

Modeling the Strategic Behavior of a Distribution Company in Wholesale Energy and Reserve Markets

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Abstract—The decision making framework in power systems has changed due to presence of distributed energy resources (DERs). These resources are installed in distribution networks to meet demand locally. Therefore, distribution companies (Discos) are able to supply energy through these resources to meet their demand at a minimum operation cost. In this framework, the Disco will change its role in the wholesale energy market from price taker to price maker. DERs can provide reserve in their normal operation; this facilitates the provision of reserves by the Disco in the wholesale reserve market. Therefore, in this paper, the strategic behavior of a Disco in wholesale energy and reserve markets is modeled as a bi-level optimization problem. In the proposed model, the operation problem of the Disco and the Independent System Operator (ISO) are modeled in the upper- and lower-level problems, respectively. Karush-Kuhn-Tucker (KKT) conditions and duality theory are used to transform the proposed nonlinear bi-level problem to a linear single level one. Numerical studies show the effectiveness of the proposed model and its solution methodology.

Index Terms—bi-level optimization problem, Disco operation problem, energy and reserve markets.

NOMENCLATURE

A. Acronyms

<i>ADN</i>	Active distribution network
<i>DER</i>	Distributed energy resource
<i>DG</i>	Distributed generator
<i>Disco</i>	Distribution company
<i>EM</i>	Energy market
<i>EPEC</i>	Equilibrium problem with equilibrium constraints
<i>Genco</i>	Generation company
<i>GA</i>	Genetic algorithm
<i>IL</i>	Interruptible load
<i>ISO</i>	Independent system operator
<i>KKT</i>	Karush-Kuhn-Tucker
<i>LP</i>	linear programming
<i>MILP</i>	Mixed-integer linear programming
<i>MINLP</i>	Mixed-integer non-linear programming
<i>MCS</i>	Monte Carlo simulation

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<i>MG</i>	Microgrid
<i>MPEC</i>	Mathematical program with equilibrium constraints
<i>NLP</i>	Non-linear programming
<i>PDF</i>	Probability distribution function
<i>PSO</i>	Particle swarm optimization
<i>PV</i>	Photovoltaic
<i>RER</i>	Renewable energy resource
<i>Ret.</i>	Retailer
<i>RM</i>	Reserve market
<i>RWM</i>	Roulette wheel mechanism
<i>SQP</i>	Sequential quadratic programming
<i>VPP</i>	Virtual power plant
<i>WT</i>	Wind turbine

B. Indices

i/j	Number of DGs/ILs
k	Number of batteries
n/m	Number of Gencos/retailers
t	Time interval
ω	Set of scenarios

C. Parameters

C_i^{DG}	Cost of DG (\$/MWh)
C_j^{IL}	Cost of IL (\$/MWh)
\underline{E}_k^{batt}	Minimum energy storage of a battery unit (MWh)
\overline{E}_k^{batt}	Maximum energy storage of a battery unit (MWh)
$e_k^{batt,ini}$	Initial energy stored in a battery unit (MWh)
\overline{P}_k^{batt}	Maximum charging/discharging power of a battery unit (MW)
\overline{P}^{Dis}	Maximum power sold/purchased by the Disco (MW)
\overline{P}_i^{DG}	Maximum power generation of a DG unit (MW)
$P_i^{DG,ini}$	Initial power generation of a DG unit (MW)
\overline{P}_j^{IL}	Maximum amount of load curtailment (MW)
\overline{P}_n^{Genco}	Maximum power generation of a Genco (MW)

P_j^L	Energy demand (MW)
$P_{\omega,t}^{PV,Forecast}$	PV forecasted output (MW)
$\overline{P}_m^{Ret.}$	Maximum power purchased by a retailer (MW)
$P_{\omega,t}^{WT,Forecast}$	WT forecasted output (MW)
\overline{R}^{Dis}	Maximum reserve provided by the Disco (MW)
\overline{R}_n^{Genco}	Maximum reserve provided by a Genco (MW)
$\overline{R}_m^{Ret.}$	Maximum reserve provided by a retailer (MW)
$\overline{R}_t^{Sys.}$	Reserve required of the system (MW)
RDN_i^{DG}	Ramp down rate of a DG unit (MW/h)
RUP_i^{DG}	Ramp up rate of a DG unit (MW/h)
$\eta_k^{batt,ch}$	Charging efficiency of a battery unit
$\eta_k^{batt,dch}$	Discharging efficiency of a battery unit
η^{Trans}	Efficiency of the Disco's transformer
κ_t^{RM}	Probability of calling reserve (%)
$\Pi_{n,t}^{Genco,Offer}$	Genco's offer to sell energy (\$/MWh)
$\Pi_{n,t}^{Genco,Res}$	Genco's offer to provide reserve (\$/MWh)
Π_t^{inc}	Incentive price due to serving reserve (\$/MWh)
Π_t^{Pen}	Penalty price due to not serving reserve (\$/MWh)
$\Pi_{m,t}^{Ret.,Bid}$	Retailer bid to buy energy (\$/MWh)
$\Pi_{m,t}^{Ret.,Res}$	Retailer offer to provide reserve (\$/MWh)
π_t^{Dis,bid^*}	Optimal value of Disco's bid to buy energy (\$/MWh)
$\pi_t^{Dis,Offer^*}$	Optimal value of Disco's offer to sell energy (\$/MWh)
π_t^{Dis,Res^*}	Optimal value of Disco's offer to provide reserve (\$/MWh)
ψ_t^{Dis}	Probability of the Disco's service failure
$\psi_{n,t}^{Genco}$	Probability of a Genco's service failure
$\psi_{m,t}^{Ret.}$	Probability of a retailer's service failure
ρ_ω	Probability of a scenario

D. Variables

$e_{k,\omega,t}^{batt}$	Energy storage of a battery unit (MWh)
$p_{k,\omega,t}^{batt,ch}$	Charging power of a battery unit (MW)
$p_{k,\omega,t}^{batt,dch}$	Discharging power of a battery unit (MW)
$p_t^{Dis,in}$	Power purchased by the Disco (MW)
$p_t^{Dis,out}$	Power sold by the Disco (MW)
$p_{i,\omega,t}^{DG}$	Power generation of a DG unit (MW)
$p_{n,t}^{Genco}$	Power generated by a Genco (MW)
$p_{j,\omega,t}^{IL}$	The amount of load curtailment (MW)
$p_{\omega,t}^{PV}$	Power generation of a PV array (MW)
$p_{m,t}^{Ret.}$	Power purchased by a retailer (MW)
$p_{\omega,t}^{WT}$	Power generation of a WT unit (MW)

$r_{k,\omega,t}^{batt}$	Reserve provided by a battery unit (MW)
r_t^{Dis}	Reserve provided by the Disco (MW)
$r_{i,\omega,t}^{DG}$	Reserve provided by a DG unit (MW)
$r_{n,t}^{Genco}$	Reserve provided by a Genco (MW)
$r_{j,\omega,t}^{IL}$	Reserve provided by an IL (MW)
$r_{m,t}^{Ret.}$	Reserve provided by a retailer (MW)
$\pi_t^{Dis,bid}$	Disco's bid to buy energy (\$/MWh)
$\pi_t^{Dis,Offer}$	Disco's offer to sell energy (\$/MWh)
$\pi_t^{Dis,Res}$	Disco's offer to provide reserve (\$/MWh)
π_t^{EM}	Wholesale energy price (\$/MWh)
π_t^{RM}	Wholesale reserve price (\$/MWh)

E. Functions

$F(X^{UL})$	Objective function of the upper-level problem
$F(X_P^{LL})$	Objective function of the primal problem of the lower-level problem
$F(X_D^{LL})$	Objective function of the dual problem of the lower-level problem

I. INTRODUCTION

A. Motivation and Aim

IN passive distribution networks, distribution companies (Discos) participate in wholesale energy markets as price-takers and purchase the required energy for their networks [1], [2]. High energy cost, low reliability, and power losses are the major problems of these networks [3]. To mitigate these problems, distributed energy resources (DERs) including distributed generators (DGs), energy storage, and demand side management are used to meet the demand of distribution networks locally. In the presence of these resources, distribution networks can be considered active distribution networks (ADNs). In such framework, the operation problem of the Disco has changed so that the Disco has other possibilities to meet its demand including optimal energy trading with DER owners and optimal scheduling of their own DERs.

Therefore, the role of the Disco in the wholesale energy market has changed and the Disco behaves as a price maker. Moreover, DERs are fast-response resources that enable the Disco to participate in the wholesale reserve market. The aim of this paper is to model the strategic behavior of a Disco in the presence of DERs in both wholesale energy and reserve markets.

B. Literature Review and Contribution

The operation problem of a Disco as a price-taker has been studied in several works. An appropriate decision making framework is presented for the operation problem of a Disco in day-ahead and real-time electricity markets in [4]. Optimal decisions of a Disco, including power purchased from the market, optimal scheduling of DGs and interruptible loads (ILs) are determined based on day-ahead electricity prices and they are considered as parameters in the second stage, which

