This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3078655, IEEE Transactions on Smart Grid



# Exploiting the Potentials of HVAC Systems in Transactive Energy Markets

Fargol Nematkhah, *Member, IEEE*, Shahab Bahrami, *Member, IEEE*, Farrokh Aminifar, *Senior Member, IEEE*, and João P. S. Catalão, *Senior Member, IEEE*

***Abstract*—Transactive energy (TE) is a viable framework to tackle the load-generation mismatch in energy systems with high penetration of renewable energy resources (RERs). In this paper, we propose a TE framework for prosumers with heating, ventilation, and air conditioning (HVAC) systems to address real-time power shortage in a residential microgrid. Our framework consists of two phases. First, to mitigate load-generation mismatch, we develop an *online* appliance scheduling method to determine the optimal operation schedule of each prosumer's appliances. In particular, we apply receding horizon optimization (RHO) to tackle the load and renewable generation uncertainties and to better match the real-time power consumption of the appliances with the priorly-purchased power from the day-ahead market. Second, in case that there still exists power shortage at the microgrid level, a TE market based on pay-as-market clearing price (MCP) is proposed among prosumers to reduce the power consumption of their HVAC systems. We capture the competition among the participating prosumers as a non-cooperative game and develop an algorithm to achieve the Nash equilibrium, while considering prosumers' willingness to participate in the TE market. Extensive simulations are performed to demonstrate the efficiency of our proposed TE framework.**

***Keywords*: Transactive energy, HVAC system, appliances scheduling, energy market, game theory.**

## I. Introduction

Proliferation of renewable energy resources (RERs) in distribution grids has shifted the concept of consumer toward prosumer, as a user with electricity generation capability. The stochastic nature of RERs, however, has exposed system operators to new challenges in balancing supply and demand. A prosumer generally prefers to match his demand with the uncertain generation of his RER. Moreover, prosumers utilize distributed loads based on self-interest. Hence, it may not be feasible to maintain the supply-demand balance in a centralized manner. Transactive Energy (TE) has been introduced as a viable solution to address power mismatch at the retail level by aligning prosumer behavior with the grid conditions. Gridwise Architecture Council defines TE as: "a set of economic and control mechanisms that allow the dynamic balance of supply and demand across the entire electrical infrastructure using value as a key operational parameter" [1]. Based on this definition, TE primarily aims to provide power system operators with a dynamic supply-demand balance where installation of costly fast-ramping units is not required to offset RERs' generation changes.

F. Nematkhah and F. Aminifar are with the Department of Electrical and Computer Engineering, The University of Tehran, Tehran, Iran. S. Bahrami is with the Department of Electrical and Computer Engineering, The University of British Columbia, Vancouver, Canada. J. P. S. Catalão is with the Faculty of Engineering of the University of Porto and INESC TEC, 4200-465 Porto, Portugal. email: {fargol.nematkhah, faminifar}@ut.ac.ir, bahramis@ece.ubc.ca, catalao@ubi.pt.

There exist major challenges in exploiting TE frameworks to balance supply and demand. First, prosumers have uncertainty in some parameters, such as their appliance specifications, RER generation, electricity prices, and yet, they need to satisfy inter-temporal operational constraints of their appliances. Hence, real-time load scheduling has to be deployed. Second, prosumers in the TE market should decide on their price bids based on the discomfort arising from changes in their energy use. Thus, the market participants should be able to evaluate their discomfort in terms of monetary values. Third, the decisions of prosumers in the TE market are coupled due to the joint impact of their bids on the market clearing price (MCP). Thus, a prosumer should take into account the actions of others in his own decision-making process.

There have been some efforts in the literature to tackle the issue of generation-load mismatch, and we divide the related works into three main threads. The first line of research is concerned with scheduling the flexible loads to tackle power mismatches by applying various methods such as heuristic optimization (e.g., particle swarm optimization), robust optimization, fuzzy decision-making, and model-predictive control [2]–[5]. Also recently, model-free approaches, particularly learning techniques, have gained momentum since they do not require information on the stochastic process behind uncertain variables [6]–[8]. However, the performance of the aforementioned methods highly depends on the efficiency of the pricing mechanisms to reflect power mismatches on the electricity price and the price-responsiveness of the users.

