

# MPC Weights Tuning Role on the Energy Optimization in Residential Appliances

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**Abstract**—Genuine concerns regarding air pollution, climate change, and dependence on unstable and expensive supplies of fossil fuels have lead policy makers and researchers to search for alternatives to conventional petroleum-fueled combustion power plants with the purpose to reduce greenhouse gas emission. This leads to an urgent need to substitute them with alternate generating capacity or reduce the consumption during peak periods, or both. One of the options for power generation is the use of renewable energy resources, which can inject power to the grid deprived of greenhouse gas emissions. But, from the load point of view, the renewable energy resources capacity is not sufficient to supply all the required power. These points to the necessity of innovative methods, able to diminish energy consumption in different sectors, but also with the aim of reducing the domestic customer's total energy costs, greenhouse gas emissions and energy demand, especially during on-peak, while always considering the end user preferences. Hence, this paper analyses model predictive control (MPC) application in domestic appliances with the purpose of energy optimization. In this context, the research theme is focused on the relation between MPC weighting adjustment and the minimization of energy consumption. Three domestic loads are used for MPC tuning evaluation: water heater (WH), room temperature control by conditioner (AC) and refrigerator (RF).

**Keywords**—Residential buildings; Model Predictive Control; Water heating and cooling; Domestic appliances.

## NOMENCLATURE

### Acronyms

AC	Air Conditioner
BTU	British Thermal Units
HVAC	Heating, Ventilation and Air Conditioning
MPC	Model Predictive Control
TH	Thermostat
WH	Water Heater
QP	Quadratic Program
OV	Output Variable
CH	Control Horizon
RF	Refrigerator

### Parameters and Variables

$\eta$	Efficiency
$UA$	Characteristics of fiber glass

$C_p$	Specific heat of water
$C_w$	Thermal capacitance of the wall
$C_{in}$	Thermal capacitance of the indoor air
$m$	Mass of water
$Q_{eg}$	Electric power
$Q_{ac,ht}$	Thermal source
$Q_{in}$	Heat to be extracted
$R_w$	Thermal resistance of walls
$R_c$	Thermal resistance of windows
$S$	Binary variable
$SP$	Set-point
$T_w$	Temperature of the wall
$T_{in}$	Temperature in the room
$T_{inlet}$	Inlet water temperature
$T_{amb}$	Ambient temperature

## I. INTRODUCTION

Present high living standards offered by modern society have been achieved due to abundant energy to relatively low energy prices. This was possible to the large scale exploitation of high carbon coal power over the last decades. However to preserve our economic and social development model substantial changes have to be made to lower greenhouse gas emissions at world scale. The transition from high-carbon coal power to low-carbon electricity from natural gas and mostly on renewables is one solution. However, there are several challenges that inhibit for now, a complete transformation into 100% sustainable energy systems [1].

With increased demand for energy, alternative strategies have to be implemented at different levels of human activity not only at industrial sector to promote efficient use of energy but, also in the residential sector studies reported that it has been responsible for 31% of the worldwide energy needs which includes to a large extent domestic consumers [2]. This means that increasing number of electronics devices and appliances in the average home there are opportunities to obtain efficiency gains on energy usage and concerted actions can be developed to address energy saving in households. One way is introducing innovative tariff schemes based on demand response programs that help the consumer to change their energy consumption habits.

By lowering the peak demand, the utilization of the available grid capacity is improved [3]. Other approach relies on updating control technology namely domestic appliances operated with regulation temperature.

In general, in a typical residential home, the appliances with higher electricity consumption provide heating and cooling services (AC, WH, and in a more reduced scale the RF). Numbers referred to UE-27 reveal that space heating for housing contributes around 70% for household electricity bill while domestic water heating stands at 10% [4]. Effective potential for energy savings as result of adopting efficiency energy measures can reach 30% [5]. In this sense, one of the ways to help reach the objective of reducing energy consumption is trough updating control technology that operates this class of operated domestic appliances. In fact heating and cooling equipment use a conventional ON-OFF device to regulate the temperature. Due to its simplicity and low manufacturing cost this solution has been the main choice by appliance brands for decades.

