

Short-Term Hybrid Probabilistic Forecasting Model for Electricity Market Prices

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Abstract—With the integration of new power production technologies and the growing focus on dispersed production, there has been a paradigm change in the electricity sector, mostly under a renewable and sustainable way. Consequentially, challenges for profitability as well as correct management of the electricity sector have increased its complexity. The use of forecasting tools that allow a real and robust approach makes it possible to improve system operation and thus minimizing costs associated with the activities of the electric sector. Hence, the forecasting approaches have an essential role in all stages of the electricity markets. In this paper, a hybrid probabilistic forecasting model (HPFM) was developed for short-term electricity market prices (EMP), combining Wavelet Transform (WT), hybrid particle swarm optimization (DEEPSO), Adaptive Neuro-Fuzzy Inference System (ANFIS), together with Monte Carlo Simulation (MCS). The proposed HPFM was tested and validated with real data from the Spanish and Pennsylvania-New Jersey-Maryland (PJM) markets, considering the next week ahead. The model was validated by comparing the results with previously published results using other methods.

Keywords—Adaptive neuro-fuzzy inference system; Electricity market prices; Forecasting; Particle swarm optimization; Monte Carlo simulation.

I. INTRODUCTION

In competitive and liberalized markets, where renewable incorporation is prominent, the natural renewable stochasticity is completely echoed in the players' decisions, bringing additional challenges for a sustainable, profitable, and reliable operation of the electricity structure [1].

Moreover, the integration of microgeneration together with the natural evolution of renewable energy technologies leads to a paradigm shift of the electricity sector. This, in addition to other factors, make forecasting EMP tools needed more than demand series forecasting tools [2].

One way to increase the sector flexibility is by integrating innovative storage systems, where the main goal is to manage the unpredictable behavior of renewables. However, this is highly costly, the lifetime is limited, and in most of the cases prototypal systems are used [3]-[5]. The study of forecasting EMP has grown as one of the biggest research areas [6].

EMP forecasting is a critical and inevitable task for agents participating in all various activities of electricity markets.

This is especially the case with the advance of smart grids, aforementioned paradigm shift in electricity sector, and the necessary and unavoidable mitigation of human footprint impact [7]. As scientific literature shows, EMP forecasting models are categorized into several techniques [8] cordially divided in different time horizons:

- 1) Very-short-term (few seconds to few hours);
- 2) Short-term (few hours to few days); and
- 3) Long-term (few days to few months) [9].

From those techniques, in the case of hard computing, the most known models are related with auto-regressive integrated moving average (ARIMA), with or without pre-processing data [10], where a huge amount of physical data is needed, and an exact modelling of the structure is mandatory.

In contrast, soft computing models generally use an auto-learning procedure from historical information to recognize expected data with outlines present in the historical data. A wide range of models can be found, mostly related with neural network (NN) philosophy such as [11], [12] and hybrid models [13]-[16], where their goals are to take opportunity of the best features from the sets techniques that compose the forecasting model.

Nowadays, the efforts of the scientific community are focused on innovative probabilistic forecasting models, where the hybridization of different methods is common, but with the goal of more realistic and spread outputs [17]-[20]. To validate the accuracy and applicability of proposed forecasting models, the usage of similar historical datasets is necessary, not with the goal of tuning the model, but to prove its advantages among other proposed models.

For example, in [16] a hybrid forecasting model was presented and applied to forecast different time horizons (i.e., for the next day- and week-ahead EMP), considering different sets of historical data from two electricity markets commonly used to validate forecasting models.

In [7] a hybrid forecasting model was presented, combining WT, DEEPSO, and ANFIS methods to forecast the EMP series for the Spanish market (2002, 2006), and PJM market (2006), whereas different forecasting windows (i.e., between 24 hours and 168 hours ahead) with a 1-hour time-step. The model was validated by comparing with previous approaches considering the same real and historical data.