The second line of research relates to the market-based energy trading among entities (e.g., microgrids) to maintain the generation-load balance. Behavior of market participants has been studied through a wide range of methods including decomposition techniques (e.g., alternating direction method of multipliers (ADMM) and dual decomposition), game theory, and prospect theory [9]–[13]. Despite the direct and voluntary contribution of entities, the potentials of load scheduling for mitigating the power mismatch tend to remain unemployed.

The third line of research deploys both load scheduling and energy trading to fully exploit flexible loads in managing the supply-demand balance. In [14], a framework was proposed to model both the load scheduling and profit sharing problems of interconnected microgrids using Nash bargaining. In [15], the real-time energy trading among entities was described as a bi-level optimization problem which included the optimal generation/consumption schedule of entities. In [16], the optimal operation schedule of users was primarily determined through approximate dynamic programming and the resulting real-time mismatch was further managed by the energy sharing of users.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3078655, IEEE Transactions on Smart Grid



Despite proposing holistic frameworks, the aforementioned studies did not evaluate the potentials of thermally flexible loads including heating, ventilation, and air conditioning (HVAC) systems in balancing generation-load mismatches. In [17]–[22], HVAC systems are examined in both centralized and distributed demand response schemes. While these studies include discomfort cost in scheduling HVAC systems using various methods such as machine learning techniques, they do not yield an estimation of discomfort to determine users' willingness toward participating in such schemes.

In this paper, we focus on addressing the real-time power shortage by scheduling prosumers' loads and further by exploiting the flexibility of HVAC systems in a market-based environment. We propose a TE framework for a microgrid with residential prosumers. In our framework, each prosumer primarily schedules its appliances in an online manner. Subsequently, if the microgrid still faces power shortage in real-time, a TE market is launched among HVAC systems to offset the power mismatch. The RERs can partially supply the demand of prosumers. Hence, in our framework, prosumers can purchase an approximate amount of their future net load from the day-ahead electricity market. The main challenge that we address in this paper is to propose a pay-as-MCP market for participation of HVAC systems and to capture their interplay as a non-cooperative game, considering the willingness of prosumers for participation of their HVAC systems in such a market. The main contributions of this paper are as follows:

- *HVAC Scheduling Method*: We model the thermal and electrical behavior of HVAC systems and propose a price-based control method for scheduling these systems, considering prosumer's thermal preferences. This method generates a setpoint-price characteristic, which enables us to evaluate the discomfort arising from changes in power consumption of HVAC systems and thereby, the prosumer's willingness to participate in the TE market.
- *Online Appliance Scheduling*: We model and further schedule prosumer's appliances using receding horizon optimization (RHO), which enables us to address the uncertainties in the appliance scheduling problem. The proposed online method preserves prosumer's privacy and allows for decentralized decision-making.
- *Competition Among HVAC Systems*: We propose a pay-as-MCP market for participation of HVACs to mitigate real-time power shortage. The distribution network operator (DNO) takes part in the market when HVACs cannot fully offset the shortage mismatch and hence, avoids a drastic increase of MCP. Accordingly, the TE market would clear in a price less than or equal to the real-time electricity price. We capture the interactions of HVAC systems in the TE market through a non-cooperative game, which enables us to determine the optimal offer of each participant considering his thermal preferences.

The remainder of this paper is organized as follows. In Section II, the system model is described. In Section III, the problem formulation is presented. Simulation results are evaluated in Section IV and the paper is concluded in Section V.

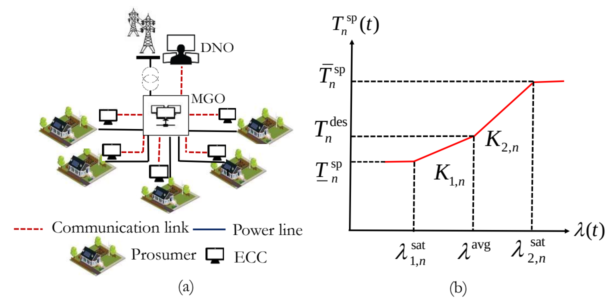


Fig. 1. (a) A residential microgrid with prosumers; (b) Setpoint-price characteristic of prosumer $n$.

