electrical, heating and cooling demands have been maximized in the presence of the unexpected uncertainties. Accordingly, it is specified that to what extent the proposed solution is robust against the increase in the load demand. Furthermore, the time resolutions of 60 min, 30 min, 15 min, and 1 min have been simulated and results are discussed to present a close-to-real operation model. Besides, the role of the energy storage system in the optimal operation of the EH, facing severe uncertainties in near real-time operation has been comprehensively investigated.

D. Paper Organization

Section II provides the conceptual model used for the EH under study. Section III demonstrates the mathematical modelling of the problem and its reformulation within the proposed robust chance-constrained framework, besides the case study modeling. Section IV presents the simulation results and a numerical stability analysis using different time discretization values. Finally, the conclusions and future prospects are given in Section V.

II. CONCEPTUAL MODEL OF THE ENERGY HUB

The EH model considered in this paper is illustrated in Fig. 1. Connections with the electrical and natural gas (NG) grids provide the input energy sources. The EH has electrical and thermal (both heating and cooling) loads. Within the EH, a local solar PV generation unit is installed.

Fig. 1. Illustration of the EH considered and modelled in this study.

 An auxiliary boiler, a combined heat and power (CHP) unit, and an electric heat pump (EHP) supply the heat load demand. The cooling load demand is supplied by an absorption chiller, together with the EHP which can operate in either heating or cooling mode. EHs can only accommodate RES and DG units within a limited size, generally a few kilowatts. Although the operating costs of such units are low, they may bring severe challenges into the EH optimal operation due to the uncertainties associated with their input energy carriers which are inherently affected by chaotic systems (e.g. meteorological effects). Therefore, in addition to the energy converters, an electrical energy storage (EES) system has been installed to mitigate the load demand and generation fluctuations in addition to decreasing the operating costs. EES systems can be managed in a way to store energy during off-peak hours, and also during hours of excess solar generation. The EES system can also be utilized to supply the required energy for the assets to meet the cooling and heating load demands. In addition, a scenariobased stochastic approach is employed to consider uncertainties in generation and load.

III. MATHEMATICAL MODELLING

The proposed EH management model is a scenario-based stochastic optimization framework, in which the optimal operation of the EH is modeled as a MILP minimization problem. The problem formulation is represented as a chance-constrained optimization model. The details of the mathematical formulation are presented in this section.

A. Chance-constrained Optimization Approach

The chance-constrained optimization approach is one of the most practical approaches that can be implemented in large-scale stochastic problems with a high uncertainty level. The generalized model for the chance-constrained optimization in this study aims at finding the optimal operating points of the hub assets. The mathematical optimization problem using the chance-constrained framework is as follows:

Min $f(x,\xi)$

subject to :
\n
$$
g(x,\xi) = 0
$$

\n $h(x,\xi) \ge 0$ (1)

Where *x* is the decision vector, ξ is the uncertainty vector, *g* and *h* represent the equality constraints and inequality constraints respectively. By applying the chance-constrained method, the equality constraint is represented as [28]:

$$
Pr(g(x,\xi)=0) \ge 1-\varepsilon
$$
 (2)

Where *ε* represents the permissible risk level to be defined. In such a case, including numerous unlikely scenarios, the robustness of the solutions in the presence of the unpredicted uncertain scenarios would be guaranteed with a certain probability. In this paper, the chance-constrained approach is adopted for solving the energy hub assets' optimal operating point determination problem in the presence of uncertainties.

B. Energy Hub Optimization Problem

The objective function to be minimized incorporates all costs of the EH, in which the optimal operation is decided based on minimizing the total costs. The EH is connected to the electrical grid with which it transacts energy, resulting in costs/revenues based on purchased/sold energy. This is also taken into consideration in the objective function, which is shown in Eq. (3) while *s* and *t* indicate the scenarios and the time intervals respectively. The objective function comprises three parts. The first part accounts for the cost of energy transaction with the electrical grid. The second part shows the energy generation cost of the energy hub assets and finally, the last part represents the electrical, heating, and cooling energy losses. It should be noticed that the problem can be solved for each season separately according to the corresponding data set of each season.