In [12] a hybrid forecasting approach was presented considering a pre-processing technique combining PSO, and fuzzy neural networks techniques to forecast and classify the EMP of Spanish electricity market. In the same trend of research, in [1], a forecasting model was proposed composed by the support vector network with an ANFIS network in order to analyze the information of the Nordpool power market in Denmark.

Considering the widespread state-of-the-art, in this work a probabilistic hybrid forecasting model is elaborated and explored for short-term EMP, combining WT as pre-processing data technique with DEEPSO with the goal to reduce the overall forecasting error by tuning the ANFIS requirements. Afterwards, the resulting forecasted data is analyzed considering an MCS; the proposed model is hereafter referred to as HPFM. The proposed HPFM was used to forecast the Spanish and PJM EMP, without exogenous data, considering the next week with time-step of 1 hour. The results were compared with other proposed models already published considering the same historical data.

The manuscript is ordered by the following sections: Section II describes the HPFM model. Section III shows the most commonly used forecasting validation tools. Section IV describes the cases studies and results. Finally, in Section V, the conclusions are provided.

II. PROPOSED HYBRID PROBABILISTIC FORECASTING MODEL

The proposed HPFM employs WT to capture some features from the random behaviour of EMP. DEEPSO, due to the differential process and hybrid features, brings better capabilities to the ANFIS structure in order to reduce the forecast error through tuning of ANFIS membership functions. The resulting data is then analyzed through MCS model where the goal is to have the capability of knowing the forecasted values range without increasing the forecast error.

A. Wavelet Transform

In current forecasting models, for the analysis of time series such as EMP and renewable power behavior, WT has been widely used because it can detect patterns and trends without losing the original information. Both above mentioned time series usually have intermittency, volatility and peak trends that are challenging to forecast. In this sense WT can be considered as a tool capable of isolating these trends from the non-stationary time series [17].

Moreover, WT are successful in power quality and transient analysis, modeling of short-term energy system disturbances, and other signal analyses in continuous or discrete domain [16].

Mathematically, data processing at several scales or resolutions is done by condensing or extending a mother function, allowing the illustration of the time series in period and occurrence domains [17]. Discrete WT (DWT), which has a reduced computational effort due to the scaling and translation process that can be done by a set of scales and positions, can be expressed as:

$$DWT_x(m, n) = 2^{-(m/2)} \times \sum_{t=0}^{T-1} x(t) \times \psi\left(\frac{t - n \times 2^m}{2^m}\right) \quad (1)$$

where T is the signal dimension, a represents a scale parameter defined by integer variable m , b represents the translation parameter dependent on integer variable n , and t represents the index of T .

DWT computation is made for the subset of scales and position chosen (i.e. considering filters called approximations or details), highlighting the information hidden by the signal $x(t)$ [8]. The multi-resolution used in this work has two signal analysis stages: decomposition and reconstruction, done by a set of filtering pairs.

B. Hybrid Particle Swarm Optimization (DEEPSO)

DEEPSO is an effective fusion mixture of evolutionary PSO with differential features. In this algorithm, the weight parameters w_{ik}^* have auto-adaptive properties which, when joined with the evolutionary process, result in several auto-adaptive indicators which produce a new result for an existing element i of the swarm in addition with a different proportion between two other experienced points of the swarm that is under evaluation [7]. In this way, the formulation is summarized below [21]:

1) *Particle's New Best Position:*

$$X_i(k) = X_i(k-1) + V_i(k); \quad i = 1, 2, \dots, N \quad (2)$$

2) *Particle's Speed Update:*

$$V_i(k) = w_{i0}^* \times V_i(k-1) + w_{i1}^* \times (X_{r1}^i(k-1) - X_{r2}^i(k-1)) + Pw_{i2}^* \times (G_{best,i}^* - X_i(k-1)) \quad (3)$$

3) *Particle's Mutation Parameters Weights:*

$$w_{ik}^* = w_{ik} + \tau N(0,1) \quad (4)$$

4) *Current Best Position with Normal Distribution:*

$$G_{best,i}^* = G_{best,i} \times (1 + w_g N(0,1)) \quad (5)$$

5) *Differential Set of Subsequent Pair of Particles Tested:*

$$f(X_{r1}^i(k-1)) < f(X_{r2}^i(k-1)) \quad (6)$$

C. Adaptive Neuro-Fuzzy Inference System

ANFIS is by nature a hybrid model combining the best features of neural networks structures and fuzzy rules with inference features. As the extensive review of the state-of-the-art demonstrates, ANFIS is capable to work with considerable length of data using low computational requirements.