Alternative control methods have been researched to address energy rational utilization of electric loads of appliances in a residential home such as residential energy monitoring and management based on fuzzy logic [6], artificial neural networks, PID control, model predictive control (MPC), among others [1].

MPC is a model-based controller concept design. It is defined as an optimal control algorithm that anticipates the future behavior of control variables based on a process model that optimizes the control signal. It is capable to control nonlinearities and at the same time satisfying constraints of the system, it has been gained acceptance in different engineering branches [7]. MPC technique based applications for the residential sector are used with the purpose to decrease peak load, improve building thermal comfort and reduce energy costs [8]. Normally, is proposed for heating, ventilation, cooling and air conditioning (HVAC) systems in order to minimize energy cost beyond a mere reduction of the used energy [9]. Other implementation aims to apply a MPC based appliance scheduling for residential building energy management purposes [10]. In [11] a MPC strategy for energy efficient buildings with thermal storage is proposed.

The formulation of control objectives as a single cost function is required as a part of MPC controller design process. Satisfying all control objectives simultaneously is not feasible since normally specific control actions have priority over other actions. Therefore, performance trade-offs have to be made among the competing control objectives. For this purpose, weight factors are applied to the process input and output variables, thus, enabling individual performance prioritization to perform the control under process constraints. Consequently, MPC uses different weights on tracking errors [12] and the control variables weight assignment is named MPC tuning.

A previous work regarding domestic loads energy optimization showed that this type of controller can compete and overpass the thermostat performance by decreasing the energy cost in typical heating and cooling applications [13].

However, the MPC tuning feature that improves the controller performance regarding minimization of the energy consumption is not explored. Hence, the objective of this research work is to provide insights about MPC weight tuning impact on domestic heating and cooling appliances in order to decrease energy usage and therefore the energy bill. Three domestic loads are used for MPC tuning evaluation: water heater (WH), room temperature control by conditioner (AC) controlled room's temperature and refrigerator (RF). A previous work regarding this subject has showed that this type of controller can overpass the thermostat performance with a final lower energy usage cost [13].

The rest of the paper is organized as follows: Section II discusses MPC theory followed by the physical models description of the domestic appliances to be simulated. Section III is dedicated to the case studies and their simulations results. Finally, Section IV provides conclusion inferred from this work.

## II. THEORY

### A. MPC controller design

The MPC is an optimization tool to solve a series of control objectives. This means the optimization process produces a sequence of optimal control actions over a finite horizon of future steps, driving the system output towards a known reference and at the same time satisfying system constraints and minimizing a specified performance criterion.

MPC formulation in state-space presents several advantages as it facilitates multivariable system representation, analysis of closed-loop properties. In this regard the system to be controlled can be described by linear time-invariant (LTI) equations as a discrete-time state space model:

$$x(k+1) = Ax(k) + Bu(k) \quad (1)$$

$$y(k) = Cx(k) \quad (2)$$

where  $x$  is the system state vector,  $u$  is the input vector,  $y$  is the output vector,  $A$  is the state matrix,  $B$  is the input matrix,  $C$  is the output matrix.

Bearing in mind that at the current control interval  $k$  the model state vector  $x(k)$  is known. Then it becomes possible to calculate the new input control vector, to be fed into the system, and at the same time, having process constraints into consideration. For the time  $k$  the MPC process predicts the plant output regarding the time  $k+P$ , where  $P$  refers to the number of samples to be estimated in the future – prediction horizon. On the other hand, a sequence of future control actions is also calculated as part of the optimization process – control horizon. The number of steps ahead is defined by  $N$ .