Min

$$
\sum_{s=1}^{N_S} \omega_s \left[\sum_{t=1}^{N_T} \left[\left(PG_{s,t}^{G2H} \lambda_t^{Buv} - PG_{s,t}^{H2G} \lambda_t^{Sell} \right) + F_{s,t}^{CHP} + F_{s,t}^{Boller} + \right] \right] \qquad (3)
$$

The optimization problem is addressed as a scenario-based stochastic optimization and *ω* is the occurrence probability of each scenario. Moreover, *f*, $PG_{s,t}^{G2H}$, and $PG_{s,t}^{H2G}$ are the operating cost, power transmitted to the EH from the electrical grid and the energy transmitted to the electrical grid from the EH respectively. Lastly, λ_i^{Sell} and λ_i^{Buy} indicate the energy selling and purchasing prices, respectively. The objective function is subject to physical and operational constraints, which are presented subsequently.

$$
F_{s,t}^{CHP} = \left(\frac{PG_{s,t}^{CHP}}{\eta_E^{CHP}} + \frac{HG_{s,t}^{CHP}}{\eta_H^{CHP}}\right) \lambda_t^{NG}
$$
(4)

$$
F_{s,t}^{Boiler} = \left(\frac{PH_{s,t}^{Boiler}}{\eta_H^{Boiler}}\right) \lambda_t^{NG}
$$
\n(5)

The operating cost of the CHP unit is represented in Eq. (4), where λ_i^{NG} is the hourly natural gas price, changing with respect to different hours and seasons of the year. Furthermore, $PG_{s,t}^{CHP}$ and $PH_{s,t}^{CHP}$ indicate the electrical power and the heat generated by the CHP unit, respectively. The generation cost of the auxiliary boiler is represented in Eq. (5) accordingly. The EES system is presented in the EH, besides other assets while its operating cost is assumed negligible. The following constraints indicate the technical limitations of the system.

$$
PS_{s,t}^{CHP,\min} I_{s,t}^{CHP} \le PG_{s,t}^{CHP} + PH_{s,t}^{CHP} \le PS_{s,t}^{CHP,\max} I_{s,t}^{CHP}
$$
(6)

$$
PG_{s,t}^{CHP,\min} I_{s,t}^{CHP} \le PG_{s,t}^{CHP} \le PG_{s,t}^{CHP,\max} I_{s,t}^{CHP} \tag{7}
$$

$$
PH_{s,t}^{CHP,\min}I_{s,t}^{CHP} \le PH_{s,t}^{CHP} \le PH_{s,t}^{CHP,\max}I_{s,t}^{CHP} \tag{8}
$$

$$
PH_{s,t}^{Boiler, Min}I_{s,t}^{Boiler} \leq PH_{s,t}^{Boiler} \leq PH_{s,t}^{Boiler, Max}I_{s,t}^{Boiler}
$$
\n
$$
(9)
$$

$$
PC^{Children,Min} I_{s,t}^{Children} \le PC_{s,t}^{Children} \le PC^{Children,Max} I_{s,t}^{Children} \tag{10}
$$

$$
PC_{s,t}^{Chiller} = PH_{s,t} COP^{Chiller} \tag{11}
$$

$$
PC^{EHP,Min}I_{s,t}^{EHP, Cooling} \le PC_{s,t}^{EHP} \le PC^{EHP,Max}I_{s,t}^{EHP, Cooling}
$$
 (12)

$$
PH^{EHP,Min}I_{s,t}^{EHP,Heating} \le PH_{s,t}^{EHP} \le PH^{EHP,Max}I_{s,t}^{EHP,Heating}
$$
 (13)

$$
0 \le I_{i,t}^{EHP,Heating} + I_{i,t}^{EHP, Cooling} \le 1
$$
\n⁽¹⁴⁾

The binary variable *I* is assigned to the model to determine the status of an asset. Constraints (6)-(8) indicate the linear mathematical formulation of the CHP unit presented in [2], [29]. It is noted that the Big-M method has been used in this paper and the feasible operating region of the CHP unit is characterized using a convex square. $I_{s,t}^{CHP}$ forces the CHP unit to operate in the feasible operating region when it is ON. The heat generated by the boiler and the chiller has been modeled in Eq. (9) and Eq. (10) respectively. The chiller absorbs heat and its output is cooling power. In this regard, the energy conversion factor of the chiller is denoted by *COP*^{Chiller} in (11). It should be considered that the EHP operates in one of the two modes: either providing the heating demand ($PH_{s,t}^{EHP}$) or cooling demand ($PC_{s,t}^{EHP}$). Hence, the EHP should operate in one mode as stated in Eqs. (12)-(14).