To this end, ANFIS model is composed of 5 layers: fuzzification process, firing strength rules, normalization, defuzzification and output; which are interconnected with the different inputs and resulting in one output. The forecast results in a Takagi-Sugeno structure [16].

D. Monte Carlo Simulation

MCS is a powerful instrument for analysis and has been in use in engineering for a long time. MCS are typically used to model an intrinsic variable of a system in which analytical formula cannot be used as a complex solution [22].

An example of MCS application can be found in [23] where the effects of renewable integration with the interruptions' probability were studied; where not only the uncertainty of components failure was analyzed but also variability of renewable integration was considered.

In this work, MCS was employed to associate and analyze, with the final result of the forecast, a range of values where the forecast can be inserted. With this, it is possible to have a probabilistic forecast result for further realistic analyses and decision making. To this end, in this proposed model an MCS with variable control was implemented, whose concept is described as computing the analysis results around the values where the possible set of results do not diverge from the expected real values [24].

By considering an input data (the historical and forecasted data) as $X = (x_1, \dots, x_n)$ the set is experimented according to its distributions. In the next step, the results of the output variable Y are computed through the performance function $Y = g(X)$ containing input variables. Afterwards, output variable samples Y are generated for statistical studies, where the features of the output variable Y are estimated. The basic structure of this model is shown in Figure 1.

Supposing that a set of N samples of random variables are generated, it means that all generated samples of the random variables constitute a set of inputs composed by, $x_i = (x_{i1}, \dots, x_{iN}), i \in N$, for the model $Y = g(X)$, i.e. [24]:

$$y_i = g(x_i) \quad , \quad i = 1, 2, \dots, N \quad (7)$$

After obtaining the output samples, the statistical study can be done in order to estimate output features Y , i.e., the mean, variance, reliability, probability of failure, the probability density function, and the cumulative density function. The meanings associated with these features are represented in the following equations; The mean:

$$\bar{Y} = \frac{1}{N} \times \sum_{i=1}^N y_i \quad (8)$$

The variance:

$$\sigma_Y^2 = \frac{1}{N-1} \times \sum_{i=1}^N (y_i - \bar{Y})^2 \quad (9)$$

The probability of failure expression in case of $g \leq 0$:

$$p_f = P\{g \leq 0\} \cong p_f = \int_{-\infty}^{+\infty} I(x) f_x(x) dx \quad (10)$$

where:

$$I(x) = \begin{cases} 1, & \text{if } g(x) \leq 0 \\ 0, & \text{if } g(x) > 0 \end{cases} \quad (11)$$

As the integral of the previous equation is only the average value of $I(x)$, the probability of failure is rewritten as:

$$p_f = \bar{Y} = \frac{1}{N} \times \sum_{i=1}^N I(x_i) = \frac{N_f}{N} \quad (12)$$

where N_f is the total of samples that have the performance function lower or equal to 0. After this step, the cumulative density function is:

$$F_y(y) = P(g \leq y) = \frac{1}{N} \times \sum_{i=1}^N I'(y_i) \quad (13)$$

where the indicator function I' is:

$$I'(x) = \begin{cases} 1, & \text{if } g(x) \leq y \\ 0, & \text{if } g(x) > y \end{cases} \quad (14)$$

Finally, the probability density function can be obtained from the numerical differentiation of CDF.