## II. System Model

Consider a microgrid comprising a set $\mathcal{N}=\{1,\dots,N\}$ of $N$ residential prosumers as shown in Fig.1 $(a)$. Each prosumer is equipped with an energy consumption controller (ECC), which carries out the computational tasks of the prosumer and acts as an interface between the prosumer and microgrid operator (MGO), who is in charge of maintaining the supply-demand balance of the microgrid. Accordingly, MGO trades power with the wholesale market through DNO and launches the TE market in case a real-time power shortage exists at the microgrid level. The ECCs, MGO, and DNO exchange power and price information through a two-way communication network. We assume that the time horizon is divided into a set $\mathcal{T}=\{1,\dots,T\}$ of $T$ time slots with equal length, e.g., $\Delta t=15$ minutes per time slot. We use the terms prosumer, ECC, and household interchangeably.

We consider a *two-settlement* wholesale electricity market structure comprising the day-ahead and real-time markets [23]. The DNO takes part in these markets as a price-taker and acts as an intermediary entity between the microgrid and the wholesale market. We assume that each ECC $n\in\mathcal{N}$ *forecasts* its *future* net load, $L_n^{\rm da}(t)$, on a day-ahead basis for all time slots $t\in\mathcal{T}$. Parameter $L_n^{\rm da}(t)$ is the day-ahead forecasted demand of prosumer $n$ at time slot $t$ that cannot be met by the local RER generation and hence, needs to be purchased from the day-ahead market. The MGO purchases aggregate day-ahead net load of the microgrid from the DNO with the day-ahead electricity price $\lambda(t)$. Let $\boldsymbol{\lambda}=(\lambda(t),\, t\in\mathcal{T})$ denote the day-ahead price vector. In real-time, on the other hand, the MGO trades power with the DNO at time slot $t$ with buying price $\gamma^{\rm b}(t)$ and selling price $\gamma^{\rm s}(t)$, which are revealed to the MGO (and ECCs) at the *beginning* of each time slot. We make the following assumption:

**Assumption 1**: The real-time buying price is greater than the day-ahead price and the real-time selling price is lower than the day-ahead price, i.e., $\gamma^{\rm s}(t)<\lambda(t)<\gamma^{\rm b}(t)$.

Assumption 1 prevents prosumers from under-subscribing and over-subscribing power in the day-ahead market.

### A. HVAC Model

For the significant role that HVACs play in our framework, we model the cooling mode of these systems, individually.

• *Operational constraints:* First, we describe the operation of HVAC systems through linear constraints. Afterwards, we explain the proposed price-based control method and the

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3078655, IEEE Transactions on Smart Grid


resulting setpoint-price characteristic. The cooling mode of HVAC $n$ can be expressed as:

$$q_n(t) = \dot{m}_n c_n \left(T_n^{\text{in}}(t) - T_n^{\text{ac}}(t)\right), \quad (1a)$$

$$\underline{q}_n \leq q_n(t) \leq \overline{q}_n, \quad (1b)$$

where $q_n(t)$ is the thermal power of HVAC system $n$ and parameters $\dot{m}_n$ and $c_n$ are the air flow rate and the specific heat of the indoor air, respectively. Variables $T_n^{\text{in}}(t)$ and $T_n^{\text{ac}}(t)$ are the indoor air temperature of household $n$ and the air temperature of the HVAC system at time slot $t$, respectively. Constraint (1b) reflects the thermal limitation of the HVAC, where $\underline{q}_n$ and $\overline{q}_n$ are the minimum and maximum thermal power for HVAC $n$, respectively. The power consumption $p_{\text{hvac},n}(t)$ of the HVAC system for prosumer $n$ at time slot $t$ can be obtained as

