# Optimal Coordinated Wind-Photovoltaic Bidding in Electricity Markets

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***Abstract*—The high penetration of wind and photovoltaic power in electricity markets will represent a major challenge in the forthcoming years. The main problem of both technologies is the high uncertainty in their production and their dependence on environmental conditions. The coordination between wind and photovoltaic power aims to lower imbalances, reducing their associated penalties. This paper describes two strategies: i) separate wind and photovoltaic strategy and ii) single wind-photovoltaic strategy. The strategies proposed are solved through stochastic mixed integer linear programming. The expected profits are maximized and they are obtained by selling the energy in the day-ahead market. The imbalances are penalized in the balancing market as well. The model is tested for a week, 168 hours, and the data used come from the Spanish electricity market. The results of the case study are discussed, comparing both strategies. Following the discussion, the most important conclusions are presented.**

***Index Terms*—Day-ahead market, photovoltaic generation, single wind-photovoltaic offer, stochastic mixed integer linear programming, wind generation.**

## NOMENCLATURE

*Indexes*

| | |
|---|---|
| $n$ | Index referring to the number of photovoltaic panels. |
| $t$ | Index referring to a period [hour]. |
| $w$ | Index referring to a scenario. |

*Parameters*

| | |
|---|---|
| $c^{PV}$ | Photovoltaic marginal cost [€/MWh]. |
| $c^{W}$ | Wind marginal cost [€/MWh]. |
| $g_{t,w}^{PV}$ | Power produced by the photovoltaic park in period *t* and scenario *w* [MW]. |
| $g_{t,w}^{W}$ | Power produced by the wind farm in period *t* and scenario *w* [MW]. |
| $\lambda_{t,w}$ | Day-ahead market price in period *t* and scenario *w* [€/MWh]. |
| $\lambda_{t,w}^{-}$ | Negative imbalance market price in period *t* and scenario *w* [€/MWh]. |
| $\lambda_{t,w}^{+}$ | Positive imbalance market price in period *t* and scenario *w* [€/MWh]. |
| $P_{MAX}$ | Maximum installed power of the combination of photovoltaic park and wind farm [MW]. |
| $P_{MAX}^{PV}$ | Maximum power of the photovoltaic park [MW]. |
| $P_{MAX}^{W}$ | Maximum power of the wind farm [MW]. |
| $prob_w$ | Probability of each scenario *w*. |

*Continuous Variables*

| | |
|---|---|
| $b_t$ | Power offer in the day-ahead market associated to the wind farm and photovoltaic park in period *t* [MW]. |
| $b_t^{PV}$ | Power offer in the day-ahead market associated to the photovoltaic park in period *t* [MW]. |
| $b_t^{W}$ | Power offer in the day-ahead market associated to the wind farm in period *t* [MW]. |
| $\Delta_{t,w}$ | Imbalance between actual wind production, actual photovoltaic production and single offer in period *t* and scenario *w* [MW]. |
| $\Delta_{t,w}^{-}$ | Negative imbalance between actual wind production, actual photovoltaic production and single offer in period *t* and scenario *w* [MW]. |
| $\Delta_{t,w}^{+}$ | Positive imbalance between actual wind production, actual photovoltaic production and single offer in period *t* and scenario *w* [MW]. |
| $\Delta pv_{t,w}$ | Photovoltaic imbalance between actual photovoltaic production and photovoltaic offer in period *t* and scenario *w* [MW]. |
| $\Delta pv_{t,w}^{-}$ | Photovoltaic negative imbalance between actual photovoltaic production and photovoltaic offer in period *t* and scenario *w* [MW]. |
| $\Delta pv_{t,w}^{+}$ | Photovoltaic positive imbalance between actual photovoltaic production and photovoltaic offer in period *t* and scenario *w* [MW]. |
| $\Delta w_{t,w}$ | Wind imbalance between actual wind production and wind offer in period *t* and scenario *w* [MW]. |
| $\Delta w_{t,w}^{-}$ | Wind negative imbalance between actual wind production and wind offer in period *t* and scenario *w* [MW]. |
| $\Delta w_{t,w}^{+}$ | Wind positive imbalance between actual wind production and wind offer in period *t* and scenario *w* [MW]. |
| $PF$ | Energy sales profit in the electricity market [€]. |

$PF_W$ Wind energy sales profit in the day-ahead market [€].