TABLE I
COMPARISON WITH DISTRIBUTION NETWORK OPERATION MODELS IN LITERATURE

Ref.	The role of Disco		DER owners		Wholesale markets		Model	Solving approach
	Price-taker	Price-maker	Disco	Private owners	Energy	Reserve		
[2]	✓	-	✓	✓	✓	-	LP	CPLEX solver
[4]	✓	-	✓	✓	✓	-	MINLP	DICOPT solver
[5]	✓	-	✓	✓	✓	-	MINLP	GA
[6]	✓	-	-	✓	✓	-	NLP	SNOPT solver
[7]	✓	-	✓	✓	✓	-	MILP	CPLEX solver
[8]	✓	-	✓	✓	✓	-	MILP	CPLEX solver
[9]	✓	-	-	✓	✓	-	MINLP	CONOPT solver
[10]	✓	-	-	✓	✓	-	MINLP	EMTP software
[11]	✓	-	✓	✓	✓	-	MINLP	MINOS solver
[12]	✓	-	-	✓	✓	-	MINLP	PSO
[13]	✓	-	✓	-	✓	-	NLP	SQP
[14]	✓	-	✓	✓	✓	-	MINLP	GA and PSO
[15]	✓	-	✓	-	✓	-	NLP	PSO
[16]	✓	-	-	✓	✓	-	MINLP	DICOPT solver
[17]	✓	-	✓	✓	✓	-	MINLP	DICOPT solver
[20]	✓	-	✓	-	✓	✓	NLP	Metaheuristic
[21]	✓	-	✓	✓	✓	✓	MINLP	DICOPT solver
[22]	✓	-	✓	✓	✓	✓	MINLP	DICOPT solver
[23]	✓	-	✓	✓	✓	✓	MILP	CPLEX solver
[24]	✓	-	-	✓	✓	✓	MILP	CPLEX solver
[32]	-	✓	✓	✓	✓	-	MINLP	Metaheuristic
[33]	-	✓	✓	✓	✓	-	MINLP	CONOPT solver
This paper	-	✓	✓	✓	✓	✓	NLP	KKT conditions and dual theory

is done according to real-time prices. This model is extended considering the uncertainty of real-time prices and demands as the risk-based stochastic model in [2]. The optimal operation of a Disco considering reconfiguration is investigated in [5]. A model of price-responsive and controllable loads is considered in the operation problem of a Disco in [6]-[8]. A new approach is presented to model the trading of energy between a Disco and a plug-in electric vehicle aggregator [9]. The short-term scheduling of energy and control resources consisting of DGs, reactive power compensators and transformers equipped with on-load tap changers is modeled as a two-stage operation problem in [10]. The effect of DG production on active and reactive power losses is investigated using appropriate indices in the short-term operation problem of a Disco [11]. The energy resource scheduling of a Disco is presented considering DGs and electric vehicles in which the effect of different electric vehicle management approaches consisting of uncontrolled charging, smart charging, etc. is investigated in the operation problem of a Disco [12]. The operation problem of a Disco in the presence of DERs is modeled from different viewpoints in [13]-[17]. The optimal decision of a Disco in a wholesale market is modeled while trading energy with microgrids (MGs) as hierarchical decision making frameworks [1], [18], [19].

Optimal decision making of a Disco in wholesale energy and reserve markets is modeled in [20] and a two-stage approach is used for solving the proposed model. The optimal decisions of a Disco in wholesale energy and reserve markets considering uncertainty of demand and wind speed are modeled in [21]. In [22]-[24] the operation problem of a

Disco in both energy and reserve markets in the presence of DERs is modeled as a stochastic optimization problem. The Disco trades energy and reserves with MGs in its network and participates in wholesale energy and reserve markets in [25].

A comprehensive study is presented for a virtual power plant (VPP) to minimize the purchased energy from the market using the optimal placement of renewable-based DGs, optimal scheduling of controllable loads and optimal operation of energy storage in [26]. Optimal operation of VPPs is modeled, where they can cooperate with each other in ADNs in [27]. The VPP's profit is maximized while participating in energy and ancillary service markets as a price-taker in [28]. A day-ahead scheduling framework for a VPP is presented in [29] while participating in joint energy and regulation reserve markets as a price-taker. The VPP provides the required energy and reserve to the markets through optimal scheduling of its renewable and fossil fuel-based DGs and optimal scheduling of electric vehicles through bilateral contracts with vehicle owners. A stochastic bi-level approach is proposed to model the offering strategy of a VPP as a price-maker in the energy market in [30]. The decision making framework of a MG aggregator is modeled while participating in day-ahead and real-time markets as a price-taker in [31].

In all these studies [1], [2], [4]-[25] the Disco is considered as a price-taker decision maker in wholesale energy and reserve markets. On the contrary, in [32] the Disco is considered as a price-maker player in the wholesale energy market while exchanging energy with DER owners. In [33] the Disco is considered as a price-maker player in day-ahead and real-time markets while trading energy with DERs in its network.

For the sake of clarity, the relevant features of the proposed model are compared with other studies reported in the literature in Table I. Since Discos are equipped with DERs and act as prosumers who can consume and produce energy simultaneously in wholesale energy and reserve markets, they can make an impact on the market equilibrium prices by changing their roles from producers to consumers and vice versa. The decision making framework of the Disco is completely new, requiring a new business model when participating in both wholesale energy and reserve markets. Therefore, the contribution of this paper is to model the decision-making framework of the Disco as a price-maker player in wholesale energy and reserve markets. This framework is modeled as a non-linear bi-level optimization problem which is transformed into a single-level mixed-integer linear programming (MILP) using Karush-Kuhn-Tucker (KKT) conditions and duality theory.

C. Paper Organization

The rest of the paper is organized as follows. The problem is described in section II. Mathematical modeling and its solution methodology are presented in section III. Numerical results are performed in section IV and conclusions and future work are shown in section V.

II. PROBLEM STATEMENT

In this paper, the strategic behavior of a Disco in wholesale energy and reserve markets is modeled as a bi-level problem. It is assumed that the Disco purchases energy from the wholesale market and sells it to the consumers at fixed prices and is also the owner of the distribution system. In the presence of DERs, the Disco has the capability of meeting its demand locally, selling energy to the wholesale energy market and providing reserves for the reserve market. Generation companies (Gencos) and retailers are the other participants of wholesale markets as shown in Fig. 1. In such a framework, consumers purchase their required electricity through the Disco or choose a retailer who has a legal license. This framework has been implemented in Ontario's power system, for example.

Fig. 2 shows the structure, decision variables and role of each player in the proposed bi-level problem. The operation problem of the Disco is modeled in the upper-level problem. The lower-level problem consists of both energy and reserve markets where the aim of the Independent System Operator (ISO) is the clearing of these two markets, maximizing social welfare. Since the behaviour of the other decision makers, Gencos and retailers, is not strategic and their bids are only modeled in the energy and reserve markets, they are modeled in the lower-level problem only. The decision variables of the Disco are: power generation of DGs, amount of load curtailment, battery power interaction, amount of stored energy in batteries, reserve provided by batteries, DGs, and ILs, offers/bids to sell/purchase energy in/from the energy market and offers to provide reserves in the reserve market. The decision variables of the lower-level problem are: power generation of Gencos, power purchased by retailers, power purchased by the Disco, power sold by the Disco, reserve provided by Disco, retailers and Gencos, and wholesale energy and reserve prices.

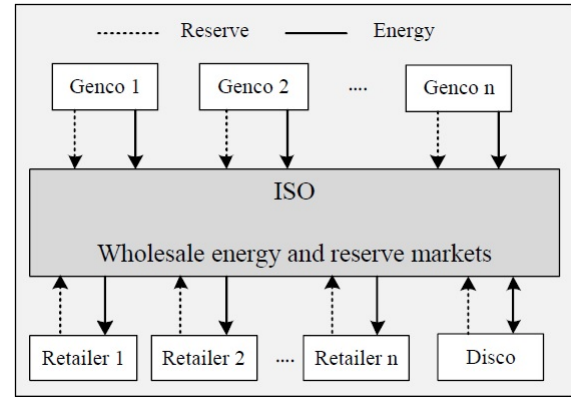


Fig. 1. Wholesale energy and reserve market participants.

In each level, there are internal and external decision variables which link the upper- and lower-level problems. After solving the upper-level problem, the external variables consisting of: offers/bids of the Disco to sell/purchase energy in/from the energy market and offers to provide reserves in the reserve market, are obtained from the optimization of the operation problem of the Disco and are passed to the lower-level problem as parameters. The external decision variables of the lower-level problem are: power purchased by the Disco, power sold by the Disco, reserves provided by the Disco and wholesale energy and reserve prices, which are passed to the Disco and from which the profit of the Disco can be calculated.

The output of the upper-level consists of the energy and reserve bids/offers and the output of the lower-level problem is the quantity of energy and reserves for the Disco. In this case, when the Disco is considered as a price-maker player in wholesale energy and reserve markets, it behaves as a marginal player in some hours and affects wholesale energy and reserve prices to maximize its profit. The behavior of the decision makers described in Fig. 2 is modeled in three steps, which are described as follows:

- A bi-level optimization approach is used, in which the operation problem of the Disco and the ISO are modeled in the upper- and lower-level problems, respectively.
- The model proposed in the previous step is a non-linear bi-level one which is transformed into a non-linear single-level problem through KKT conditions.
- Duality theory is used to linearize the model from the previous step and the resulted model becomes an MILP one.

III. MATHEMATICAL MODELING

A. Disco's Operation Problem

In this paper, the strategic behavior of the Disco is modeled in both wholesale energy and reserve markets. The problem is cast as a bi-level optimization problem in which the operation problem of the Disco is modeled in the upper-level and the model for the simultaneous energy and reserve markets is presented in the lower-level. The upper-level problem is as follows:

Equation (1) presents the objective function of the Disco's

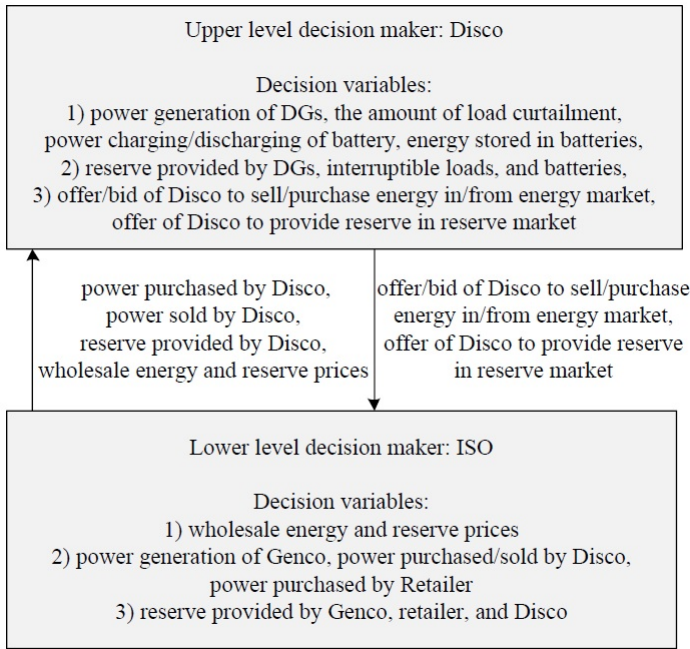


Fig. 2. The bi-level problem framework.

operation problem consisting of four main terms. Fossil-fuel based DGs and ILs belong to private owners, submitting their bids to provide energy and reserve to the Disco. The Disco considers their bids in its objective function using the first and second terms of equation (1), respectively. Since renewable energy-based DGs and energy storage are considered to belong to the Disco and their operation costs are low, the operation cost of these resources can be ignored in the objective function of the Disco [5], [31].