$$p_{\text{hvac},n}(t) = \Big(\frac{q_n(t)}{\mu_n}\Big) v_{\text{hvac},n}(t),$$

where binary variable $v_{\text{hvac},n}(t)$ indicates operation state of the HVAC system for prosumer $n$, i.e., $v_{\text{hvac},n}(t) = 1$ whenever the system is operating and $v_{\text{hvac},n}(t) = 0$, otherwise. Parameter $\mu_n$ is the product of sensible heat ratio (SHR) and coefficient of performance (CoP). Parameter SHR relates the sensible thermal load to its total amount and CoP is the ratio of removed heat to the consumed energy [24]. To describe HVAC power consumption $p_{\text{hvac},n}(t)$ through linear terms, we use the following constraints:

$$\underline{q}_n v_{\text{hvac},n}(t) \leq p_{\text{hvac},n}(t)\mu_n \leq \overline{q}_n v_{\text{hvac},n}(t), \quad (2a)$$

$$q_n(t) - \overline{q}_n(1 - v_{\text{hvac},n}(t)) \leq p_{\text{hvac},n}(t)\mu_n, \quad (2b)$$

$$p_{\text{hvac},n}(t)\mu_n \leq q_n(t) - \underline{q}_n(1 - v_{\text{hvac},n}(t)). \quad (2c)$$

We express the thermal dynamics of household $n$ through the following recursive equation [25]:

$$T_n^{\text{in}}(t+1) - T_n^{\text{in}}(t) = \left(\frac{T^{\text{out}}(t) - T_n^{\text{in}}(t)}{R_n C_n} - \frac{p_{\text{hvac},n}(t)\mu_n}{C_n}\right)\Delta t. \quad (3)$$

Let $T^{\text{out}}(t)$ describe the ambient temperature at time slot $t$. Parameters $C_n$ and $R_n$ denote the heat capacity of indoor air and the equivalent thermal resistance of household $n$, respectively. Accordingly, the first term in the right-hand side of equation (3) corresponds to the thermal interaction between household $n$ and ambient while the second term relates to the contribution of HVAC in the indoor temperature changes. We define variable $\Delta T_n(t)$ to decide the operation status of HVAC $n$ at time slot $t$ as:

$$\Delta T_n(t) = T_n^{\text{in}}(t) - T_n^{\text{sp}}(t), \quad (4)$$

where $T_n^{\text{sp}}(t)$ is the setpoint temperature of prosumer $n$ at time slot $t$. Note that HVAC system $n$ operates to maintain the indoor air temperature close to the setpoint temperature, determined based on the prosumer's thermal preference. Accordingly, HVAC system $n$ operates if $\Delta T_n(t) > 0$, and is turned off, otherwise. The following constraint determines the operation status $v_{\text{hvac},n}(t)$ as:

$$-M(1 - v_{\text{hvac},n}(t)) < \Delta T_n(t) \leq M v_{\text{hvac},n}(t), \quad (5)$$

where $M$ is a large positive constant.

• *Price-based control method:* According to constraint (5), the operation state and hence, the energy cost of an HVAC system depend on its temperature setpoint $T_n^{\text{sp}}(t)$, which reflects prosumer's thermal preference $\tau_{\text{hvac},n}$. We assume that prosumer $n$ announces its daily thermal preferences to the ECC in the form of tuple, $\tau_{\text{hvac},n} = (\underline{T}_n^{\text{sp}}, T_n^{\text{des}}, \overline{T}_n^{\text{sp}})$, where $T_n^{\text{des}}$ is the desirable air temperature, and $\underline{T}_n^{\text{sp}}$ and $\overline{T}_n^{\text{sp}}$ are the lowest and highest temperatures that prosumer $n$ can bear, respectively. Thereby, the temperature setpoint $T_n^{\text{sp}}(t)$ of HVAC system $n$ is determined on a *day-ahead* basis as well. In the proposed price-based control method, ECC $n$ adjusts the temperature setpoint $T_n^{\text{sp}}(t)$ based on the day-ahead electricity price vector $\boldsymbol{\lambda}$ to reduce the energy cost of HVAC system. Accordingly, each ECC $n \in \mathcal{N}$ over-cools the household when the electricity price is less than its average, $\lambda^{\text{avg}}$, to avoid the operation of HVAC when price exceeds $\lambda^{\text{avg}}$. Thus, ECC $n$ increases the temperature setpoint when the price is greater than the average price.