Optimal control problem is solved by using a quadratic objective function known as quadratic program (QP). The optimization cost function combines a set of performance index with regard to different control objectives which are:

$$J(z_k) = J_y(z_k) + J_u(z_k) + J_{\Delta u}(z_k) + J_\varepsilon(z_k) \quad (3)$$

where  $z_k$  is the QP decision at each control interval,  $J_y(z_k)$  minimizes output reference tracking error,  $J_u(z_k)$  refers to the control signal tracking error,  $J_u(z_k)$  is applied to regulate control signal increments and  $J_\varepsilon(z_k)$  is related with constraint violations. QP decision  $z_k$  takes the form as:

$$z_k^T = [u(k|k)^T \ u(k+1|k)^T \ \dots \ u(k+P-1|k)^T] \quad (4)$$

Assuming the number of process output variables is reduced to one, output reference tracking performance index is described as:

$$J_y(z_k) = \sum_{i=1}^P \{\omega_i^y [r(k+i|k) - y(k+i|k)]\}^2 \quad (5)$$

where  $r(k+i|k)$  is the set-point signal,  $y(k+i|k)$  is the controlled variable and an associated  $w_i^y$  – a weighting coefficient that allocates more relevance to the term.  $k$  is the current control interval and  $k+i$  is the time instant related to the future state prediction.

Control objective related to control signal tracking is:

$$J_u(z_k) = \sum_{i=1}^P \{\omega_i^u [u(k+i|k) - u_t(k+i|k)]\}^2 \quad (6)$$

where  $u(k+i|k)$  is the manipulate variable or control signal,  $u_t(k+i|k)$  is the target for control signal and  $w_i^u$  a weighting coefficient for giving more importance to this term.

The third term in Eq. (3) minimizes abrupt changes between consecutive output control signals into small steps. This means a constraint is imposed on rate of change of control signal. Therefore the performance index is formulated as:

$$J_{\Delta u}(z_k) = \sum_{i=1}^P \{\omega_i^{\Delta u} [u(k+i|k) - u_k(k+i-1|k)]\}^2 \quad (7)$$

where  $u_k(k+i-1|k)$  is a control signal from the previous control interval and  $\omega_i^{\Delta u}$  a weighting coefficient that penalizes changes in the  $u_k$ .

Finally, the fourth term is related with the constraint violation used for constraint softening

$$J_\varepsilon(z_k) = \rho_\varepsilon \varepsilon_k^2 \quad (8)$$

where  $\rho_\varepsilon$  is a constraint violation penalty weight and  $\varepsilon_k$  is a slack variable at control interval  $k$ .

A finite control horizon  $M$  parameter establishes the number of parameters used to predict future control trajectory. This means given the state variable vector  $x(k_i)$ , the future state variables are evaluated for a number of samples  $P$  called as the prediction horizon.

By applying  $\omega_i^y$  with a higher value than  $\omega_i^u$  leads the model output to closely follow the set-point signal. On contrary, if  $\omega_i^y$  is decreased the difference from the reference tracking to the plant output will rise.

To stimulate the controller to use smaller increments on control signal the  $\omega_i^{\Delta u}$  must be increased. Therefore, the cost function to be minimized has the following constraints specified as:

$$\begin{aligned} y_{min}(i) - \varepsilon_k V_{min}^y(i) &\leq y(k+i|k) \\ &\leq y_{max}(i) + \varepsilon_k V_{max}^y(i), i = 1:P \end{aligned} \quad (9)$$

$$\begin{aligned} u_{min}(i) - \varepsilon_k V_{min}^u(i) &\leq u(k+i-1|k) \\ &\leq u_{max}(i) + \varepsilon_k V_{max}^u(i), i = 1:P \end{aligned} \quad (10)$$

where  $y_{min}$  and  $y_{max}$  represents the lower and upper limit of process future outputs respectively. The parameters  $V$  are dimensionless controller constants. With a very large  $V_{min}$  and  $V_{max}$  the constraints are easier to satisfy. The second equation in constraints (10), the  $u_{min}$  and the  $u_{max}$  are the lower and upper bounds for the control signal.

### B. Household

Building material properties dictate thermal response and consequently its energy consumption behavior. Thus, retention of warm/cold air in the house depends on thermal conductivity characteristics of the materials used on the floor, roof, windows and walls. As a whole, thermal performance is defined by the house geometry and the number of rooms.