$PF_{PV}$ Photovoltaic energy sales profit in the day-ahead market [€].

*Binary Variables*

$j_{t,w}$ 0/1 variable, which is equal to 1 if the imbalance in period $t$ is negative, otherwise it is 0 for a positive imbalance.

$j^W_{t,w}$ 0/1 variable, which is equal to 1 if the wind imbalance in period $t$ is negative, otherwise it is 0 for a positive imbalance.

$j^{PV}_{t,w}$ 0/1 variable, which is equal to 1 if the photovoltaic imbalance in period $t$ is negative, otherwise it is 0 for a positive imbalance.

## I. Introduction

Current policies concerning the penetration of renewable energy resources such as wind and photovoltaic energy are strongly based both on the Kyoto Protocol agreement and more recently on Fukushima incident in Japan. The trends of renewable energy penetration in electricity markets are shown in [1]. The incorporation of renewable energy is considered to be challenging due to the rising uncertainty in generation. Hence, this paper aims to model the offer that renewable generators should send to the day-ahead market. In European electricity markets there are various mechanisms to penalize participants, such as the balancing market [2].

Wind and photovoltaic energy are strongly characterized by uncertainty due to the weather conditions. For this reason, a decision-making tool can be useful for renewable generators to determine the optimal offers through the use of several forecasting techniques [3] and stochastic programming [4].

Wind energy offers are studied in [5]. Some works explore the combination of different technologies such as wind generation and storage with hydro-pumping capabilities in order to compensate the variation in production [6]. Normally, photovoltaic generation is associated with a storage system [7], especially for residential customers [8]. Photovoltaic production is focused on micro-grids under uncertainty [9].

Photovoltaic generation has not yet been explored in strategic terms as is the case of offering energy in electricity markets. This paper proposes two strategies for the integration of renewable energy in day-ahead electricity markets. The first approach develops wind and photovoltaic models as separate offers and the second approach presents the combination of both technologies using only a single offer.

The main contributions of this paper are:

- A new approach to reduce the volatility of both wind and photovoltaic sales profits is presented.
- Two types of new strategies for wind and photovoltaic technologies through stochastic mixed integer linear programming in the day-ahead market are presented.
- An analysis of the coordination between wind and photovoltaic power for selling their energy in the day-ahead market is shown.

This paper is organized as follows: Section II describes the problem and both strategies, Section III presents the mathematical formulation divided into two parts, Section IV defines the case study to test the model, Section V shows the results and Section VI summarizes the relevant conclusions of the paper.

## II. Problem description

Wind and photovoltaic production are characterized by their high generation uncertainty. New production management strategies can mitigate the uncertainty of the incomes derived from energy sales, even some strategies can increase the profits by reducing the volatility.

Hence, the combination of different technologies can compensate the imbalances of both technologies, leading to lower penalties. In this paper, two strategies are proposed: i) separate wind and photovoltaic offering strategy and ii) coordinated wind-photovoltaic offering strategy (single offering strategy).

The first strategy is portrayed in Fig. 1. This strategy presents two different optimizations, ensuring the best offer for each individual technology.

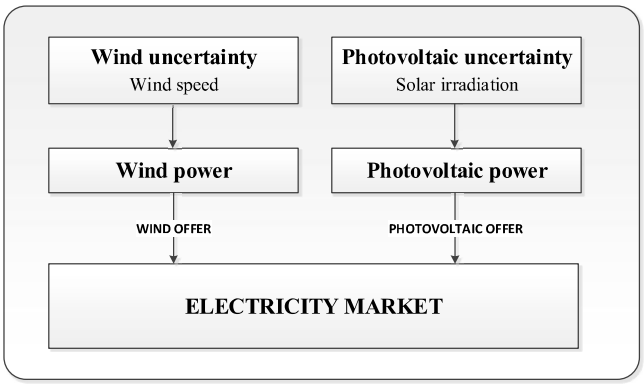


Fig. 1. Separate wind and photovoltaic offering strategy.

The coordinated offering strategy is depicted in Fig. 2. The production coordination of both technologies presents a single offer in the electricity market.

## III. Mathematical Formulation

The proposed optimization problem refers to the maximization of profit. The objective function and constraints are presented and analyzed in this section. The separate offering strategy and the single offering strategy are modeled using stochastic mixed integer linear programming.