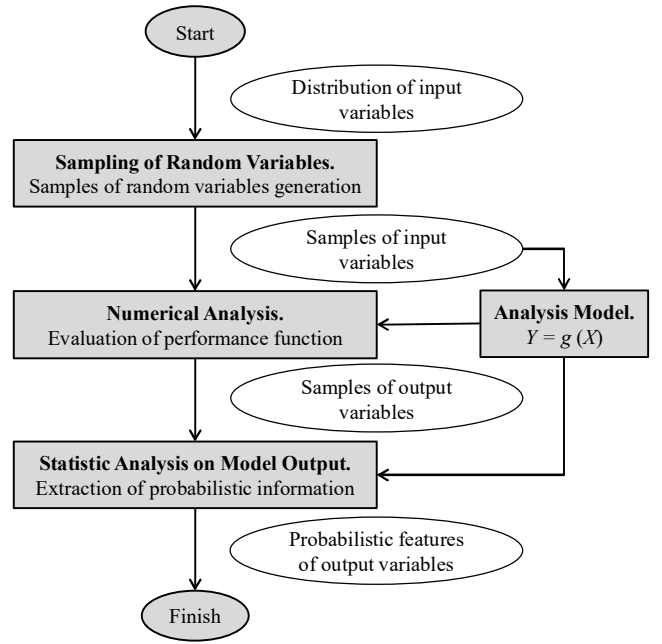


Fig. 1. MCS flowchart structure.

E. Probabilistic Hybrid Forecasting Model

The flowchart and main concept of PHFM is illustrated in Figure 2. In brief, the algorithm can be addressed taking in consideration the next steps:

Step 1: Start the PHFM model with a historical data of EMP taking as window frames the forecasting time-frame (168 hours for each set chosen);

Step 2: Select the historical data that will be decomposed by the WT;

Step 3: Select the parameters of the DEEPSO (Table I);

Step 4: Select the set of weeks that will be used in DEEPSO to obtain the necessary features to tuning and increase the performance of ANFIS model;

Step 5: Select the parameters of the ANFIS (Table I);

Step 6: Select the inputs of each iteration of the ANFIS method;

Step 7: Calculate the forecasting errors with the different error measurements criterions to authenticate the advances proposed PHFM model.

Step 7.1: If the criterion error goal is not achieved, start again the Step 7.

Step 7.2: If the criterion error goal is not found in step 7, jump to step 4 in order to find another set of solution.

Step 7.3: if the best forecasting results is found, or the number the iteration is reached, saved the latest best record and go to step 8.

Step 8: Use the inverse of WT transform to include the data previously filtered in the forecasted output.

Step 9: Obtain the analysis result using the MCS; print the forecasting results and finish.

III. FORECASTING VALIDATION

In order to analyze the forecasting EMP results obtained by the proposed PHFM model/approach with other available and authenticated short-term models/approaches, under the same input historic EMP sets, the Mean Absolute Percentage Error (MAPE) measure is frequently used.

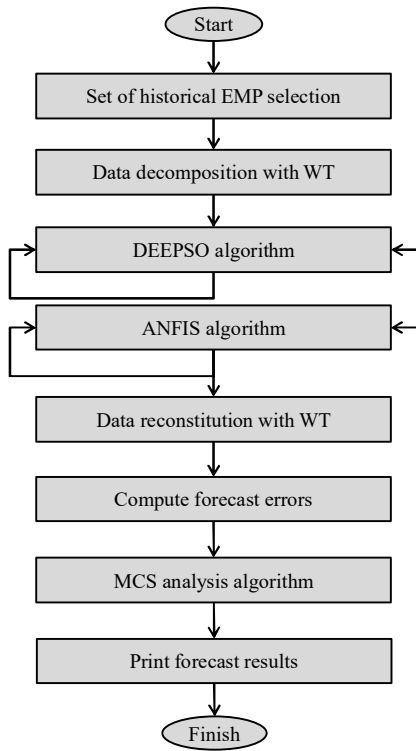


Fig. 2. HPFM flowchart structure.