In this study, three rooms are modeled, while only one is equipped with a temperature regulation system. Single room dynamic model takes into account the outside environment,  $T_{amb}$ , and the thermal characteristics of the room. The AC power unit is represented as  $Q_{ac,ht}$  thermal source, while the heat to be extracted is represented as  $Q_{in}$  thermal source. Thermal equations are derived from [14] as:

$$\frac{dT_w}{dt} = \frac{Q_s}{C_w} + \frac{T_{out}}{R_w C_w} + \frac{T_{in}}{R_w C_w} - \frac{2T_w}{R_w C_w} \quad (11)$$

$$\frac{dT_{in}}{dt} = \frac{(Q_{in} - Q_{ac,ht}) S(t)}{C_{in}} - \frac{T_{in}}{C_{in}} \left( \frac{1}{R_w} + \frac{1}{R_c} \right) - \frac{T_w}{R_w C_{in}} \quad (12)$$

### C. Water heater

Physical description model of the WH takes into account the mass of water ( $m$ ), specific heat of water ( $C_p$ ), characteristics of fiber glass ( $C_w$ ,  $UA$ ), gas or electric rated power ( $Q_{e,g}$ ), and the efficiency ( $\eta$ ).

Energy transit equation for WT has the following expression [15]:

$$\frac{dT_w}{dt} = \frac{m C_p}{C_w} T_{inlet} + \frac{UA}{C_w} T_{amb} - \frac{UA + m C_p}{C_w} T_w + Q_{e,g} \eta \quad (13)$$

### D. Air conditioner

The AC unit model is represented by an input-output power block that receives a certain amount of energy  $Q_{out}$  (cooling capacity in terms of BTU) to remove heat  $Q_{in}$  from the air inside the room. In the model, AC energy efficiency is equal to one.

## III. CASE STUDIES

### A. Testing Methodology

This section is dedicated to present the main results by adjusting different MPC weights and analyze its effects in order to minimize the energy consumption of the studied household appliances. For the calculation of energy cost prices charged for the residential market in Canada are used during a 24 hour period. The daily electricity fee is classified in three levels of prices according to different demand periods: off-hours, mid-hours and on-peak hours.

The case studies are comprised of three electrical loads, namely heating and cooling. For each one of them it is measured the energy, the energy cost and temperature variation plotted as function of weight selection relative to the manipulated variable and process output as part of the cost function.

*i) Household*

A small AC 8900 BTU unit is utilized to lower the temperature in a residence in a room. The AC operation is externally operated by the MPC system, which includes a temperature sensing device. MPC control specifications are a temperature reference of 23°C and with a +/- 1° tolerance band. This means that the control temperature interval is restricted between 22.5°C and 23.5°C. Data collection reports to the AC power consumption of the room, room temperature and outdoor temperature – depicted as a disturbance source of the system. An outdoor temperature profile is generated representing a typical summer day. Considerable thermal amplitude is introduced to function as a disturbance source in the temperature equation model of the room.

*ii) Water heater*

The WH device has a use pattern related to the residents’ daily hygiene since it functions as a hot water tank. Hence, it is expected that during the night the water is heated at a slower rate, while at peak-hour the temperature control system has to react quick enough to compensate the hot water output. A rated resistive element of 4.5kW is used with the purpose to heat the water. The storage capacity is 184 L in terms of tank net volume. MPC is initialized with a set point (SP) of 55 °C with a +/-1.5 °C band. WH outside air temperature is fixed at 23 °C.

*iii) Refrigerator*

Storing food at relatively low temperature is nowadays common in very household. The RF performs this task using a classic temperature control based on a thermostatic relay. The main disturbance for changing the temperature settings inside of the RF is due to the door opening. That is, the number of times and the time it is kept open increases the temperature inside of the equipment. For simulation purpose the equipment is modeled with a rated power of 0.23kW. The MPC system is configured to maintain the internal temperature between 3.9°C and 5.1°C. Disturbance events for MPC evaluation are generated at 10 pm for 1 hour and at 14 pm for an equal period. Fig. 1 shows the disturbances sources for the all the 3 cases.

*B. Simulation results*

*i) Air conditioner*

The Fig. 2 represents the energy profile with of the MPC operated AC. The different combination between weights leads only to limit values. On the other hand, since there are three different tariffs during the same day, the objective is to try to lower the energy cost during ON peak hours, as the Fig. 3 clearly shows that certain combination of weights benefits the home user by reducing the energy consumption in the studied 24 hours period. In Fig. 4 is shown that the objective of maintaining the temperature between the limits if fulfilled.