The objective function maximizes the expected profits. The profits come from selling the energy in the day-ahead market.

In the next step, the models are presented separately with their respective objective functions and two blocks of constraints each.

The constraints are split into two blocks: the ones needed to evaluate the offer (offer constraints) and the ones needed to calculate the imbalances (imbalance constraints).

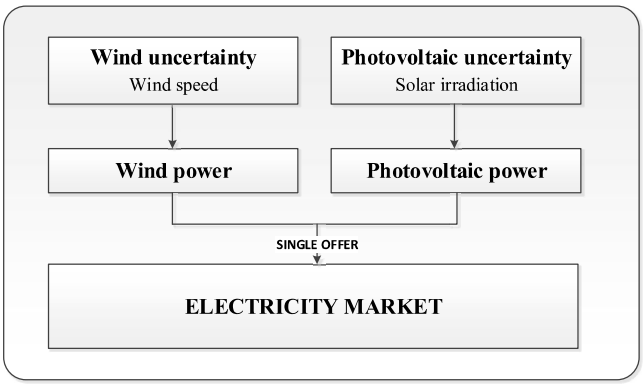


Fig. 2. Single wind-photovoltaic offering strategy.

### A. Separate wind and photovoltaic offering strategy

The objective function is divided into two parts: wind profits and photovoltaic profits. Also, the offer constraints and the imbalance constraints are shown and described in the forthcoming subsections.

*1) Objective function of the separate wind and photovoltaic offering strategy:* The objective function maximizes the profits of selling energy coming from wind generation ($PF_W$) and photovoltaic generation ($PF_{PV}$) in the day-ahead market for each period $t$. The differences between generation and offer (imbalance) are penalized in the balancing market.

$$max \;\; PF_W + PF_{PV}. \tag{1}$$

where

$$PF_W = \sum_w prob_w \sum_t \Big[\lambda_{t,w} \cdot b_t^W + \lambda_{t,w}^+ \cdot \Delta w_{t,w}^+ - \lambda_{t,w}^- \cdot \Delta w_{t,w}^- - c^W \cdot g_{t,w}^W\Big]; \tag{2}$$

$$PF_{PV} = \sum_w prob_w \sum_t \Big[\lambda_{t,w} \cdot b_t^{PV} + \lambda_{t,w}^+ \cdot \Delta pv_{t,w}^+ - \lambda_{t,w}^- \cdot \Delta pv_{t,w}^- - c^{PV} \cdot g_{t,w}^{PV}\Big]. \tag{3}$$

The revenues come from the energy sold in the day-ahead market and the positive imbalances. On the other hand, the costs consist of the negative imbalances and the marginal costs. The marginal costs are calculated according to [10].

*2) Offer constraints:* the generators have to offer their energy in the day-ahead market for each hour. The limits of both generators are between zero and the maximum power, as shown in (4) to (7).

$$b_t^W \geq 0; \tag{4}$$

$$b_t^W \leq P_{MAX}^W; \tag{5}$$

$$b_t^{PV} \geq 0; \tag{6}$$

$$b_t^{PV} \leq P_{MAX}^{PV}. \tag{7}$$

*3) Imbalance constraints:* Imbalances can be positive or negative. They are calculated as the difference between the generation and the offer in (8) and (14). An imbalance is positive when the generation is higher than the offer, and negative when the offer is higher than the generation. The constraints calculate the value of the wind and photovoltaic imbalances, which can be positive or negative as seen in (9) and (15), respectively. The limits of the positive imbalances and the negative imbalances are the maximum wind power and the maximum photovoltaic power as seen in (10), (11), (16) and (17). The lower limit of the imbalances is zero as seen in (12), (13), (18) and (19).