$$
PC_{s,t}^{EHP} = PG_{s,t} COP^{EHP, Cooling}
$$
\n
$$
(15)
$$

$$
PH_{s,t}^{EHP,Heating} = PG_{s,t} COP^{EHP,Heating}
$$
\n
$$
(16)
$$

$$
0 \le PH_{s,t}^{Heater} \le PH_{s,t}^{Heater,Max} I_{s,t}^{Heater}
$$
\n
$$
(17)
$$

$$
PH_{s,t}^{Heater} = PG_{s,t} COP^{Heater}
$$
\n
$$
(18)
$$

$$
Eng_{s,t}^{EES} = Eng_{s,t-1}^{EES} + PG_{s,t}^{EES,Ch} \eta^{EES,Ch.} - PG_{s,t}^{EES,Dis.} / \eta^{EES,Dis.}
$$
\n(19)

$$
Eng^{EES,Min} \leq Eng_{s,t}^{EES} \leq Eng^{EES,Max}
$$
\n(20)

$$
Eng_{t=0}^{EES} = Eng_{t=24}^{EES}
$$
 (21)

$$
0 \le PG_{s,t}^{EES,Ch.} \le PG_{s,t}^{EES,Ch.,Max} I_{s,t}^{EES,Ch.}
$$
\n
$$
(22)
$$

$$
0 \le PG_{s,t}^{EES, Dis.} \le PG_{s,t}^{EES, Dis. \text{Max}} I_{s,t}^{EES, Dis.} \tag{23}
$$

$$
0 \le I_{s,t}^{EES,Ch.} + I_{s,t}^{EES,Dis.} \le 1 \tag{24}
$$

$$
PG_{s,t}^{EES} = PG_{s,t}^{EES,Dis.} - PG_{s,t}^{EES,Ch.}
$$
\n(25)

$$
0 \le PG_{s,t}^{G2H} \le PL^{Max} I_{s,t}^{G2H} \tag{26}
$$

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$$
0 \le PG_{s,t}^{H2G} \le PL^{Max} I_{s,t}^{H2G} \tag{27}
$$

$$
0 \le I_{s,t}^{G2H} + I_{s,t}^{H2G} \le 1
$$
\n(28)

Equations (15) and (16) depict the electricity to heat or cooling energy conversion model, where *COP*^{EHP, Heating} and *EHP Cooling* , *COP* indicate the capacity of transforming electricity to heat and cooling power respectively. The constraint, relating to the heat generated by the electric heater is stated in Eq. (17) while the conversion of electricity to heat is represented by Eq. (18). The constraints of the EES unit are presented in Eqs. (19)-(25) [30], [31], while Eqs. (26)-(28) model the energy transacted between the EH and the electrical grid. It is noteworthy that a more detailed model for the operation of EES systems can be found in [32] and [33]. PL^{Max} denotes the capacity of the feeder connecting the EH to the electrical grid. The power balance constraints in Eqs. (29)-(31) are of paramount importance for the electrical, heating, and cooling power. In this respect, the CHP, EES, and PV units, as well as the electrical grid are capable of supplying the required electric energy, while the CHP, EHP, electric heater and the boiler units are supposed to generate heat. The absorption chiller and the EHP are utilized for the cooling power procurement. It is noted that the constraints have been formulated in the robust chance-constraint framework. Moreover, the loading factors of the electrical, heat and cooling load demand have been assigned to the model to take into account the worst loading conditions while they are assumed identical and intended to be maximized.