TABLE I. DEEPSO AND ANFIS CONSIDERATIONS

| | Parameters | Sort or Magnitude |
|--------|------------------------------------|------------------------|
| WT | WT direction | "row" |
| | (Re)Decomposition level | 3 |
| | WT Mother function | "Db3", "Db4" |
| | Analysis noise tool | "sqtwolog", "minimaxi" |
| | Rescaling thresholds | "one", "sln", "mln" |
| DEEPSO | Sharing information probability | 0.1 |
| | Early inertia weight | 0.01-0.9 |
| | Ending inertia weight | 0.01-0.1 |
| | Starting swarm cognitive weights | 1-4 |
| | Starting swarm spreading process | 1-4 |
| | Starting spreading acceleration | 1-4 |
| | Population size | 168 |
| | Minimum point of new location | Set of Min. inputs |
| | Maximum point of new location | Set of Max. inputs |
| | Cognitive parameter | 0.1 |
| ANFIS | Iterations per simulation | 50-1000 |
| | Membership rules | 2-15 |
| | Number of iteration per simulation | 2-50 |
| | Membership function bell | "pimf", "trimf" |

The MAPE criterion (%) is generally expressed as [11]:

$$MAPE = \frac{100}{N} \times \sum_{n=1}^N \frac{|\hat{p}_n - p_n|}{\bar{p}} \quad (15)$$

$$\bar{p} = \frac{1}{N} \times \sum_{i=1}^N p_n \quad (16)$$

in which \hat{p}_n is the PHFM output at time n , p_n is the EMP data at time n , \bar{p} is the EMP average result for the forecasting data, and N is the dimension number of input data.

From the similar definition resulted from MAPE, the uncertainty of the PHFM is computed considering the (weekly) error variance:

$$\sigma_{e,n}^2 = \frac{1}{N} \times \sum_{n=1}^N \left(\frac{|\hat{p}_n - p_n|}{\bar{p}} - e_n \right)^2 \quad (17)$$

$$e_n = \frac{1}{N} \times \sum_{i=1}^N \frac{|\hat{p}_n - p_n|}{\bar{p}} \quad (18)$$

IV. CASE STUDIES AND RESULTS

The PHFM under analysis provides forecast results for the next week with a frame of 1 hour, considering 6 weeks of historical data. To this end, the proposed PHFM used the well-known real data from the Spanish EMP of year 2002 and for the PJM EMP for winter season of 2006. More details are available in [16].

Moreover, in coherence with published and validated studies, no exogenous data were taken into account, for the fairness and clear comparison. Figures 3-7 provide the numerical results for the different periods' weeks of the year (spring, summer, fall and winter) of the Spanish EMP, and for the week of 22-28 February of the PJM market.

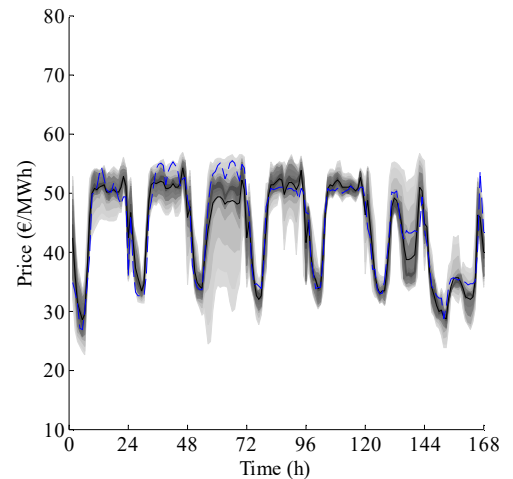


Fig. 3. EMP results from Spring week 2002 for the Spanish market: black dense line represents the calculated EMP results; blue dashed line represents the real EMP.