$$\Delta w_{t,w} = g_{t,w}^W - b_t^W; \tag{8}$$

$$\Delta w_{t,w} = \Delta w_{t,w}^+ - \Delta w_{t,w}^-; \tag{9}$$

$$\Delta w_{t,w}^- \leq P_{MAX}^W \cdot j_{t,w}^W; \tag{10}$$

$$\Delta w_{t,w}^+ \leq P_{MAX}^W \cdot (1 - j_{t,w}^W); \tag{11}$$

$$\Delta w_{t,w}^- \geq 0; \tag{12}$$

$$\Delta w_{t,w}^+ \geq 0; \tag{13}$$

$$\Delta pv_{t,w} = g_{t,w}^{PV} - b_t^{PV}; \tag{14}$$

$$\Delta pv_{t,w} = \Delta pv_{t,w}^+ - \Delta pv_{t,w}^-; \tag{15}$$

$$\Delta pv_{t,w}^- \leq P_{MAX}^{PV} \cdot j_{t,w}^{PV}; \tag{16}$$

$$\Delta pv_{t,w}^+ \leq P_{MAX}^{PV} \cdot (1 - j_{t,w}^{PV}); \tag{17}$$

$$\Delta pv_{t,w}^- \geq 0; \tag{18}$$

$$\Delta pv_{t,w}^+ \geq 0. \tag{19}$$

### B. Coordinated wind-photovoltaic offering strategy

The objective function and constraints of the coordinated strategy or single offer (SO) are presented in the next section.

*1) Objective function of the coordinated wind-photovoltaic offering strategy:* The profits gained from selling energy from wind and photovoltaic production in the day-ahead market are represented by $b_t$ as a single offer for each period $t$. Hence, the decision is made through $b_t$.

$$max \;\; PF; \tag{20}$$

where

$$PF = \sum_w prob_w \sum_t \Big[\lambda_{t,w} \cdot b_t + \lambda_{t,w}^+ \cdot \Delta_{t,w}^+ - \lambda_{t,w}^- \cdot \Delta_{t,w}^- - c^W \cdot g_{t,w}^W - c^{PV} \cdot g_{t,w}^{PV}\Big]. \tag{21}$$

The marginal costs are divided into two parts in this strategy: the photovoltaic marginal costs and the wind marginal costs. They are calculated according to [10].

*2) Offer constraints:* The value of the offer variable, $b_t$, is constrained between the sum of the maximum wind and photovoltaic power and zero, as expressed by (22) and (23). $P_{MAX}$ is the sum of $P_{MAX}^{W}$ and $P_{MAX}^{PV}$ as in (24).

$$b_t \geq 0; \tag{22}$$

$$b_t \leq P_{MAX}; \tag{23}$$

$$P_{MAX} = P_{MAX}^{W} + P_{MAX}^{PV}. \tag{24}$$

*3) Imbalance constraints:* The imbalances are calculated as the difference between generation and offer of both technologies (25).

$$\Delta_{t,w} = (g_{t,w}^{W} + g_{t,w}^{PV}) - b_t; \tag{25}$$

$$\Delta_{t,w} = \Delta_{t,w}^{+} - \Delta_{t,w}^{-}. \tag{26}$$

The upper limit of the imbalances is $P_{MAX}$, as in (27) and (28).

$$\Delta_{t,w}^{-} \leq P_{MAX} \cdot j_{t,w}; \tag{27}$$

$$\Delta_{t,w}^{+} \leq P_{MAX} \cdot (1 - j_{t,w}). \tag{28}$$

The lower limit of the imbalances is zero, as expressed by (29) and (30).

$$\Delta_{t,w}^{-} \geq 0; \tag{29}$$

$$\Delta_{t,w}^{+} \geq 0. \tag{30}$$

## IV. Case Study

The case study comprises a wind farm and a photovoltaic park. The generators are located in Navarre, Northern Spain. The simulations refer to the Spanish electricity market for a time frame of 168 hours.

The total wind capacity is 50 MW and the wind marginal cost is equal to €17/MWh [10]. The photovoltaic park has a total capacity of 50 MW and a marginal cost of €23.6/MWh [10].

### A. Conversion of wind speed and solar irradiation to energy

The conversion of wind speed is as in (31). The expression is defined as [11].

$$P(v) = 0.5 \cdot c_p(v) \cdot \rho \cdot A \cdot v^3. \tag{31}$$

where $A$ is the area swept by the wind turbine rotor, $\rho$ is the air density, and $c_p$ is the overall efficiency of the wind turbine as a function of wind speed.