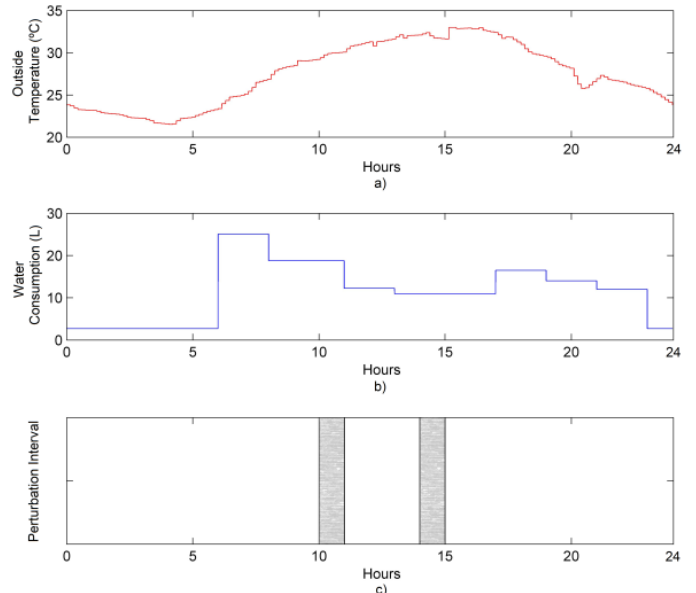


Fig. 1. Disturbances sources of a) outside temperature (AC), b) Water consumption (WH) and c) fridge open dor event intervals.

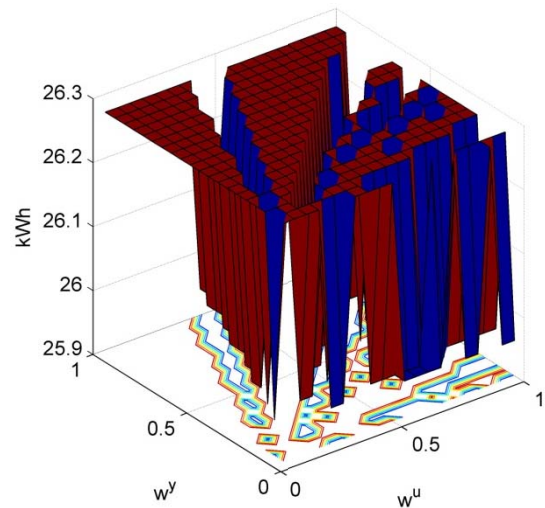


Fig. 2. AC: Energy output vs weights tuning.

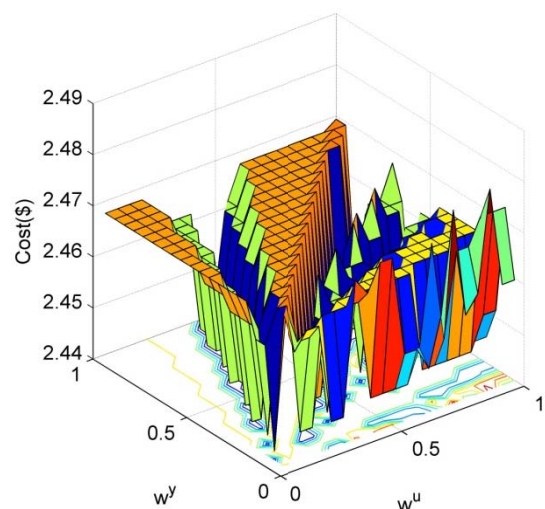


Fig. 3. AC: Energy cost vs weights tuning.