$$
Pr\left[\begin{bmatrix}PG_{s,t}^{G2H} + PG_{s,t}^{CHP} + PG_{s,t}^{EES} + PG_{s,t}^{PV} \\ -(1 + \alpha^{Elec.})P_{s,t}^{Load} - W_{s,t}^{Elec.} \end{bmatrix}\right] \ge 1 - \varepsilon
$$
(29)

$$
Pr\left[\begin{bmatrix}PH_{s,t}^{Boiler} + PH_{s,t}^{Heater} + PH_{s,t}^{CHP} + PH_{s,t}^{EHP,Heating} \\ -\left(1+\alpha^{Heating}\right)PH_{s,t}^{Load} - W_{s,t}^{Heating} \end{bmatrix}\right] \ge 1-\varepsilon
$$
 (30)

$$
Pr\left[\begin{bmatrix}PC_{s,t}^{Children} + PC_{s,t}^{EHP, Collins} \end{bmatrix} \ge 1 - \varepsilon \begin{pmatrix}-(1+\alpha^{Cooling})PC_{s,t}^{Cooling}\end{pmatrix} \ge 1 - \varepsilon \end{pmatrix} \ge 1 - \varepsilon \tag{31}
$$

The proposed chance-constrained model in this paper aims to provide the optimal operating points of the hub assets for the unpredicted uncertain conditions. The load balance equations are reformulated in such a way to provide the robust solutions in the presence of all unpredicted scenarios. Therefore, the hub operator can guarantee the load demand supply with a scaling factor for each type of loads. The corresponding loading factors, α^{Elec} , $\alpha^{Hearing}$ and $\alpha^{Cooling}$ are supposed to be identical to represented the worst case scenarios in this study. In this paper, the Big M approach is adopted to solve the chance-constrained stochastic optimization problem. In order to achieve the maximum loading conditions, new binary variables are suggested to handle the feasibility of the equality constraints, i.e. load balances in this paper. Therefore, the load balance equations can be reformulated as:

$$
\[PG_{s,t}^{G2H} + PG_{s,t}^{CHP} + PG_{s,t}^{EES} + PG_{s,t}^{PV}\] - (1 + \alpha^{Elec.}) P_{s,t}^{Load} - W_{s,t}^{Elec.} \leq M z_{s,t}^{Elec.} \tag{32}
$$

$$
\left[PH_{s,t}^{Boiler} + PH_{s,t}^{Heater} + PH_{s,t}^{CHP} + PH_{s,t}^{EHP,Heating}\right] - \left(1 + \alpha^{Heating}\right)PH_{s,t}^{Load} - W_{s,t}^{Heating} \le M z_{s,t}^{Heating}
$$
(33)

$$
\left[PC_{s,t}^{Children} + PC_{s,t}^{EHP, Coding} \right] - \left(1 + \alpha^{Cooling}\right) PC_{s,t}^{Load} - W_{s,t}^{Cooling} \le M z_{s,t}^{Cooling} \tag{34}
$$

The corresponding binary variables in (32)-(34), $z_{s,t}^{Elec.}$ Z ^{*Elec*}., , $Z_{s,t}^{Heating}$, and $Z_{s,t}^{Cooling}$ are proposed to convert the electricity, heating and cooling load balance equations respectively to the equivalent deterministic balance equations. The parameter *M* should be sufficiently large to avoid any violation occurrence whenever it happens. In case of violation, the corresponding binary variables will be '1'. Hence, the number of activated scenarios due to the violations must be less than the predefined risk index:

$$
\sum_{s=1}^{N_S} \sum_{t=1}^{N_T} \omega_s z_{s,t}^{Elec.} \leq \varepsilon \qquad , \ \ z_{s,t}^{Elec.} \in \{0,1\} \tag{35}
$$

$$
\sum_{s=1}^{N_S} \sum_{t=1}^{N_T} \omega_s z_{s,t}^{Heating} \le \varepsilon \quad , \quad z_{s,t}^{Heating} \in \{0,1\} \tag{36}
$$

$$
\sum_{s=1}^{N_S} \sum_{t=1}^{N_T} \omega_s z_{s,t}^{Cooling} \le \varepsilon \quad , \quad z_{s,t}^{Cooling} \in \{0,1\}
$$

Equations $(35)-(37)$ confirm that the probability of the violated scenarios should be less than the predefined risk index targeted by the hub operator.