The irradiation reflects the variability of the photovoltaic resource. For each period, the solar irradiation is determined through (32),where the total power output in MW, $P^{PV}$, is calculated as in [9].

$$P^{PV} = (\sum_{n=1}^{N} \eta_n^{PV} \cdot A_n^{PV} \cdot G)/10^6. \tag{32}$$

In (32) $\eta_n^{PV} = 0.143$ is the panel efficiency of each array, $A_n^{PV} = 1.6 \cdot 12$ is the panel area of each array $[m^2]$, $G$ is the solar irradiation from known data $[W/m^2]$ and $N = 18200$ is the total number of arrays present in the photovoltaic park. The photovoltaic generation is calculated for 168 hours.

### B. Scenarios

Uncertainty is introduced through parameters such as wind generation, photovoltaic generation, market prices, positive imbalance market prices and negative imbalance market prices. The scenarios span from July to September, 2014, based on the Spanish electricity market [12].

There are three scenario trees, two for the separate model and one for the coordinated model.

The separate model scenario tree is composed of two types of nodes, the first one comprises price and wind generation nodes, where the total number of scenarios, 12·12, is equal to 144. The second type of node comprises price and photovoltaic generation nodes, the total number of scenarios being 12·12, i.e., 144 scenarios.

On the other hand, the coordinated scenario tree is divided into three types of nodes, the first type comes from prices, the second from photovoltaic generation and the third from wind generation, so the total number of scenarios is 12·12·12, i.e., 1728 scenarios.

The scenarios for wind generation, photovoltaic generation, market prices, positive imbalance market prices and negative imbalance market prices are shown in Figs. 3-7.

The model is programmed in MATLAB [13] and GAMS [14] in a computer with a processor at 3.1 GHz and 256 GB of RAM. The CPU time is different for each model due to the number of scenarios. The CPU time for the separate offer is 50 seconds and for the coordinated model is 12 min. The CPU times are different due to the high number of scenarios of the coordinated model, but still acceptable for the time horizon of 168 hours.

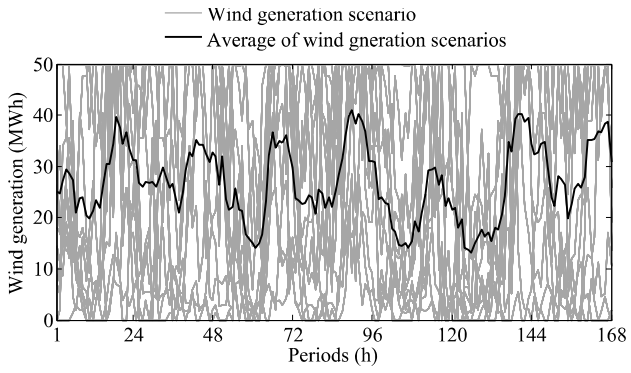


Fig. 3. Wind generation scenarios and the average of wind generation scenarios from July to September, 2014.

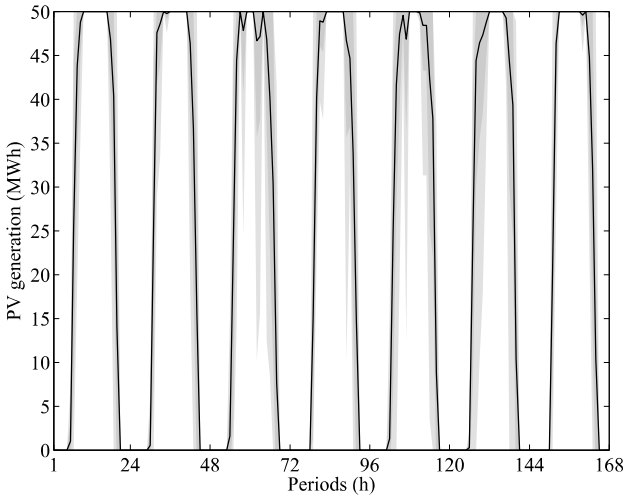


Fig. 4. The maximum, average+standard deviation, average, average-standard deviation and minimum of the photovoltaic generation scenarios from July to September, 2014.