The third term is used to model the profit of the Disco from energy trading in the wholesale energy market, and its profit from providing reserve to the wholesale reserve market is modeled in the last term. The reserve exchange includes income from reserve provision and delivering energy after the reserve call, including the penalty for not being ready to deliver the required reserve amount. The profit model of the Disco is similar to the decision makers' profit from providing reserve to the wholesale reserve market as proposed in [34].

$$\begin{aligned} \min F(X^{UL}) = & \sum_t \sum_\omega \rho_\omega \sum_i (C_i^{DG} p_{i,\omega,t}^{DG} + C_i^{DG} \kappa_t^{RM} r_{i,\omega,t}^{DG}) \\ & - \sum_t \sum_\omega \rho_\omega \sum_j (C_j^{IL} p_{j,\omega,t}^{IL} + C_j^{IL} \kappa_t^{RM} r_{j,\omega,t}^{IL}) - \\ & \sum_t (p_t^{Dis,out} - p_t^{Dis,in}) \pi_t^{EM} - \sum_t (r_t^{Dis} \pi_t^{RM} + \\ & \Pi_t^{inc} \kappa_t^{RM} r_t^{Dis} (1 - \psi_t^{Dis}) - \Pi_t^{Pen} \kappa_t^{RM} r_t^{Dis} \psi_t^{Dis}) \quad (1) \end{aligned}$$

The Disco is equipped with renewable energy resources (RERs) whose operation is based on possible scenarios that can be forecasted. Therefore, the upper-level optimization problem of the Disco is modeled based on possible scenarios. However, the Disco exchanges both energy and reserves in the wholesale market based on the expected amount. Equation (2) represents the expected amount of energy and reserve ex-

changes of the Disco that are used in the market optimization problem. This equation is used to justify the relation between possible scenarios and outcomes. For the reserve equation, it could be possible to use the worst scenario instead of the expected amount, but this would increase the operation cost of the Disco. It should be noted that, in the proposed model, the Disco decides the amount of reserves to supply in the wholesale market, therefore, choosing the worst scenario will not impact on the total reserve of the system.

$$\begin{aligned} p_t^{Dis,out} &= \sum_\omega \rho_\omega p_{\omega,t}^{Dis,out}, \quad p_t^{Dis,in} = \sum_\omega \rho_\omega p_{\omega,t}^{Dis,in}, \\ r_t^{Dis} &= \sum_\omega \rho_\omega r_{\omega,t}^{Dis} \quad (2) \end{aligned}$$

1) *Energy and reserve balance constraints:* Equations (3) and (4) represent the energy and reserve balance of the Disco. The reserve provided by the Disco in the reserve market is supplied by DGs, ILs, and batteries as described in (4).

$$\begin{aligned} p_{\omega,t}^{Dis,in} \eta^{Trans} - p_{\omega,t}^{Dis,out} / \eta^{Trans} + \sum_i p_{i,\omega,t}^{DG} + p_{\omega,t}^{PV} + p_{\omega,t}^{WT} \\ + \sum_k (-p_{k,\omega,t}^{batt,ch} / \eta_k^{batt,ch} + p_{k,\omega,t}^{batt,dch} \eta_k^{batt,dch}) \\ = \sum_j (P_j^L - p_{j,\omega,t}^{IL}) \quad (3) \end{aligned}$$

$$r_{\omega,t}^{Dis} = \sum_i r_{i,\omega,t}^{DG} + \sum_j r_{j,\omega,t}^{IL} + \sum_k r_{k,\omega,t}^{batt} \quad (4)$$

2) *DG constraints:* The detailed operational modeling of fossil-based DGs is described in (5)-(10). The sum of the energy and reserves provided by DGs is lower than their maximum generation as described in (5). The ramp-up and ramp-down limits of DG are modeled in (8) and (9). In these equations, for $t = 1$, the initial DG power generation ($p_{i,\omega,t-1}^{DG,ini}$) is considered instead of $p_{i,\omega,t-1}^{DG}$. Moreover, the reserve provided by DGs is lower than the ramp-up limit, as modeled in (10).

$$p_{i,\omega,t}^{DG} + r_{i,\omega,t}^{DG} \leq \bar{P}_i^{DG} \quad (5)$$

$$0 \leq p_{i,\omega,t}^{DG} \quad (6)$$

$$0 \leq r_{i,\omega,t}^{DG} \quad (7)$$

$$p_{i,\omega,t}^{DG} - p_{i,\omega,t-1}^{DG} \leq RUP_i^{DG} \quad \forall i, t, \omega \quad (8)$$

$$p_{i,\omega,t}^{DG} - p_{i,\omega,t-1}^{DG} \leq RDN_i^{DG} \quad \forall i, t, \omega \quad (9)$$

$$r_{i,\omega,t}^{DG} \leq RUP_i^{DG} \quad (10)$$

3) *IL constraints:* Equations (11)-(13) are used to model the IL constraints in the Disco's problem. A bid-based mechanism for IL is considered wherein the consumers submit their bids in terms of the maximum amount of load curtailment to provide energy and reserve and associated prices on an hourly basis, as proposed in [2], [19].

$$p_{j,\omega,t}^{IL} + r_{j,\omega,t}^{IL} \leq \bar{P}_j^{IL} \quad (11)$$

$$0 \leq p_{j,\omega,t}^{IL} \quad (12)$$

$$0 \leq r_{j,\omega,t}^{IL} \quad (13)$$

4) *RERs constraints:* As RERs are belong to the Disco, the latter forecasts their output power and, then, the operational

constraints of RERs are modeled based on these forecasted power outputs in (14) and (15). Moreover, the uncertain behavior of these resources is modeled using appropriate scenarios as described in detail in Appendix A.

$$0 \leq p_{\omega,t}^{PV} \leq P_{\omega,t}^{PV,Forecast} \quad (14)$$

$$0 \leq p_{\omega,t}^{WT} \leq P_{\omega,t}^{WT,Forecast} \quad (15)$$

5) *Energy storage constraints*: The operational modeling of energy storage to provide energy and reserves is presented in (16)-(22). The minimum and maximum limits of power charging, power discharging and energy of the battery are described as equations (16)-(18), respectively. The energy and power limitations of the battery to provide energy and reserves are modeled in equations (19)-(21), respectively. In equation (19), the sum of the scheduled reserve of the battery and the scheduled power discharged is limited to the energy stored in the battery. In fact, this equation ensures that the battery has sufficient energy stored to deliver the scheduled reserves at that time step, but does not account for a prolonged activation of battery-based reserves. The energy balance of the battery is described in (22). In this equation, for $t = 1$, the initial energy stored in the battery ($e_k^{batt,ini}$) is considered instead of $e_{k,\omega,t-1}^{batt}$.

$$0 \leq p_{k,\omega,t}^{batt,ch} \leq \bar{P}_k^{batt} \quad (16)$$

$$0 \leq p_{k,\omega,t}^{batt,dch} \leq \bar{P}_k^{batt} \quad (17)$$

$$\underline{E}_k^{batt} \leq e_{k,\omega,t}^{batt} \leq \bar{E}_k^{batt} \quad (18)$$

$$(r_{k,\omega,t}^{batt} + p_{k,\omega,t}^{batt,dch}) / \eta_k^{batt,dch} \leq e_{k,\omega,t}^{batt} \quad (19)$$

$$r_{k,\omega,t}^{batt} - p_{k,\omega,t}^{batt,ch} + p_{k,\omega,t}^{batt,dch} \leq \bar{P}_k^{batt} \quad (20)$$

$$r_{k,\omega,t}^{batt} \leq \bar{P}_k^{batt} \quad (21)$$

$$e_{k,\omega,t}^{batt} = e_{k,\omega,t-1}^{batt} + p_{k,\omega,t}^{batt,ch} - p_{k,\omega,t}^{batt,dch} \quad \forall k, t, \omega \quad (22)$$

In equations (1)-(22), X^{UL} is the decision variable vector of the Disco (upper-level) problem defined as: $X^{UL} = [p_{i,\omega,t}^{DG}, r_{i,\omega,t}^{DG}, p_{j,\omega,t}^{IL}, r_{j,\omega,t}^{IL}, p_{\omega,t}^{PV}, p_{\omega,t}^{WT}, e_{k,\omega,t}^{batt}, p_{k,\omega,t}^{batt,ch}, p_{k,\omega,t}^{batt,dch}, \pi^{Dis,offer}, \pi^{Dis,bid}, \pi^{Dis,res}]$.

Based on the proposed model for the Disco, its external decision variables, i.e. $[\pi^{Dis,offer}, \pi^{Dis,bid}, \pi^{Dis,res}]$ are determined and passed on to the lower-level problem, where they are parameters as $[\pi^{Dis,offer*}, \pi^{Dis,bid*}, \pi^{Dis,res*}]$.

B. Modeling Energy and Reserve Markets

In the lower-level problem, the Disco participates in both wholesale energy and reserve markets simultaneously. Both markets are operated by the ISO and consist of retailers and Gencos. Retailers participate in the energy market to supply their customers. The customers are considered as smart players who reduce their load demands based on retailers' incentive-based demand response programs. The Gencos supply the energy required by the system and participate in the reserve market to guarantee system adequacy in case of contingency. The lower-level optimization problem of the proposed bi-level problem is as follows.