IV. CASE STUDY SIMULATIONS

A. Energy Hub Characteristics

To test and validate the proposed optimization framework for the EH energy management, the model is simulated using the data of the EH in [8], which was illustrated in Fig. 1. The EH in this paper is a commercial building with the peak electricity demand about 500 kW for summer weekdays and the installed capacity of PV panel is 30 kW. The size of the absorption chiller and electrical heater are 75 kW and 300 kW respectively, while the EES has a rated capacity of 300 kWh with 10 kW charging and discharging capability at each hour. The EHP size is 200 kW and it can work either in heating or cooling mode. The CHP characteristics are reported in [34] and [35] with the maximum and minimum values of power generation limits of 375 kW and 100 kW respectively. Besides, the lower and upper bound of the heat generation of the CHP unit are 125 kW and 0 kW respectively. The CHP unit has a convex feasible operating region and its electrical and thermal efficiencies are 0.55 and 0.45 respectively. The capacity of the boiler is 200 kW and its thermal efficiency is 0.5. The rated capacity of the power transformer is 300 kW and there is no limit on the capacity of the gas pipeline.

In this section, the analyses performed on the case study are presented and discussed in detail. The optimization is performed for a day-ahead operation with a granularity of 60 minutes (i.e., one-hour time step).

B. Input Scenarios

Seasonal variations have a critical influence on EHs due to the presence of multiple carriers, RES-based generation, and most importantly thermal networks alongside the electrical one. Therefore, it is of paramount importance to test the

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proposed model separately for winter and summer. For each season, it is also necessary to account for the uncertainties associated with the loads and PV generation. In this study, two different seasons, i.e. summer and winter, have been considered to prove that the proposed method is working well.

For each season, 12 weeks have been considered and the first and the last week data are removed. For 10 weeks, the dataset is clustered according to the 5 working days and 2 days for weekends. Therefore, there are 50 working days and according to the mean-variance and correlation matrix of the

cooling, heating, electricity and PV power generation, scenario generation has been carried out.

The scenario generation is adopted according to [36] and scenario reduction technique is based on the method proposed in [30], applied to the generated scenarios to achieve the 10 scenarios. As a result, the dataset generated has a size of $24\times1000\times4$. Accordingly, there are 1000 scenarios for each day, including 4 daily datasets for electricity, heating, cooling and PV power generation.

Fig. 2. The 10 scenarios for: (a) summer electrical load demand, (b), summer cooling load demand, (c) summer heating load demand, (d) summer PV power generation, (e) winter electrical load demand, (f) winter cooling load demand, (g) winter heating load demand, and (h) winter PV power generation.

Fig. 3. Hourly electricity and natural gas prices, (a) summer working days; (b), winter working days

C. Simulation Results

An overview of the obtained optimal asset operation strategy is shown in Table I and the subsequent figures and tables. The simulation results show that the CHP unit operates with the maximum capacity to generate electricity without any heating power due to the relatively high electricity tariff and low NG price. Moreover, the cooling load demand is supplied through the absorption chiller along with the EHP. As the electricity price is high, the heater is not used to supply the heat load demand and instead the boiler generates the heat. It is worth noting that the EHP is used only for a few hours and due to the operation limitation of such a unit, it operates only in the cooling mode. In winter, the electrical and cooling load demands reduce which in turn mitigates the electricity price compared to summer. The boiler is permanently operated together with the CHP unit to supply the heat load demand while the heater is only used during some hours. The EHP is quite inactive in winter and it can be considered for the annual maintenance, as the cooling load demand is at its minimum amount and the absorption chiller is used to supply the demand.

Unit	Asset Operation Strategy		
	Summer	Winter	
CHP Heat	Not Used	Fully Used	
CHP Elec.	Fully Used	Fully Used	
EHP Heat	Not Used	Not Used	
EHP Cool	Partially Used	Not Used	
Boiler	Fully Used	Fully Used	
Chiller Fully Used		Partially Used	
EES	Fully Used	Fully Used	
Heater	Not Used	Partially Used	

TABLE I. OPTIMAL OPERATION STRATEGY FOR HUB ASSETS

Furthermore, an analysis has been carried out to specify the role of the EES system in mitigating the EH's operating cost. On the other hand, the NG price would be high, and thus, it is mainly used to supply the heat load demand. The CHP unit is used in winter in a way to simultaneously generate electricity and heat. The generation of this asset is almost constant at each hour in all scenarios. This means that the energy hub operator can fix the hourly operating point of the CHP unit for each scenario. The boiler permanently operates together with the CHP unit to supply the heat load demand while as it has been previously mentioned, the heater is used only over some hours.