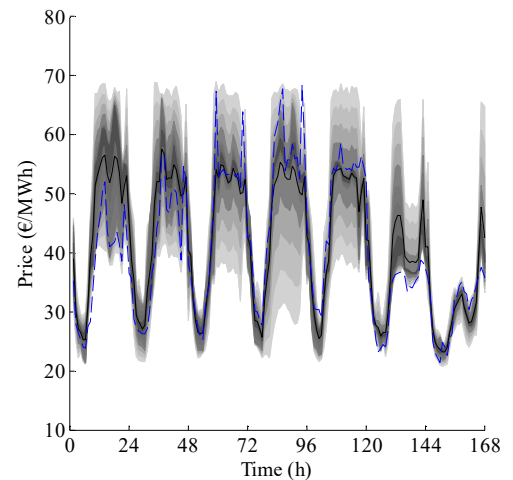


Fig. 4. EMP results from Summer week 2002 for the Spanish market: black dense line represents the calculated EMP results; blue dashed line represents the real EMP.

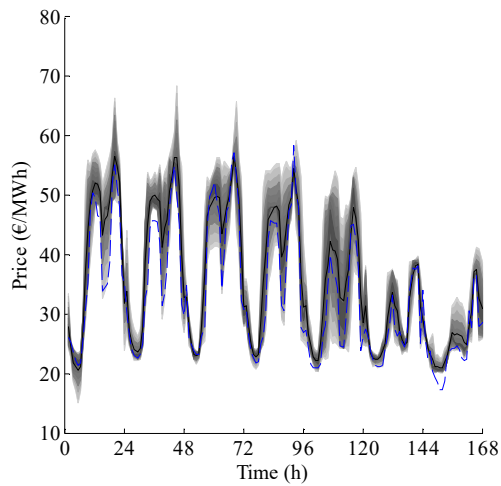


Fig. 5. EMP results from Fall week 2002 for the Spanish market: black dense line represents the calculated EMP results; blue dashed line represents the real EMP.

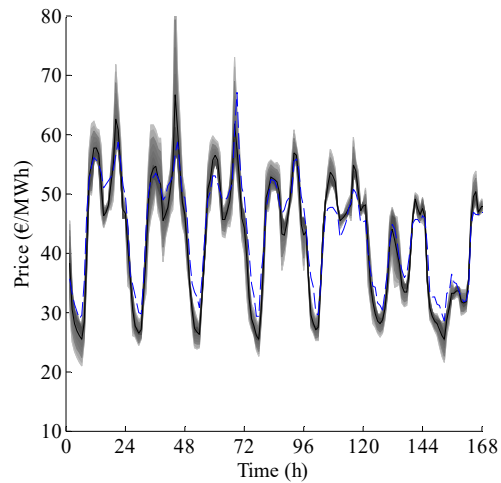


Fig. 6. EMP results from Winter week 2002 for the Spanish market: black dense line represents the calculated EMP results; blue dashed line represents the real EMP.

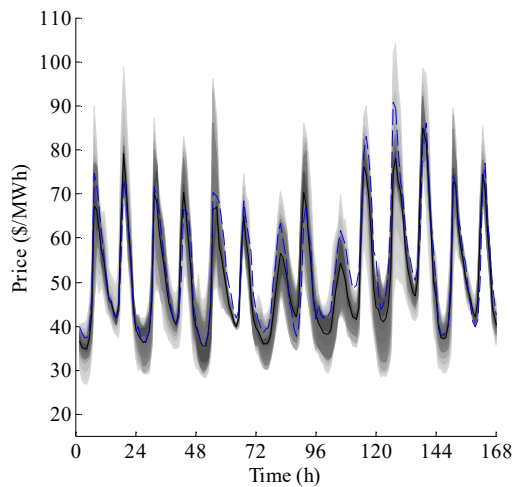


Fig. 7. EMP results from Winter week 2002 for the PJM market: black dense line represents the calculated EMP results; blue dashed line represents the real EMP.

From the numerical results obtained it is possible to observe how PHFM model addressed the uncertainty of the different EMP and different seasons under analysis. Moreover, the precision of the forecasted values throughout the week under analysis is observed. Also, from the MCS analysis it is observed how the forecasting values may fluctuate due to the related uncertainty and stochasticity incorporated in the historical data.