Equation (23) represents the objective function of the ISO

to clear both energy and reserve markets simultaneously. This equation consists of seven terms. Gencos offers to sell energy and retailers bids to buy their required energy in the energy market, which are modeled as first and second terms, respectively. Financial trading of the Disco in the energy market is modeled as shown in the third and fourth terms. The marginal reserve costs of retailers, Gencos, and the Disco are modeled in the other terms, respectively. In this equation the offers/bids of the Disco to sell/purchase energy to/from the energy market and its offers to provide reserve to the reserve market are parameters that are not fixed but come from solving the upper-level problem. Since the behaviour of Gencos and retailers is not strategic, offers/bids of Gencos/retailers to sell/purchase energy in the energy market and their offers to provide reserve in the reserve market are considered as fixed parameters in this equation.

$$\begin{aligned} \min F(X_P^{LL}) = & \sum_t \left(\sum_n \Pi_{n,t}^{Genco,Offer} p_{n,t}^{Genco} - \right. \\ & \sum_m \Pi_{m,t}^{Ret.,Bid} p_{m,t}^{Ret.} + \pi_t^{Dis,Offer*} p_t^{Dis,out} - \pi_t^{Dis,Bid*} p_t^{Dis,in} \\ & + \sum_m \left(\Pi_{m,t}^{Ret.,Res} r_{m,t}^{Ret.} + \Pi_t^{inc} \kappa_t^{RM} r_{m,t}^{Ret.} (1 - \psi_{m,t}^{Ret.}) - \right. \\ & \left. \Pi_t^{Pen} \kappa_t^{RM} r_{m,t}^{Ret.} \psi_{m,t}^{Ret.} \right) + \sum_n \left(\Pi_{n,t}^{Genco,Res} r_{n,t}^{Genco} + \right. \\ & \left. \Pi_t^{inc} \kappa_t^{RM} r_{n,t}^{Genco} (1 - \psi_{n,t}^{Genco}) - \Pi_t^{Pen} \kappa_t^{RM} r_{n,t}^{Genco} \psi_{n,t}^{Genco} \right) + \\ & \left. \left(\pi_t^{Dis,Res*} r_t^{Dis} + \Pi_t^{inc} \kappa_t^{RM} r_t^{Dis} (1 - \psi_t^{Dis}) - \right. \right. \\ & \left. \left. \Pi_t^{Pen} \kappa_t^{RM} r_t^{Dis} \psi_t^{Dis} \right) \right) \quad (23) \end{aligned}$$

In equations (23), X_P^{LL} is the decision variable vector of the ISO (lower-level) primal problem defined as: $X_P^{LL} = [p_{n,t}^{Genco}, r_{n,t}^{Genco}, p_{m,t}^{Ret.}, r_{m,t}^{Ret.}, p_t^{Dis,out}, p_t^{Dis,in}, r_t^{Dis}]$.

1) *Energy and reserve balance constraints*: Energy and reserve balance constraints are modeled in (24) and (25), respectively.

$$\sum_m p_{m,t}^{Ret.} - \sum_n p_{n,t}^{Genco} + p_t^{Dis,in} - p_t^{Dis,out} = 0 : \pi_t^{EM} \quad (24)$$

$$\sum_m r_{m,t}^{Ret.} + \sum_n r_{n,t}^{Genco} + r_t^{Dis} = \bar{R}_t^{Sys.} : \pi_t^{RM} \quad (25)$$

2) *Retailers' constraints*: Operational constraints of retailers in order to participate in both energy and reserve markets are modeled in (26)-(28).

$$0 \leq p_{m,t}^{Ret.} : \underline{\mu}_{m,t}^{Ret.} \quad (26)$$

$$p_{m,t}^{Ret.} + r_{m,t}^{Ret.} \leq \bar{P}_m^{Ret.} : \bar{\mu}_{m,t}^{Ret.} \quad (27)$$

$$0 \leq r_{m,t}^{Ret.} \leq \bar{R}_m^{Ret.} : \underline{\mu}_{m,t}^{Ret.,Res}, \bar{\mu}_{m,t}^{Ret.,Res} \quad (28)$$

3) *Gencos' constraints*: Operational constraints of Gencos in order to participate in both energy and reserve markets are modeled in (29)-(31).

$$0 \leq p_{n,t}^{Genco} : \underline{\mu}_{n,t}^{Genco} \quad (29)$$

$$p_{n,t}^{Genco} + r_{n,t}^{Genco} \leq \bar{P}_n^{Genco} : \bar{\mu}_{n,t}^{Genco} \quad (30)$$

$$0 \leq r_{n,t}^{Genco} \leq \bar{R}_n^{Genco} : \underline{\mu}_{n,t}^{Genco,Res}, \bar{\mu}_{n,t}^{Genco,Res} \quad (31)$$

4) *Disco's constraints*: Operational constraints of the Disco in order to participate in both energy and reserve markets are modeled in (32)-(35).

$$0 \leq p_t^{Dis,out} : \underline{\mu}_t^{Dis,out} \quad (32)$$

$$p_t^{Dis,out} + r_t^{Dis} \leq \bar{P}^{Dis} : \underline{\mu}_t^{Dis,out} \quad (33)$$

$$0 \leq p_t^{Dis,in} \leq \bar{P}^{Dis} : \underline{\mu}_t^{Dis,in}, \bar{\mu}_t^{Dis,in} \quad (34)$$

$$0 \leq r_t^{Dis} \leq \bar{R}^{Dis} : \underline{\mu}_t^{Dis,Res}, \bar{\mu}_t^{Dis,Res} \quad (35)$$

π and μ are the dual variables for equality and non-equality constraints of the lower-level problem, respectively, that are shown on the right hand side of each equation. The dual variables of power and reserve balance constraints (π) are equal to the wholesale energy and reserve prices, respectively. The proposed model could be modified to consider other Discos and aggregators in the network. In the proposed model, the distribution network is considered as a single bus and power losses are ignored and the transmission network and its load flow are not considered. These assumptions reduce the size and improve the computational tractability of the model.

C. Mathematical Program with Equilibrium Constraints

The model proposed in the previous section is a non-linear bi-level optimization problem. Since the decision variables of the upper-level problem are considered as parameters in the lower-level problem, they can be replaced with their KKT conditions [19], [35]-[37]. For this purpose, at first, the Lagrangian function is formed as proposed in [19]. KKT conditions including some constraints are described as follows.

$$-\Pi_{m,t}^{Ret.,Bid} + \pi_t^{EM} - \underline{\mu}_{m,t}^{Ret.} + \bar{\mu}_{m,t}^{Ret.} = 0 \quad (36)$$

$$\Pi_{n,t}^{Genco,Offer} - \pi_t^{EM} - \underline{\mu}_{n,t}^{Genco} + \bar{\mu}_{n,t}^{Genco} = 0 \quad (37)$$

$$-\pi_t^{Dis,Bid^*} + \pi_t^{EM} - \underline{\mu}_t^{Dis,in} + \bar{\mu}_t^{Dis,in} = 0 \quad (38)$$

$$\pi_t^{Dis,Offer^*} - \pi_t^{EM} - \underline{\mu}_t^{Dis,out} + \bar{\mu}_t^{Dis,out} = 0 \quad (39)$$

$$\Pi_{m,t}^{Ret.,Res} + \Pi_t^{inc} \kappa_t^{RM} (1 - \psi_{m,t}^{Ret.}) - \Pi_t^{Pen} \kappa_t^{RM} \psi_{m,t}^{Ret.} - \pi_t^{RM} + \bar{\mu}_{m,t}^{Ret.} - \underline{\mu}_{m,t}^{Ret.,Res} + \bar{\mu}_{m,t}^{Ret.,Res} = 0 \quad (40)$$

$$\Pi_{n,t}^{Genco,Res} + \Pi_t^{inc} \kappa_t^{RM} (1 - \psi_{n,t}^{Genco}) - \Pi_t^{Pen} \kappa_t^{RM} \psi_{n,t}^{Genco} - \pi_t^{RM} + \bar{\mu}_{n,t}^{Genco} - \underline{\mu}_{n,t}^{Genco,Res} + \bar{\mu}_{n,t}^{Genco,Res} = 0 \quad (41)$$

$$\pi_t^{Dis,Res^*} + \Pi_t^{inc} \kappa_t^{RM} (1 - \psi_t^{Dis}) - \Pi_t^{Pen} \kappa_t^{RM} \psi_t^{Dis} - \pi_t^{RM} + \bar{\mu}_t^{Dis,out} - \underline{\mu}_t^{Dis,Res} + \bar{\mu}_t^{Dis,Res} = 0 \quad (42)$$

$$0 \leq p_{m,t}^{Ret.} \perp \underline{\mu}_{m,t}^{Ret.} \geq 0 \quad (43)$$

$$0 \leq (\bar{P}_m^{Ret.} - p_{m,t}^{Ret.} - r_{m,t}^{Ret.}) \perp \bar{\mu}_{m,t}^{Ret.} \geq 0 \quad (44)$$

$$0 \leq r_{m,t}^{Ret.} \perp \underline{\mu}_{m,t}^{Ret.,Res} \geq 0 \quad (45)$$

$$0 \leq (\bar{R}_m^{Ret.} - r_{m,t}^{Ret.}) \perp \bar{\mu}_{m,t}^{Ret.,Res} \geq 0 \quad (46)$$

$$0 \leq p_{n,t}^{Genco} \perp \underline{\mu}_{n,t}^{Genco} \geq 0 \quad (47)$$

$$0 \leq (\bar{P}_n^{Genco} - p_{n,t}^{Genco} - r_{n,t}^{Genco}) \perp \bar{\mu}_{n,t}^{Genco} \geq 0 \quad (48)$$

$$0 \leq r_{n,t}^{Genco} \perp \underline{\mu}_{n,t}^{Genco,Res} \geq 0 \quad (49)$$

$$0 \leq (\bar{R}_n^{Genco} - r_{n,t}^{Genco}) \perp \bar{\mu}_{n,t}^{Genco,Res} \geq 0 \quad (50)$$

$$0 \leq p_t^{Dis,out} \perp \underline{\mu}_t^{Dis,out} \geq 0 \quad (51)$$

$$0 \leq (\bar{P}^{Dis} - p_t^{Dis,out} - r_t^{Dis}) \perp \bar{\mu}_t^{Dis,out} \geq 0 \quad (52)$$

$$0 \leq p_t^{Dis,in} \perp \underline{\mu}_t^{Dis,in} \geq 0 \quad (53)$$

$$0 \leq (\bar{P}^{Dis} - p_t^{Dis,in}) \perp \bar{\mu}_t^{Dis,in} \geq 0 \quad (54)$$

$$0 \leq r_t^{Dis} \perp \underline{\mu}_t^{Dis,Res} \geq 0 \quad (55)$$

$$0 \leq (\bar{R}^{Dis} - r_t^{Dis}) \perp \bar{\mu}_t^{Dis,Res} \geq 0 \quad (56)$$

Equations (36)-(42) are stationarity constraints which are obtained from the first order derivatives of the Lagrangian function with respect to the decision variables of the lower-level problem. There are seven decision variables in the primal problem of lower-level problem as shown in X_P^{LL} whose stationarity constraints are obtained based on these variables. Equations (43)-(56) are complementary slackness constraints, where each of them is described in (57).

$$0 \leq a \perp b \geq 0 \Rightarrow$$

$$a \geq 0, b \geq 0, a \leq M_1 U, b \leq M_2 (1 - U) \quad (57)$$

where M_1 and M_2 are large enough values and U is a binary variable [37]. Therefore, the bi-level problem is transformed into a single-level one generating a mathematical program with equilibrium constraints (MPEC).