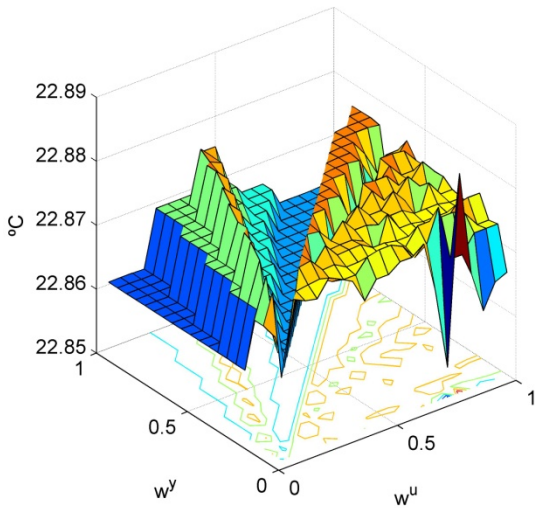


Fig. 4. AC: temperature vs weights tuning.

*ii) Water heater*

By observing Fig. 5 it can be seen that there are four levels of energy consumption. For the lower level of energy consumption the combinations of weights requires that the model output weight tuning has to be calibrated with a reduced value while there are no limits for the manipulated variable weight tuning. In case of Fig. 6 doesn't follow the energy profile since it reveals a peak in the cost of energy which is a result of the influence of the price tariff. Fig. 7 shows that specification of the regulation is satisfied.

*iii) Refrigerator*

Unlike the case studies verified previously, the simulation of Fig. 8 points out that the minimum value of energy is reached with the weight of the system output null when only two weights are admissible on the side of the manipulated value. In Fig. 9 is shown that the maximum value of the energy cost is coincident with maximum value of the energy output. In the previous cases the implementation of the MPC has shown that the specification limits are followed which does not happens in this case as can be seen in the Fig. 10.

It becomes evident that in this case aiming the minimum of energy consumption has a penalizing effect in maintaining the temperature in the wanted regulated intervals.

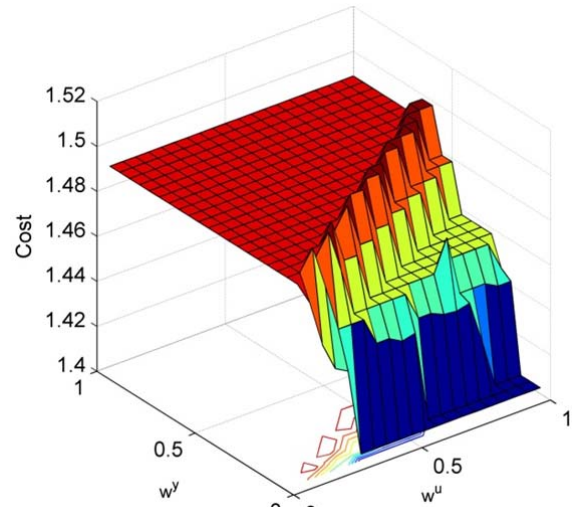


Fig. 6. WH: Energy cost vs weights tuning.

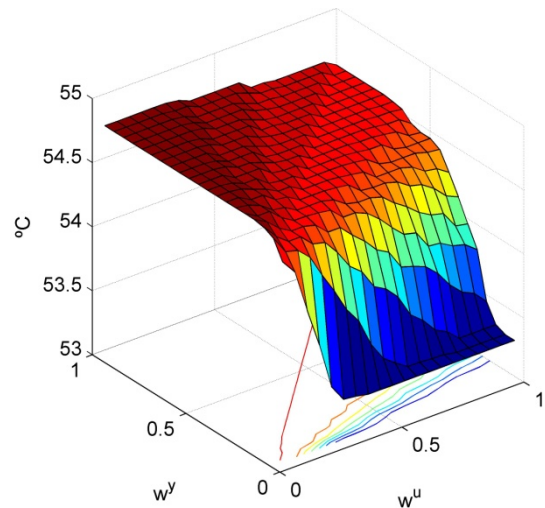


Fig. 7. WH: Water temperature vs weights tuning.