Now, we describe the relation between setpoint temperature $T_n^{\text{sp}}(t)$ and the day-ahead electricity price $\lambda(t)$ through the setpoint-price characteristic. Fig.1 $(b)$ depicts the setpoint-price characteristic of prosumer $n$. ECC $n$ sets the setpoint temperature to the desirable temperature $T_n^{\text{des}}$, when $\lambda(t) = \lambda^{\text{avg}}$. Accordingly, $T_n^{\text{sp}}(t)$ decreases to values within the interval $[\underline{T}_n^{\text{sp}}, T_n^{\text{des}})$, whenever $\lambda(t) < \lambda^{\text{avg}}$; and is set to values within the interval $(T_n^{\text{des}}, \overline{T}_n^{\text{sp}}]$, whenever $\lambda(t) > \lambda^{\text{avg}}$. We also introduce saturation prices $\lambda_{1,n}^{\text{sat}}$ and $\lambda_{2,n}^{\text{sat}}$ to better express the setpoint-price characteristic. To respect prosumer's thermal preferences, for prices less than $\lambda_{1,n}^{\text{sat}}$ and greater than $\lambda_{2,n}^{\text{sat}}$, setpoint temperature is respectively set to $\underline{T}_n^{\text{sp}}$ and $\overline{T}_n^{\text{sp}}$, ignoring the price value. For price values, $\lambda_{1,n}^{\text{sat}} \leq \lambda(t) \leq \lambda_{2,n}^{\text{sat}}$, setpoint temperature $T_n^{\text{sp}}(t)$ is a piecewise linear function with two line segments. Variables $K_{1,n}$ and $K_{2,n}$ denote the slopes of each line segment in the setpoint-price characteristic of prosumer $n$. For the price vector, $\boldsymbol{\lambda}$, we define vector $\boldsymbol{\delta} = (\delta(t),\ t \in \mathcal{T})$, where parameter $\delta(t) = 1$ if $\lambda(t) > \lambda^{\text{avg}}$ and $\delta(t) = 0$ if $\lambda(t) \leq \lambda^{\text{avg}}$. We can express the setpoint-price characteristic of prosumer $n$ by the following constraints:

$$\underline{T}_n^{\text{sp}} \leq T_n^{\text{sp}}(t) \leq \overline{T}_n^{\text{sp}}, \quad (6a)$$

$$T_n^{\text{sp}}(t) \geq \left(T_n^{\text{des}} + K_{1,n}(\lambda(t) - \lambda^{\text{avg}})\right)(1 - \delta(t)), \quad (6b)$$

$$T_n^{\text{sp}}(t) \leq \left(T_n^{\text{des}} + K_{2,n}(\lambda(t) - \lambda^{\text{avg}})\right)\delta(t) + \overline{T}_n^{\text{sp}}(1 - \delta(t)). \quad (6c)$$

Note that if prosumer $n$ prefers thermal comfort over cost reduction of its HVAC system, he would set both values of $\underline{T}_n^{\text{sp}}$ and $\overline{T}_n^{\text{sp}}$ to the desirable temperature, $T_n^{\text{des}}$. Accordingly, the setpoint temperature would only be set to the desirable temperature, $T_n^{\text{sp}}(t) = T_n^{\text{des}}$, for all time slots $t$ and irrespective of the day-ahead electricity price.

### B. Household Appliances Model

We assume that prosumer $n$ has the set of electrical appliances $\mathcal{A}_n$. Appliance $a \in \mathcal{A}_n$ becomes awake at time slot $t_{a,n}^{\text{s}}$, when it is ready to be operated. Let $\mathcal{A}_n^{\text{awk}}(t) \subseteq \mathcal{A}_n$ denote the set of awake appliances in household $n$ at time slot $t$. ECC $n$ is responsible of scheduling the operation of awake appliances.