D. Mixed-Integer Linear Programming Model

The resulting model from the previous subsection is a nonlinear MPEC. The non-linear terms of equation (1) are replaced with linear expressions, as described in detail in

Appendix B and the resulting MILP model is described as follows:

$$\begin{aligned}
 \min \sum_t & \left[\sum_{\omega} \rho_{\omega} \left(\sum_i (C_i^{DG} p_{i,\omega,t}^{DG} + C_i^{DG} \kappa_t^{RM} r_{i,\omega,t}^{DG}) \right. \right. \\
 & \left. \left. + \sum_j (C_j^{IL} p_{j,\omega,t}^{IL} + C_j^{IL} \kappa_t^{RM} r_{j,\omega,t}^{IL}) \right) \right. \\
 & - \Pi_t^{inc} \kappa_t^{RM} r_t^{Dis} (1 - \psi_t^{Dis}) + \Pi_t^{Pen} \kappa_t^{RM} r_t^{Dis} \psi_t^{Dis} \\
 & + \sum_n \Pi_{n,t}^{Genco,Offer} p_{n,t}^{Genco} - \sum_m \Pi_{m,t}^{Ret.,Bid} p_{m,t}^{Ret.} \\
 & + \sum_m (\Pi_{m,t}^{Ret.,Res} r_{m,t}^{Ret.} + \Pi_t^{inc} \kappa_t^{RM} r_{m,t}^{Ret.} (1 - \psi_{m,t}^{Ret.}) \\
 & - \Pi_t^{Pen} \kappa_t^{RM} r_{m,t}^{Ret.} \psi_{m,t}^{Ret.}) + \sum_n (\Pi_{n,t}^{Genco,Res} r_{n,t}^{Genco} \\
 & + \Pi_t^{inc} \kappa_t^{RM} r_{n,t}^{Genco} (1 - \psi_{n,t}^{Genco}) - \Pi_t^{Pen} \kappa_t^{RM} r_{n,t}^{Genco} \psi_{n,t}^{Genco}) \\
 & \left. + \sum_n (\bar{P}_n^{Genco} \bar{\mu}_{n,t}^{Genco} + \bar{R}_n^{Genco} \bar{\mu}_{n,t}^{Genco,Res}) \right. \\
 & \left. + \sum_m (\bar{P}_m^{Ret.} \bar{\mu}_{m,t}^{Ret.} + \bar{R}_m^{Ret.} \bar{\mu}_{m,t}^{Ret.,Res}) - \bar{R}_t^{Sys.} \pi_t^{RM} \right] \quad (58)
 \end{aligned}$$

subject to:

$$(2)-(22), (24), (25), (36)-(56).$$

IV. NUMERICAL RESULTS

Numerical results are presented in the next three subsections. First, the required data for the optimization is presented. Then, the results are shown and the behavior of the Disco and the other players is investigated in wholesale energy and reserve markets. Finally, a sensitivity analysis is performed with respect to the incentive/penalty for serving/not serving reserves and the number of scenarios.

A. Data

To evaluate the effectiveness of the proposed model, a hypothetical network is considered composed of 10 Gencos and 10 retailers. Required data for these players including demand bids, production offers, reserve offers, and maximum energy provided by Gencos are provided in Table II. The maximum reserve provided by each Genco in the reserve market is 10% of its maximum power generation. The load pattern of all retailers is similar and is obtained by multiplying the base load (250 MW) by the correction factor (α_t), which is shown in Table III. The maximum amount of reserves provided by each retailer in the reserve market is 10% of its demand in each hour. The incentive price for providing reserve and the penalty for not serving reserve are other required data are given in Table III.

The probability of the required reserves to be called by the ISO depends on the uncertain behaviour of the demand and the line and generator outage probability, which is described as the calling reserve probability. To determine the value of this probability in real systems, historical data from line limits, line and generator outages and demand can be used. In this paper, the mean value of a normal distribution function

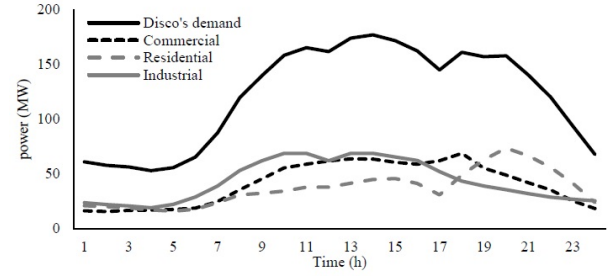


Fig. 3. Different types of Disco's demands.

in each time step, as in [38], models the calling reserve probability with some modifications, where the calling reserve probability in the operation problem of the Disco is shown in Table III.

The service failure probability for the Disco and Gencos is 0.04 as proposed in [25] and [39], respectively. Moreover, the probability of retailer's service failure is considered equal to the probability of the Disco's service failure in this paper. This value indicates the inability of decision makers, i.e., Gencos, retailers, and the Disco to produce energy in the reserve market when being called by the ISO. For the Disco, this inability may be caused by a distribution network failure or by the DERs' failure, for the retailers when their consumers were not able to reduce their load demand, and for the Gencos by their generators' failures. Therefore, this parameter can be estimated for each decision maker in real systems using the failure rate of each piece of equipment used to provide energy when being called by the ISO.

Technical and economic data of DERs, which are required by the Disco to determine optimal decisions in wholesale energy and reserve markets, are shown in Table IV. The Disco has 15 fossil-fueled DGs (5 from each of the 3 types of DG), and 5 batteries. Moreover, the Disco has industrial, residential, and commercial loads whose consumptions are shown in Fig. 3. The maximum amount of load curtailment for each load is 10% of it in each period and the price of IL is considered as in [19]. The maximum capacity and efficiency of the Disco's transformer are 200 MW and 0.95, respectively.

The capacities of both photovoltaic (PV) arrays and wind turbines (WTs) are 3.3 MW. The uncertainties of these resources are modeled as described in detail in Appendix A. The resulting optimization model has 21,313 equations, 12,937 single variables, and 2,064 discrete variables. The problem is implemented in GAMS using the CPLEX 12 solver [40].

B. Results

The simulation results are shown in Figs. 4 to 8. The clearing prices of wholesale energy and reserve markets are shown in Fig. 4. Energy and reserve balance for wholesale markets are shown in Figs. 5 and 6. The optimal scheduling of Disco's DERs to meet its demand and the optimal trading in the wholesale markets are described in Figs. 7 and 8. The results show that the demand in hours 1-13, 22-24 is met by Genco 1, Genco 2, and Gencos 5-9, as these Gencos have low marginal costs. Therefore, the prices of the wholesale

energy market in these hours are low. In hours 14-21, the demand increases and Genco 10 is added to meet the demand of the system. Due to the high marginal cost of this Genco, wholesale energy prices are higher in these hours. In hours 12, 13 and 22-24, the wholesale energy price is \$45/MWh, which is the marginal price of retailer 9. Thus, this retailer acts as a marginal player in these hours. However, these prices are determined in these hours according to the strategic behavior of the Disco. In fact, in these hours, the Disco reduces the energy purchased from the market and meets its demand with the optimal scheduling of the DERs. This behavior of the Disco decreases the demand of the whole system as well as the wholesale energy prices. Meanwhile, if the Disco purchases the required demand in these hours from the market, this demand is met by Genco 10 which increases the wholesale energy price from \$45/MWh to \$48/MWh. Therefore, the wholesale energy price decreases in these hours due to the strategic behavior of the Disco which acts as a marginal player in these hours.

The wholesale reserve prices are equal to the marginal reserve cost of Genco 10 in hours 1, 2, 11-14, 18-24, of Genco 7 in hours 3-10, of Genco 4 in hours 15, 17 and of retailer 10 in hour 16. In hours 1, 2, and 11, Genco 7 and Genco 9 reach the maximum reserve provided and the remaining required reserve is provided by the Disco. Thus, the Disco acts as a marginal player in the reserve market in these hours and increases the wholesale reserve prices to the marginal reserve cost of Genco 10. In hour 15, Genco 7, Genco 9, and Genco 10 reach the maximum reserves and the remaining required reserve of the system is provided by the Disco. Thus, the Disco acts as a marginal player in the reserve market in this hour in which the Disco increases the wholesale reserve prices to the marginal reserve cost of Genco 4. In other hours, the wholesale reserve prices are determined due to the offers of Gencos and retailer 10. In hours 1, 2, 11, and 15, the Disco provides the reserve required by the system from the DERs in its network and meets its demand by purchasing power from the energy market. In fact, this behaviour of the Disco in these hours occurs when it participates in energy and reserve markets at the same time.