The hourly electricity tariffs for weekdays during summer and winter are provided in Fig. 3. As can be seen, the electricity tariff for summer includes three periods, while the number of periods for winter is two. The peak hours for summer start at 11:00 and continue until 18:00. The energy price during winter has a smooth trend during 8:00-21:00. It is worth mentioning that the selling and buying prices have been considered to be the same in summer to encourage the EH to sell the surplus electricity to the grid. It means that the grid services and taxes are excluded in the pricing mechanism and it brings some incentives for the EH operator to benefit from participation in the market. The selling price of energy to the grid is 0.9 of the energy purchase price in winter as there is a surplus in the utility grid energy. Moreover, considering the same prices for energy exchange with the grid provides some benefits to the grid operator to buy from one end-user and sell it to another one without any additional cost. It is noteworthy that the simulation has been done for three different cases as: i) $\alpha = 0$ and $\varepsilon = 0$. ii) Optimally determining α for $\varepsilon = 0$. iii) Optimally determining α for $\varepsilon = 0.05$ and $\varepsilon = 0.10$.

Table II addresses the daily operating cost of the hub serving the loads including/excluding the impacts of the energy storage device. The difference between these two scenarios confirms that the energy storage device can effectively reduce the daily operating cost. The generation of this asset is almost constant at each hour in all scenarios. This means that the energy hub operator can fix the hourly operating point of the CHP unit for each scenario. The boiler permanently operates together with the CHP unit to supply the heat load demand while as it has been previously mentioned, the heater is used only over some hours. Moreover, an analysis has been carried out to specify the role of the EES system in mitigating the energy hub's operating cost. EES system would be able to store energy over the offpeak hours when the electricity price is low and contributes to supplying the electrical load demand when the energy price is high. Fig. 4 depicts the stored energy in the battery for the energy hub operation in summer and winter for different time resolutions. The simulation results show that in the presence of EES, the charging and discharging of the battery will be smoothed moderately by increasing the time resolution. This fact confirms that increasing the time resolution can definitely provide a narrow band for charging and discharging of the battery during the operational horizon. Therefore, the trend of storing energy will be smooth. The energy stored in the battery during winter and summer implicitly follows the energy prices. For a summer working day, the electricity price is considerably high between 11:00- 18:00, while during winter, the working day's electricity tariff is much more flat.

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Fig. 4. Numerical stability test demonstrated by convergence of the results for the EES SoC for the summer (left) and winter (right), for $\Delta t = 60, 30, 15,$ and 1 minutes in (a), (b), (c) and (d) respectively.

Season	Resolution (min.)	Daily Expected Operational Cost (S)		
		With EES	Without EES	
Summer	60	4229.9881	4377.9881	
Winter	60	5658.9593	5687.5520	
Summer	30	4293.9313	4368.9313	
Winter	30	5697.0823	5711.0330	
Summer	15	4326.5583	4364.0114	
Winter	15	5711.7873	5718.8525	
Summer	1	4358.6645	4361.1553	
Winter		5720.2720	5720.7428	

TABLE II. OPTIMAL DAILY OPER ATION COST

TABLE III. EXECUTION TIME

The peak interval of a winter working day occurs between 8:00-21:00. Moreover, the electricity and heating demands are considerably high during the evening and night, while the PV power generation is zero at this time interval. The electric heater is partially used and all the mentioned factors result in the strategic discharging of the EES during the evening and night-time intervals. The simulation results have been further elaborated, particularly concerning the peak period. The main premise of the provided comparison is to validate the numerical stability of the mathematical formulation by proving that the solution for the SoC profile converges to a "real-time" solution as the duration of each time interval tends to zero ($\Delta T \rightarrow 0$). In other words, the approximation error tends to zero as the duration tends to zero, verifying the numerical stability of the mathematical model.

D. Numerical Stability Analysis (Effect of Time Step Size)

In almost all previously published works, the use of a onehour time step was assumed *a priori* without proper justification or validation thereof. The choice of an adequate time step size is of paramount importance for two reasons:

• A very large time step would often yield inaccurate and thereby unreliable results.