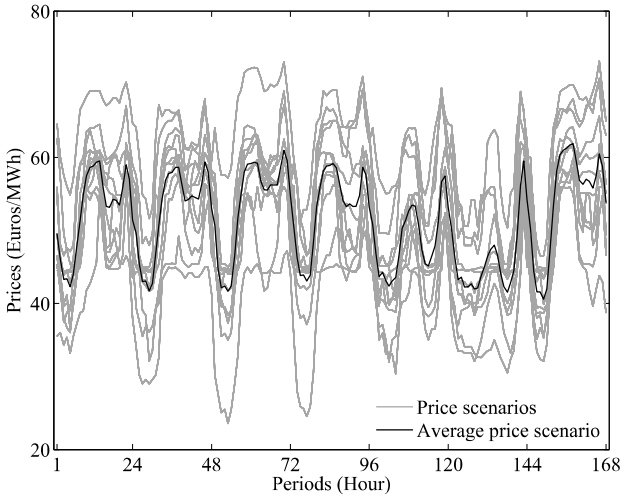


Fig. 5. Price scenarios and average price scenarios from July to September, 2014.

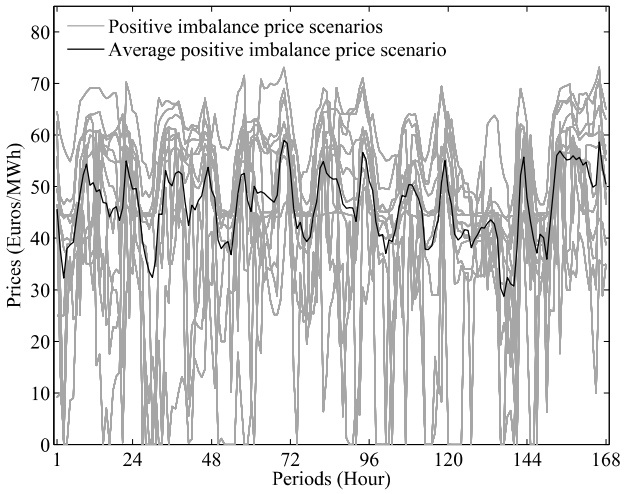


Fig. 6. Positive imbalance price scenarios and average positive imbalance price scenarios from July to September, 2014.

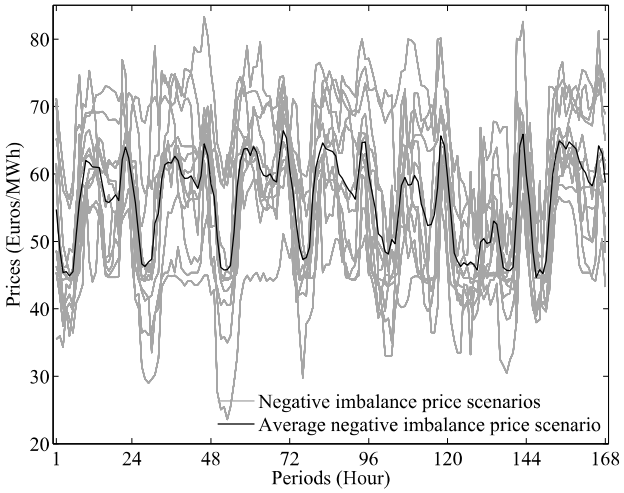


Fig. 7. Negative imbalance price scenarios and average negative imbalance price scenarios from July to September, 2014.

## V. Results

The most important variables, i.e. offers, imbalances and profits for each model and technology are presented in this section.

The offers introduced in Fig. 8 are obtained with respect to the scenarios depicted in Figs. 3-7. The coordinated strategy offers more energy than the separate offer in 41 hours, less energy in 11 hours, and the same energy in 116 hours. Furthermore, the coordinated offers follow the separate offers.

The more representative increases of the offers in the coordinated strategy are in hours with a high market price like hours 85 and 160.

On the other hand, the imbalances are portrayed in Fig. 9 for scenario 1. The imbalances of the coordinated strategy are lower than the ones in the separate strategy, shown in Fig. 9. The reduction of the imbalances in the coordinated strategy does not increase the profit considerably, but the standard deviation of the profits is reduced by 13.7%, as shown in Table I.

The imbalances of the separate strategy come from wind and photovoltaic generation, being evaluated with different variables for wind, $\Delta w_{t,w}$ and photovoltaic generation, $\Delta pv_{t,w}$. However, in the coordinated strategy, the offer is evaluated with one variable for both technologies $b_t$, and the imbalance only uses one variable, $\Delta_{t,w}$, for both technologies. Hence, the coordinated strategy can absorb more generation volatility.