The results obtained for the case study show that the Disco

TABLE II
TECHNICAL AND ECONOMIC DATA OF GENCOs AND RETAILERS

Genco No.	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10
\bar{P}_n^{Genco} [MW]	120	120	150	300	150	300	620	600	350	500
$\Pi_n^{Genco, Bid}$										
[\$/MWh]	40	40	65	50	23	35	40	33	39	48
$\Pi_n^{Genco, Res}$										
[\$/MWh]	35	35	60	35	18	30	15	28	14	28
Retailer No.	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10
$\bar{P}_m^{Ret.}$ [MW]	250	250	250	250	250	250	250	250	250	250
$\Pi_m^{Ret., Bid}$										
[\$/MWh]	75	70	65	60	57	53	50	50	45	40
$\Pi_m^{Ret., Res}$										
[\$/MWh]	70	65	60	55	52	48	45	45	40	35

TABLE III
OTHER REQUIRED DATA COMMON FOR ALL PLAYERS

Hour	κ_t^{RM}	Π_t^{inc}, Π_t^{Pen}	α_t	Hour	κ_t^{RM}	Π_t^{inc}, Π_t^{Pen}	α_t
1	2.1	72.84	0.902	13	5.5	79.21	1.018
2	1.5	67.24	0.873	14	4.9	78.82	1.192
3	1.3	62.12	0.764	15	5	77.02	1.25
4	1.5	62.08	0.727	16	7.8	75.18	1.338
5	1.9	61.5	0.64	17	9.7	74.15	1.25
6	2.4	62.05	0.494	18	15.6	75.6	1.163
7	2.9	65.98	0.494	19	15.5	77.11	1.105
8	5.5	71.1	0.494	20	15.1	79.42	1.105
9	7.8	73.37	0.582	21	15.4	81.38	1.076
10	8.1	78	0.756	22	15.6	83.58	1.018
11	8.3	79.24	0.814	23	9.4	80.8	0.96
12	8.4	78.33	0.931	24	2.4	74.77	0.96

TABLE IV
TECHNICAL AND ECONOMIC DATA OF DISCO'S DERs

Resources	Parameters				
	C_i^{DG}	\bar{P}_i^{DG}	RUP_i^{DG}	RDN_i^{DG}	$p_i^{DG, ini}$
DG1	37	4	1	1	0
DG2	40	5	1.25	1.25	0
DG3	35	5.5	1.375	1.375	0
	\bar{P}_k^{batt}	\underline{E}_k^{batt}	\bar{E}_k^{batt}	$e_k^{batt, ini}$	$\eta_k^{batt, ch/dch}$
Battery	1	0.5	2.5	1	0.95

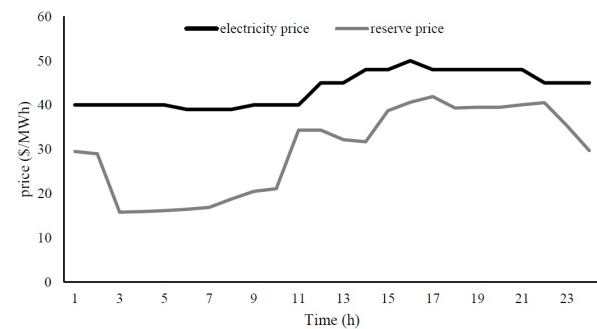


Fig. 4. Wholesale energy and reserve prices.

acts as a strategic prosumer in wholesale energy and reserve markets. The Disco acts as a strategic consumer in some hours (12, 13 and 22-24), where the purchased energy has an impact on wholesale energy prices and acts as a strategic producer in some hours (1, 2, 11 and 15), for which the reserve provided has an impact on wholesale reserve prices.

C. Sensitivity Analysis

In this subsection, a sensitivity analysis with respect to two parameters, incentive/penalty due to serving/not serving reserve and number of scenarios is performed. First, the effect of the incentive/penalty due to serving/not serving reserve on the results and the behavior of the decision makers is evaluated. For this purpose, the values of this parameter in the previous subsection are considered as base values (1 p.u.). The sensitivity analysis on this parameter is shown in Fig. 9 and Table V. As shown, when the values of Π_t^{inc}, Π_t^{Pen} increase,

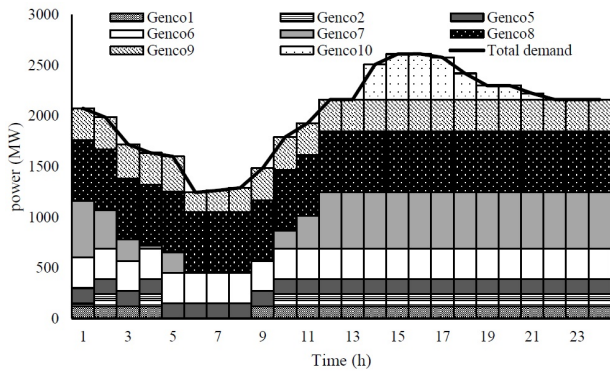


Fig. 5. Power generation of Gencos to meet the demand.

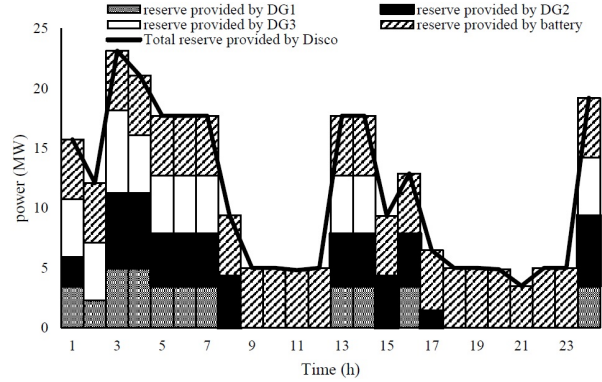


Fig. 8. The amount of reserve provided by DERs for the Disco.

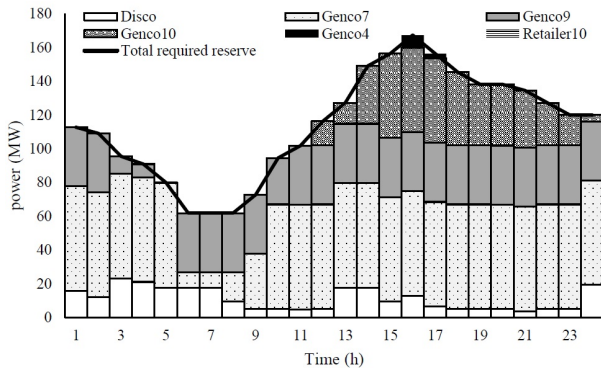


Fig. 6. Reserve provided by each player.

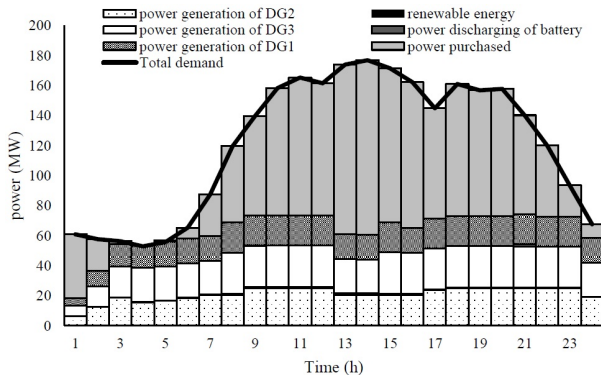


Fig. 7. Optimal decisions of the Disco to meet the demand.

where the total cost is provided in Table V.

Since the Disco's problem uses a stochastic framework, the sensitivity of this problem with respect to the number of scenarios is performed and the results are shown in Table VI. The output powers of PV arrays and WTs are considered as uncertain parameters in this problem. In the base case, whose results are shown in the previous section, 3 scenarios are generated to model the uncertain behavior of each parameter in each time step and 9 scenarios are obtained from the scenario tree model for these parameters. To evaluate the sensitivity of the proposed model to the number of scenarios, 5, 10, 20, and 50 scenarios are generated for each uncertain parameter, leading to 25, 100, 400, and 2500 scenarios using the scenario tree model. Since the Disco is modeled as a strategic player in wholesale energy and reserve markets, the wholesale energy and reserve prices are important decision variables determined in the proposed model. By increasing the number of scenarios, the wholesale energy and reserve prices do not change and are equal to the ones shown in Fig. 4. Moreover, the decision variables of the Disco's problem are determined so that its total cost is minimized. By increasing the number of scenarios, the variations of the total cost of the Disco are very low as shown in Table VI. Therefore, increasing the number of scenarios does not change the outcome of the decision-making process. However, the size of the problem, including the number of equations and single variables, increases.

V. CONCLUSIONS AND FUTURE WORK

In this paper, the strategic behavior of a Disco in both energy and reserve markets is modeled. For this purpose, a bi-level optimization model is developed in which the upper- and lower-level decision makers are the Disco and the ISO, respectively. KKT conditions and duality theory are used to transform the proposed non-linear bi-level model into an MILP. The results show that the strategic behavior of the Disco in both energy and reserve markets decreases the Disco's operation costs. In some hours, the Disco uses its DERs to meet the demand and the power purchased from the wholesale energy market decreases. In these hours, the wholesale energy price decreases and the Disco acts as a strategic player. In some hours in which some Gencos reach their maximum

the wholesale reserve prices increase. It should be noted that the wholesale energy prices are constant in this way. When the wholesale reserve prices increase, the marginal reserve provision cost of the Disco increases and, thus, the amount of reserves provided by the Disco to the wholesale reserve market decreases. On the other hand, the reserves provided by the Disco in hours 1, 2, 11, 15 do not change due to its strategic behavior. Meanwhile, in other hours, the reserves provided by the Disco decrease. Therefore, in the hours in which the reserve provided by the Disco decreases, the Disco uses its DERs to meet its demand and, therefore, the power purchased by the Disco in the wholesale energy market decreases. This behavior of the Disco modifies its profit in the reserve market,

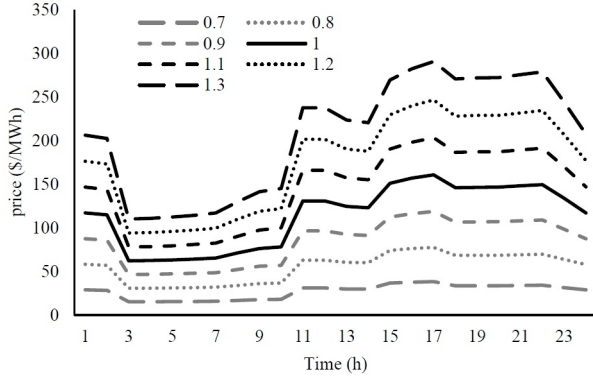


Fig. 9. Wholesale reserve prices for different values of incentive/penalty due to serving/not serving reserves.

amount of reserves that can be provided to the market, the Disco acts as a strategic player in this market and increases the wholesale reserve prices to the marginal reserve cost of other Gencos, obtaining more profit from this market. A sensitivity analysis on incentive/penalty to serve/not serve reserve shows that this parameter has an important impact on the behavior of the Disco. High values of this parameter cause that the amount of reserves provided to the market by Gencos and retailer 10 increase in some hours and, thus, the reserves provided by the Disco in these hours decrease.

In the proposed model, retailers and Gencos are truthful bidders for the strategic decision making of the Disco. As future work, it is suggested to model the strategic behaviour of the Disco in wholesale energy and reserve markets, where the other decision makers such as Gencos, retailers and other Discos also behave strategically. For this purpose, the problem can be modeled using a bi-level model in which the operation problem of each decision maker could be modeled in the upper-level problem and different energy and reserve market clearing scenarios could be represented in the lower-level problems. Then, the lower-level problems could be replaced with their KKT conditions and the proposed bi-level model for each decision maker could be transformed into an MPEC. The MPECs of all decision makers could be put together to produce an equilibrium problem with equilibrium constraints (EPEC) which provides a solution to balance out the objective function of decision makers.