Tables II - IV, present the evaluation between PHFM with other previous hybrid models with intelligence features, in the scientific community, regarding MAPE criterion, and weakly error variance criterion, for the Spanish and PJM markets, in each season.

From the errors outcomes obtained, PHFM generally outperform, in most of the situations under analysis, the hybrid models under analysis and comparison. In this sense, and with the introduction of exogenous data in a future analysis, it can be expected that the forecasting errors considering the same cases studies may decrease, increasing the robustness and expected application in real-life scenarios.

The proposed PHFM was developed on a common PC equipped with i3-2310 CPU with 2.10GHz speed, 4GB RAM, using MATLAB 2016b platform. The average computation time to obtain a single forecasting result is around 1 minute.

TABLE II. MAPE OUTCOMES FOR SPANISH EMP (%)

| [7], [11], [16] | Spring | Summer | Fall | Winter | Average |
|---------------------|--------|--------|-------|--------|---------|
| NN (2007) | 5.36 | 11.40 | 13.65 | 5.23 | 8.91 |
| HIS (2009) | 6.06 | 7.07 | 7.47 | 7.30 | 6.97 |
| MICNN (2012) | 4.28 | 6.47 | 5.27 | 4.51 | 5.13 |
| EPA (2011) | 4.10 | 6.39 | 6.40 | 3.59 | 5.12 |
| HPM (2016) | 3.70 | 6.16 | 6.28 | 3.55 | 4.92 |
| HEA (2014) | 3.33 | 5.38 | 4.97 | 4.29 | 4.18 |
| PHFM (2018) | 4.13 | 5.21 | 4.77 | 4.48 | 4.65 |

TABLE III. WEEKLY ERROR VARIANCE OUTCOMES FOR SPANISH EMP

| [7], [11], [16] | Spring | Summer | Fall | Winter | Average |
|---------------------|--------|--------|--------|--------|---------|
| NN (2007) | 0.0018 | 0.0109 | 0.0136 | 0.0017 | 0.0070 |
| HIS (2009) | 0.0049 | 0.0029 | 0.0031 | 0.0034 | 0.0036 |
| MICNN (2012) | 0.0014 | 0.0033 | 0.0022 | 0.0014 | 0.0021 |
| EPA (2011) | 0.0016 | 0.0048 | 0.0032 | 0.0012 | 0.0027 |
| HPM (2016) | 0.0016 | 0.0037 | 0.0032 | 0.0008 | 0.0019 |
| HEA (2014) | 0.0011 | 0.0026 | 0.0014 | 0.0008 | 0.0015 |
| PHFM (2018) | 0.0016 | 0.0021 | 0.0010 | 0.0011 | 0.0014 |

TABLE IV. MAPE AND WEEKLY ERROR VARIANCE OUTCOMES FOR PJM EMP

| [7], [11], [16] | MAPE (%) | Variance |
|--------------------|----------|----------|
| HIS (2009) | 7.30 | 0.0031 |
| EPA (2011) | 6.40 | 0.0032 |
| HEA (2014) | 3.08 | 0.0017 |
| PHFM (2018) | 5.88 | 0.0026 |

V. CONCLUSION

In this work, a HPFM approach was proposed for short-term EMP forecasting and performed on two real markets' historical data previously used in other validated published proposals for a fair and clear comparison. The HPFM outcomes represent a combination of WT (which provides the analysis of the data without losing its essential features), DEEPSO (which due to its hybrid structure provides augmented features of the ANFIS by modifying the inputs and membership functions), and MCS analysis (which provides a probabilistic outcome of how the forecasted values may be spread along the time horizon). The application of the proposed HPFM was shown to be proficient, aiding in the reduction of uncertainty. The results obtained from both cases studies revealed the maturity of the HPFM, through the observation of the outcomes from MAPE and weekly error variance criterions.

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