The profits, standard deviations, offers and imbalances for each strategy are presented in Table I.

The coordination of the technologies reduces the standard deviation of the profits, reducing their risk or volatility. This reduction of the standard deviation by coordination is due to the reduction of the imbalances. Therefore, the increase in the offer and the lower imbalance in the coordinated strategy involves a slightly higher profit and a lower standard deviation of the profits.

## VI. Conclusions

Two stochastic mixed integer linear models have been described, whose offers are optimized to maximize the profit from selling wind and photovoltaic production in an electricity market. This new approach increased the profits and reduces their risk. Also, the imbalances are decreased to the advantage of the electric system.

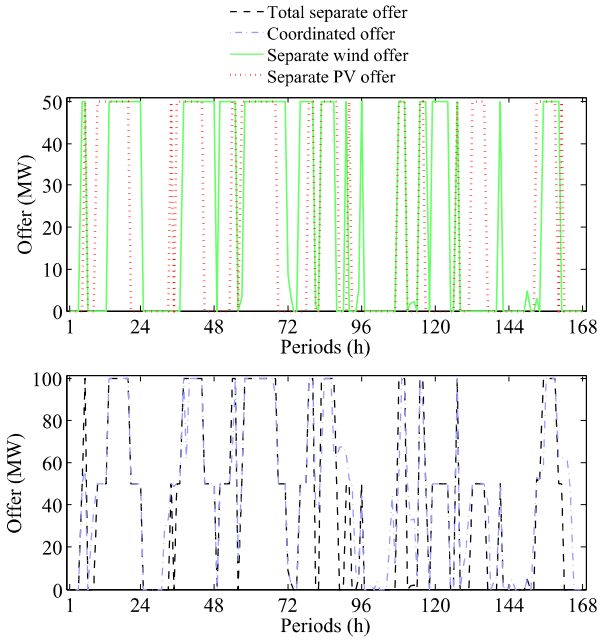


Fig. 8. Offers for separate offer, coordinated offer, wind offer and photovoltaic offer of separate offer per period.

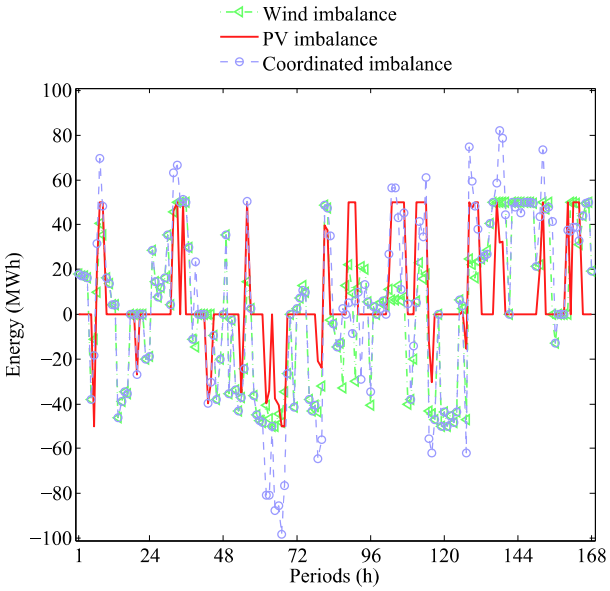


Fig. 9. Imbalances of the wind generator, the photovoltaic generator and the coordinated model in the scenario 1.

TABLE I
TOTAL PROFITS, STANDARD DEVIATION OF THE TOTAL PROFITS, AVERAGE POSITIVE IMBALANCES AND AVERAGE NEGATIVE IMBALANCES FOR BOTH STRATEGIES.

| | Wind power | PV power | Total W & PV | Coordinated |
|---|---|---|---|---|
| **Profits (€)** | 143064.7 | 113994.8 | 257059.5 | 257207.5 |
| **Standard deviation of the profits (€)** | 43808.6 | 7555.1 | 51363.7 | 44313.5 |
| **Total offer (MW)** | 3832.8 | 3200 | 7032,8 | 7758.1 |
| **Average positive imbalance (MW)** | 2437.3 | 1493.7 | 3931 | 2903.5 |
| **Average negative imbalance (MW)** | 1761.4 | 465.6 | 2227 | 1924.8 |

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