APPENDIX A

MODELING OF THE UNCERTAINTIES OF PV ARRAYS AND WTS

The uncertainties in the inputs from PV arrays and WTs are modeled by generating appropriate scenarios. Power generation is modeled based on the hourly historical data of the site under study [41] and the detailed features of the units. In order to model the intermittent generation of the units, a typical day with 24-h time periods is taken into account. The data associated with the same time periods of the day are used to obtain the probability distribution functions (PDFs) corresponding to each hour. Wind speed distribution is

TABLE V
DISCO'S BEHAVIOR AND RESERVE PROVISION BY PLAYERS DUE TO DIFFERENT VALUES OF Π_t^{inc} , Π_t^{Pen} .

Π_t^{inc} , Π_t^{Pen} (p.u.)	Disco's total cost	Disco' revenue from RM	Total power purchased by Disco	Total res. provided by Disco
0.7	114797	7572.8	1485	291.7
0.8	114811	7488.61	1474.3	281.5
0.9	114544	7626.7	1463.4	271.4
1	114536	7589	1458.7	265.93
1.1	114717	7293.58	1447.6	252.48
1.2	114577	7430.2	1447.5	252.14
1.3	114578	7352.18	1442.4	246.05

Total Reserve Provision

	Genco4	Genco7	Genco9	Genco10	Retailer10
0.7	2.585	1210.3	840	394.54	0
0.8	2.585	1248.5	805.31	401.24	0
0.9	0	1221.2	832.55	406.79	7.3977
1.0	2.902	1281.3	773.19	408.4	7.3977
1.1	4.363	1226.9	832.55	411.92	10.898
1.2	0	1251.6	807.88	412.26	15.261
1.3	4.363	1226.9	832.35	418.35	10.898

TABLE VI

SENSITIVITY OF THE PROBLEM TO THE NUMBER OF SCENARIOS

Number of scenarios	Disco's total cost (p.u.)	Number of equations	Number of single variables
9	1	21313	12937
25	0.9996	44737	23305
100	0.9946	154537	71905
400	0.996	593737	266305
2500	0.9966	3668137	1627105

regularly considered by a Weibull distribution [42]. The PDF of the wind speed is represented in (59).

$$f_v(v) = \frac{k}{c} \left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^k\right] \quad (59)$$

where $c > 0$ and $k > 0$ are the scale and shape factors, respectively.

The PDF is divided into N_{ws} scenarios and the probability of each scenario is calculated by (60).

$$prob_\omega = \int_{WS_\omega}^{WS_{\omega+1}} f_v(v) dv, \quad \omega = 1, 2, \dots, N_{ws} \quad (60)$$

where WS_ω is the wind speed of the ω^{th} scenario. The power generated, $P_{GW}(\omega)$, corresponding to a specific wind speed, WS_ω , can be obtained from (61) in which A , B , and C are constants calculated according to [42].

$$P_{GW}(\omega) = \begin{cases} 0 & 0 \leq WS_\omega < V_c \text{ or } WS_\omega > V_{c0} \\ P_r(A + B WS_\omega + C WS_\omega^2) & V_c \leq WS_\omega \leq V_r \\ P_r & V_r \leq WS_\omega \leq V_{c0} \end{cases} \quad (61)$$

where V_c , V_{c0} , and V_{cr} represent the cut-in, cut-out, and rated speeds, respectively.

The hourly solar irradiance data for the site under study have been used to generate a Beta PDF [43] for each hour. Thus, the PDF of the solar irradiance is calculated in (62).

$$f_b(s) = \begin{cases} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) \Gamma(\beta)} s^{\alpha-1} (1-s)^{\beta-1} & : 0 \leq s \leq 1; \alpha, \beta \geq 0 \\ 0 & : \text{otherwise.} \end{cases} \quad (62)$$

where $f_b(\cdot)$ is the Beta distribution function and α and β are the parameters of the Beta function for each time period, which can be determined using historical data.

Similarly, Beta PDFs are split into N_s scenarios for which the occurrence probability of each scenario during any specific hour is expressed in (63).

$$\text{prob}_i^s = \int_{S_i}^{S_{i+1}} f_s(s) ds_i \quad i = 1, 2, \dots, N_s \quad (63)$$

where S_i and prob_i^s denote the solar radiation and probability occurrence of interval i , respectively.

After generating the related PDFs, different realizations of the solar irradiance and wind speed are generated using the roulette wheel mechanism (RWM) [44] and Monte Carlo simulation (MCS) [45], separately. In this case, N_s and N_{ws} scenarios are generated for solar irradiance and wind speed, respectively. Each scenario has its own probability of occurrence.

A large number of scenarios may contribute to a more accurate model of the random variables. Nevertheless, this would increase the computational burden of the problem. Finally, a fast-forward scenario reduction method based on Kontorwish distance [46] is used to reduce the number of scenarios, while providing a reasonable approximation of the random variables of the system. At the end, three scenarios are obtained for power generation of WTs and PV arrays separately in each time step. The set of scenarios is obtained from the two-stage scenario tree model [19], [22]. Therefore, in each time step, there are 9 scenarios where each scenario consists of the power generation of PV arrays and WTs and has its own probability of occurrence.

APPENDIX B

OBJECTIVE FUNCTION LINEARIZATION

To linearize the objective function of the upper-level problem, the dual problem of the lower-level problem is constructed as (64).

$$\begin{aligned} \max F(X_D^{LL}) = & \bar{R}_t^{Sys.} \pi_t^{RM} - \bar{P}_m^{Ret.} \bar{\mu}_{m,t}^{Ret.} - \bar{R}_m^{Ret.} \bar{\mu}_{m,t}^{Ret.,Res} \\ & - \bar{P}_n^{Genco} \bar{\mu}_{n,t}^{Genco} - \bar{R}_n^{Genco} \bar{\mu}_{n,t}^{Genco,Res} - \bar{P}^{Dis} \bar{\mu}_t^{Dis,out} \\ & - \bar{P}^{Dis} \bar{\mu}_t^{Dis,in} - \bar{R}^{Dis} \bar{\mu}_t^{Dis,Res} \end{aligned} \quad (64)$$

subject to:

$$(36)-(42).$$

where, X_D^{LL} is decision variables vector of the dual problem of lower-level problem defined as:

$$X_D^{LL} = [\pi_t^{EM}, \pi_t^{RM}, \underline{\mu}_{m,t}^{Ret.}, \bar{\mu}_{m,t}^{Ret.}, \underline{\mu}_{n,t}^{Genco}, \bar{\mu}_{n,t}^{Genco}, \underline{\mu}_t^{Dis,in}, \bar{\mu}_t^{Dis,in}, \underline{\mu}_t^{Dis,out}, \bar{\mu}_t^{Dis,out}, \underline{\mu}_t^{Ret.,Res}, \bar{\mu}_t^{Ret.,Res}, \underline{\mu}_t^{Genco,Res}, \bar{\mu}_t^{Genco,Res}, \underline{\mu}_t^{Dis,Res}, \bar{\mu}_t^{Dis,Res}].$$

According to strong duality theory, the objective functions of the primal and dual problems are equal at the optimal values of their decision variables, where the resulting relation between them is described in (65).

$$\begin{aligned} F(X_P^{LL}) = F(X_D^{LL}) \\ \Rightarrow & \pi_t^{Dis,Offer*} p_t^{Dis,out} - \pi_t^{Dis,Bid*} p_t^{Dis,in} + \pi_t^{Dis,Res*} r_t^{Dis} \\ & = - \sum_n \Pi_{n,t}^{Genco,Offer} p_{n,t}^{Genco} + \sum_m \Pi_{m,t}^{Ret.,Bid} p_{m,t}^{Ret.} \\ & - \sum_m (\Pi_{m,t}^{Ret.,Res} r_{m,t}^{Ret.} + \Pi_t^{inc} \kappa_t^{RM} r_{m,t}^{Ret.} (1 - \psi_{m,t}^{Ret.}) \\ & - \Pi_t^{Pen} \kappa_t^{RM} r_{m,t}^{Ret.} \psi_{m,t}^{Ret.}) - \sum_n (\Pi_{n,t}^{Genco,Res} r_{n,t}^{Genco} \\ & + \Pi_t^{inc} \kappa_t^{RM} r_{n,t}^{Genco} (1 - \psi_{n,t}^{Genco}) - \Pi_t^{Pen} \kappa_t^{RM} r_{n,t}^{Genco} \psi_{n,t}^{Genco}) \\ & - (\Pi_t^{inc} \kappa_t^{RM} r_t^{Dis} (1 - \psi_t^{Dis}) - \Pi_t^{Pen} \kappa_t^{RM} r_t^{Dis} \psi_t^{Dis}) \\ & + \bar{R}_t^{Sys.} \pi_t^{RM} - \bar{P}_m^{Ret.} \bar{\mu}_{m,t}^{Ret.} - \bar{R}_m^{Ret.} \bar{\mu}_{m,t}^{Ret.,Res} \\ & - \bar{P}_n^{Genco} \bar{\mu}_{n,t}^{Genco} - \bar{R}_n^{Genco} \bar{\mu}_{n,t}^{Genco,Res} - \bar{P}^{Dis} \bar{\mu}_t^{Dis,out} \\ & - \bar{P}^{Dis} \bar{\mu}_t^{Dis,in} - \bar{R}^{Dis} \bar{\mu}_t^{Dis,Res} \end{aligned} \quad (65)$$

The procedure to transform the left-hand side of equation (65) to the nonlinear expressions of equation (1) is described next. First, the stationarity constraints (38), (39), and (42) related to $p_t^{Dis,in}$, $p_t^{Dis,out}$, and r_t^{Dis} , respectively, are rewritten as equations (66)-(68).