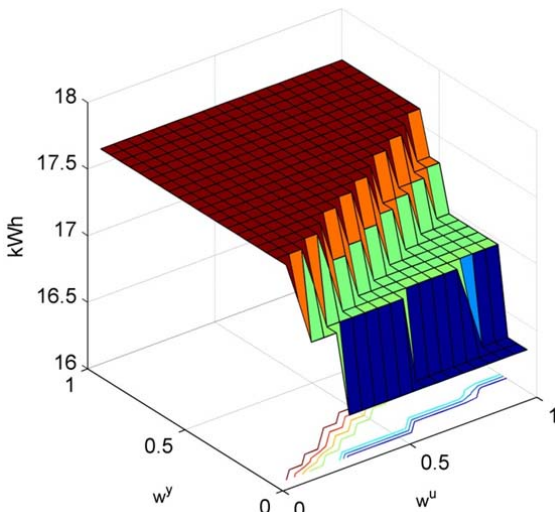


Fig. 5. WH: Energy output vs weights tuning.

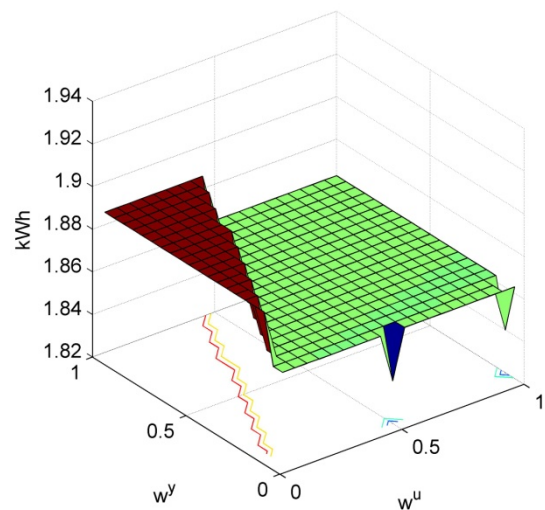


Fig. 8. Refrigerator: Energy output vs weights tuning.



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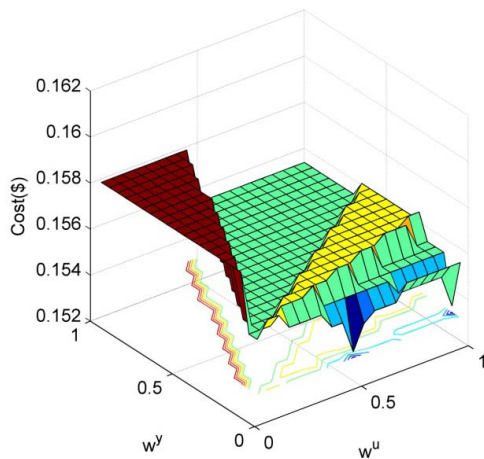


Fig. 9. Refrigerator: Energy cost vs weights tuning.

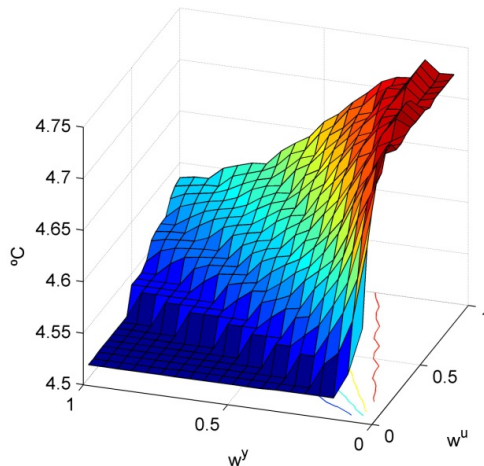


Fig. 10. Refrigerator: Inside temperature vs weights tuning.

## IV. CONCLUSION

This paper has focused on using MPC techniques as an alternative way of improving energy usage of residential households. Three domestic loads were simulated in order to observe the impact of adjusting MPC weights. The simulation outcomes have showed that each domestic appliance requires personalized weights tuning in order to reach the target to reduce at a minimum the energy consumption. On the other hand, it is proven that fact that having a multi-tariff system, the curve of the costs changes significantly when compared with the energy one. However, for the MPC weight tuning calculation it is sufficient to observe the energy curve. Nonetheless, in one of the cases the objective to minimize the energy consumption had a negative effect in maintaining the temperature in the required regulated intervals.