• More importantly, if the numerical approximation is not properly formulated, the model would diverge as the discretization variable tends to zero (in this case, time).

Therefore, all simulations have been repeated by considering smaller time steps of 30 minutes, 15 minutes and 1 minute. As Fig. 4 shows, the numerical stability is confirmed by observing that the results converge as the time step size tends to zero. Furthermore, Tables II and III show the total daily operation cost, and computational time, respectively, for all cases and time steps. It is shown that while the computational time increases significantly, the decrease in operating cost is not worthwhile, particularly for the 1-minute resolution.

E. Chance-constrained and Loadability Analysis

A sensitivity analysis has been carried out in this section to specify the impacts of the EES system and the risk level toleration. The results obtained from the simulation are represented in Table IV. As this table shows, in case no risk can be tolerated in summer, the EES system raises the flexibility of the model since the value of *α* has increased by 0.017 compared to the case without any EES system. The flexibility can be further increased by 0.142 for $\varepsilon = 0.1$, but at a substantially higher operating cost, i.e. tolerating \$8321.49 more compared to the case with $\varepsilon = 0$. The value of electrical and cooling load demands can considerably fluctuate and the increased flexibility can be obtained due to the low heating load demand in summer, the spare capacity of the CHP unit, and the capability of this unit to provide the required electricity, and also tolerating a higher cost. The fluctuations of the electrical and heating load demands are substantial in winter and the role of the EES system is more highlighted in increasing the flexibility. In case no risk can be tolerated, the EES system would raise the flexibility by 0.19 with \$132.19 more cost. Unlike summer, the fluctuations in the load demand and a higher risk level would considerably increase the total operating cost. For example, in case an EES system is installed, the cost increases by \$21240.96 for $\varepsilon = 0.1$ compared to the case with $\varepsilon = 0$.

TABLE IV. CHANCE-CONSTRAINED MODEL ANALYSIS

However, it should be noted that a higher flexibility would be achieved in winter in comparison with summer so that this increase in the cost can increase the flexibility by 0.33 in case the EES system is installed.

V. CONCLUSION AND FUTURE WORKS

This paper proposed an optimization framework for the optimal management of energy hub resources, considering different energy carriers and uncertainties associated with loads and generation from renewable energy resources. The proposed model was investigated as a stochastic chanceconstrained optimization problem and the flexibility of the proposed model was assessed in two different case studies. In the first case study, the effectiveness of the proposed model was studied for different time resolutions and the simulation results confirmed that the suggested model can effectively be used for close-real-time frameworks. A numerical stability analysis was conducted to validate the selection of the time resolution and independence of the results from time step size. By reducing the time discretization (time step size from 60 minutes to 30 and then 15 minutes and 1 minute), the electrical energy storage device was capable of reducing the fluctuations using the optimal charging/discharging fluctuations using the optimal charging/discharging functionalities. This issue did verify that the implemented close-to-real-time operation model would significantly leverage the capabilities of the energy storage assets of the energy hub. In the second case study, the flexibility of the operating points in the presence of unexpected uncertainties was also evaluated. The simulation results showed that for the given configuration of the energy hub, the flexibility of serving the demand is acceptable. The loadability index for winter was greater than summer since the thermal load in this season was the dominated loads and there were different technologies available to serve the thermal load. Therefore, the flexibility of the energy hub was much higher compared to that of summer. The main restriction in this season related to the electricity provision by CHP, PV and grid transformer. Furthermore, the cooling load in this season was considerable and there were two options for serving the cooling load, i.e. absorption chiller and electrical heat pump and both of them were working at the highest capacity during this season. Therefore, the cooling and electricity demands constraints were activated in the chance-constrained problem.

The future work to the current study can be done in the following areas:

- a) Developing a model predictive control (MPC)-based approach for semi-real-time operation purposes, where both device-layer and system-level controllers must be coordinated to meet the system's need in shorter time intervals, considering different technical/security constraints.
- b) Augmenting the model by adding the forecasting toolbox for increasing the accuracy of the operation.
- c) Developing the proposed problem using a decentralized model, and addressing different energy hubs in real distribution networks with a peer-to-peer trading market paradigm.

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