$$\pi_t^{Dis,Bid*} = \pi_t^{EM} - \underline{\mu}_t^{Dis,in} + \bar{\mu}_t^{Dis,in} \quad (66)$$

$$\pi_t^{Dis,Offer*} = \pi_t^{EM} + \underline{\mu}_t^{Dis,out} - \bar{\mu}_t^{Dis,out} \quad (67)$$

$$\begin{aligned} \pi_t^{Dis,Res*} = & -\Pi_t^{inc} \kappa_t^{RM} (1 - \psi_t^{Dis}) + \Pi_t^{Pen} \kappa_t^{RM} \psi_t^{Dis} \\ & + \pi_t^{RM} - \bar{\mu}_t^{Dis,out} + \underline{\mu}_t^{Dis,Res} - \bar{\mu}_t^{Dis,Res} \end{aligned} \quad (68)$$

Then, both sides of equations (66)-(68) are multiplied by their variables as follows.

$$\begin{aligned} p_t^{Dis,in} \pi_t^{Dis,Bid*} = & p_t^{Dis,in} \pi_t^{EM} - p_t^{Dis,in} \underline{\mu}_t^{Dis,in} \\ & + p_t^{Dis,in} \bar{\mu}_t^{Dis,in} \end{aligned} \quad (69)$$

$$\begin{aligned} p_t^{Dis,out} \pi_t^{Dis,Offer*} = & p_t^{Dis,out} \pi_t^{EM} + p_t^{Dis,out} \underline{\mu}_t^{Dis,out} \\ & - p_t^{Dis,out} \bar{\mu}_t^{Dis,out} \end{aligned} \quad (70)$$

$$\begin{aligned} r_t^{Dis} \pi_t^{Dis,Res*} = & -r_t^{Dis} \Pi_t^{inc} \kappa_t^{RM} (1 - \psi_t^{Dis}) + \\ & r_t^{Dis} \Pi_t^{Pen} \kappa_t^{RM} \psi_t^{Dis} + r_t^{Dis} \pi_t^{RM} - r_t^{Dis} \bar{\mu}_t^{Dis,out} + \\ & r_t^{Dis} \underline{\mu}_t^{Dis,Res} - r_t^{Dis} \bar{\mu}_t^{Dis,Res} \end{aligned} \quad (71)$$

The left-hand side of equation (65) is obtained from equations (69)-(71) as follows.

$$\begin{aligned}
 & p_t^{Dis,out} \pi_t^{Dis,Offer*} - p_t^{Dis,in} \pi_t^{Dis,Bid*} + r_t^{Dis} \pi_t^{Dis,Res*} \\
 & = p_t^{Dis,out} \pi_t^{EM} + p_t^{Dis,out} \mu_t^{Dis,out} - p_t^{Dis,out} \mu_t^{Dis,out} \\
 & \quad - p_t^{Dis,in} \pi_t^{EM} + p_t^{Dis,in} \mu_t^{Dis,in} - p_t^{Dis,in} \mu_t^{Dis,in} \\
 & \quad - r_t^{Dis} \Pi_t^{inc} \kappa_t^{RM} (1 - \psi_t^{Dis}) + r_t^{Dis} \Pi_t^{Pen} \kappa_t^{RM} \psi_t^{Dis} \\
 & \quad + r_t^{Dis} \pi_t^{RM} - r_t^{Dis} \mu_t^{Dis,out} + \\
 & \quad r_t^{Dis} \mu_t^{Dis,Res} - r_t^{Dis} \mu_t^{Dis,Res} \quad (72)
 \end{aligned}$$

Due to the complementary slackness constraints, some relations between primal and dual variables of the lower-level problem are obtained.

$$0 \leq p_t^{Dis,out} \perp \mu_t^{Dis,out} \geq 0 \implies p_t^{Dis,out} \mu_t^{Dis,out} = 0 \quad (73)$$

$$\begin{aligned}
 0 \leq (\bar{P}^{Dis} - p_t^{Dis,out} - r_t^{Dis}) \perp \mu_t^{Dis,out} \geq 0 \implies \\
 \bar{P}^{Dis} \mu_t^{Dis,out} = (p_t^{Dis,out} + r_t^{Dis}) \mu_t^{Dis,out} \quad (74)
 \end{aligned}$$

$$0 \leq p_t^{Dis,in} \perp \mu_t^{Dis,in} \geq 0 \implies p_t^{Dis,in} \mu_t^{Dis,in} = 0 \quad (75)$$

$$\begin{aligned}
 0 \leq (\bar{P}^{Dis} - p_t^{Dis,in}) \perp \mu_t^{Dis,in} \geq 0 \implies \\
 \bar{P}^{Dis} \mu_t^{Dis,in} = p_t^{Dis,in} \mu_t^{Dis,in} \quad (76)
 \end{aligned}$$

$$0 \leq r_t^{Dis} \perp \mu_t^{Dis,Res} \geq 0 \implies r_t^{Dis} \mu_t^{Dis,Res} = 0 \quad (77)$$

$$\begin{aligned}
 0 \leq (\bar{R}^{Dis} - r_t^{Dis}) \perp \mu_t^{Dis,Res} \geq 0 \implies \\
 \bar{R}^{Dis} \mu_t^{Dis,Res} = r_t^{Dis} \mu_t^{Dis,Res} \quad (78)
 \end{aligned}$$

Equation (72) is simplified using equations (73)-(78) and the resulting relation is obtained.

$$\begin{aligned}
 & p_t^{Dis,out} \pi_t^{Dis,Offer*} - p_t^{Dis,in} \pi_t^{Dis,Bid*} + r_t^{Dis} \pi_t^{Dis,Res*} \\
 & = p_t^{Dis,out} \pi_t^{EM} - p_t^{Dis,in} \pi_t^{EM} + r_t^{Dis} \pi_t^{RM} \\
 & \quad - r_t^{Dis} \Pi_t^{inc} \kappa_t^{RM} (1 - \psi_t^{Dis}) + r_t^{Dis} \Pi_t^{Pen} \kappa_t^{RM} \psi_t^{Dis} \\
 & \quad - \bar{P}^{Dis} \mu_t^{Dis,in} - \bar{P}^{Dis} \mu_t^{Dis,out} - \bar{R}^{Dis} \mu_t^{Dis,Res} \quad (79)
 \end{aligned}$$

Equation (65) is rewritten using equation (79) and the resulting equation is obtained.

$$\begin{aligned}
 & p_t^{Dis,out} \pi_t^{EM} - p_t^{Dis,in} \pi_t^{EM} + r_t^{Dis} \pi_t^{RM} \\
 & \quad - r_t^{Dis} \Pi_t^{inc} \kappa_t^{RM} (1 - \psi_t^{Dis}) + r_t^{Dis} \Pi_t^{Pen} \kappa_t^{RM} \psi_t^{Dis} \\
 & \quad - \bar{P}^{Dis} \mu_t^{Dis,in} - \bar{P}^{Dis} \mu_t^{Dis,out} - \bar{R}^{Dis} \mu_t^{Dis,Res} = \\
 & \quad - \sum_n \Pi_{n,t}^{Genco,Offer} p_{n,t}^{Genco} + \sum_m \Pi_{m,t}^{Ret.,Bid} p_{m,t}^{Ret.} \\
 & \quad - \sum_m (\Pi_{m,t}^{Ret.,Res} r_{m,t}^{Ret.} + \Pi_t^{inc} \kappa_t^{RM} r_{m,t}^{Ret.} (1 - \psi_{m,t}^{Ret.}) \\
 & \quad - \Pi_t^{Pen} \kappa_t^{RM} r_{m,t}^{Ret.} \psi_{m,t}^{Ret.}) - \sum_n (\Pi_{n,t}^{Genco,Res} r_{n,t}^{Genco} \\
 & \quad + \Pi_t^{inc} \kappa_t^{RM} r_{n,t}^{Genco} (1 - \psi_{n,t}^{Genco}) - \Pi_t^{Pen} \kappa_t^{RM} r_{n,t}^{Genco} \psi_{n,t}^{Genco}) \\
 & \quad - (\Pi_t^{inc} \kappa_t^{RM} r_t^{Dis} (1 - \psi_t^{Dis}) - \Pi_t^{Pen} \kappa_t^{RM} r_t^{Dis} \psi_t^{Dis}) \\
 & \quad + \bar{R}_t^{Sys} \pi_t^{RM} - \bar{P}_m^{Ret.} \mu_{m,t}^{Ret.} - \bar{R}_m^{Ret.} \mu_{m,t}^{Ret.,Res} \\
 & \quad - \bar{P}_n^{Genco} \mu_{n,t}^{Genco} - \bar{R}_n^{Genco} \mu_{n,t}^{Genco,Res} - \bar{P}^{Dis} \mu_t^{Dis,out} \\
 & \quad - \bar{P}^{Dis} \mu_t^{Dis,in} - \bar{R}^{Dis} \mu_t^{Dis,Res} \quad (80)
 \end{aligned}$$

$$\begin{aligned}
 & p_t^{Dis,out} \pi_t^{EM} - p_t^{Dis,in} \pi_t^{EM} + r_t^{Dis} \pi_t^{RM} = \\
 & \quad - \sum_n \Pi_{n,t}^{Genco,Offer} p_{n,t}^{Genco} + \sum_m \Pi_{m,t}^{Ret.,Bid} p_{m,t}^{Ret.} \\
 & \quad - \sum_m (\Pi_{m,t}^{Ret.,Res} r_{m,t}^{Ret.} + \Pi_t^{inc} \kappa_t^{RM} r_{m,t}^{Ret.} (1 - \psi_{m,t}^{Ret.}) \\
 & \quad - \Pi_t^{Pen} \kappa_t^{RM} r_{m,t}^{Ret.} \psi_{m,t}^{Ret.}) - \sum_n (\Pi_{n,t}^{Genco,Res} r_{n,t}^{Genco} \\
 & \quad + \Pi_t^{inc} \kappa_t^{RM} r_{n,t}^{Genco} (1 - \psi_{n,t}^{Genco}) - \Pi_t^{Pen} \kappa_t^{RM} r_{n,t}^{Genco} \psi_{n,t}^{Genco}) \\
 & \quad + \bar{R}_t^{Sys} \pi_t^{RM} - \bar{P}_m^{Ret.} \mu_{m,t}^{Ret.} - \bar{R}_m^{Ret.} \mu_{m,t}^{Ret.,Res} \\
 & \quad - \bar{P}_n^{Genco} \mu_{n,t}^{Genco} - \bar{R}_n^{Genco} \mu_{n,t}^{Genco,Res} \quad (81)
 \end{aligned}$$

The nonlinear expressions of equation (1), which is on the left-hand side of equation (81), is replaced with linear expressions.

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