Now we describe the operation of prosumer appliances. We define the appliance specifications as follows:

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3078655, IEEE Transactions on Smart Grid



**Definition 1** (*Appliance Specifications*): The specifications for the awake appliance $a$ in household $n$ is defined by vector $\boldsymbol{s}_{a,n}=(D_{a,n},\, \underline{p}_{a,n},\, \overline{p}_{a,n},\, E_{a,n})$, where $D_{a,n}$ denotes the total number of time slots it takes for appliance $a$ to finish its current task, parameters $\underline{p}_{a,n}$ and $\overline{p}_{a,n}$ denote the lower and upper limits for power consumption, and parameter $E_{a,n}$ denotes the total energy required to complete the current task.

The appliance specifications inform ECC about the task of that appliance based on the following assumption:

**Assumption 2**: The appliance specifications, $\boldsymbol{s}_{a,n}$, for appliance $a$ are revealed to ECC $n$ when the appliance awakes. We consider the deadline $t^{\text{e}}_{a,n}$ for awake appliance $a$ to finish its task. This parameter is revealed to ECC when the appliance wakes up. In current time slot $t$, we define variables $d_{a,n}(t)$ and $e_{a,n}(t)$ to denote the *remaining* time slots and *required* energy for appliance $a$ to complete its current task before the deadline $t^{\text{e}}_{a,n}$. Note that we have $d_{a,n}(t) = D_{a,n}$ and $e_{a,n}(t)=E_{a,n}$ when appliance $a$ becomes awake at time slot $t=t^{\text{s}}_{a,n}$ and update $d_{a,n}(t)$ and $e_{a,n}(t)$ at $t^{\text{s}}_{a,n}< t \le t^{\text{e}}_{a,n}$ as $d_{a,n}(t)=d_{a,n}(t-1)-v_{a,n}(t-1)$ and $e_{a,n}(t)=e_{a,n}(t-1)-p_{a,n}(t-1)$, respectively. We have the following constraints for all awake appliances $a\in\mathcal{A}^{\text{awk}}_n$ at time slot $t$

$$\textstyle\sum_{t}^{t^{\text{e}}_{a,n}} p_{a,n}(t)\Delta t = e_{a,n}(t), \tag{7a}$$

$$\textstyle\sum_{t}^{t^{\text{e}}_{a,n}} v_{a,n}(t) = d_{a,n}(t), \tag{7b}$$

$$\underline{p}_{a,n} v_{a,n}(t) \le p_{a,n}(t) \le \overline{p}_{a,n} v_{a,n}(t), \tag{7c}$$

where variable $p_{a,n}(t)$ denotes the power consumption of appliance $a$ at time slot $t$. Binary variable $v_{a,n}(t)$ indicates the operation state of appliance $a$, i.e., $v_{a,n}(t)=1$ whenever the appliance is operating and $v_{a,n}(t)=0$, otherwise. Thus, equalities (7a) and (7b) guarantee that the assigned task of appliance $a$ is finished within the time interval $[t,\, t^{\text{e}}_{a,n}]$ and during $d_{a,n}(t)$ time slots.

## III. Problem Formulation

In this section, we discuss the day-ahead HVAC scheduling problem and the real-time generation-load balancing scheme. The former derives the optimal setpoint-price characteristic slopes and the operation schedule of HVAC system $n$. The latter schedules awake appliances of prosumer $n$ in an online manner and further studies prosumer's decision-making in the TE market, considering its willingness to market participation.

### *A. HVAC Scheduling Problem*

ECC $n$ aims to schedule its HVAC system in order to minimize the energy cost, $c^{\text{hvac}}_n(t) = p_{\text{hvac},n}(t)\lambda(t)\Delta t$ for $t \in \mathcal{T}$. Considering that prosumer $n$ usually decides the thermal preferences, $\tau_{\text{hvac},n}$, on a daily basis, ECC $n$ schedules its HVAC system in day-ahead. The energy cost, $c^{\text{hvac}}_n(t)$, depends on the temperature setpoint, $T^{\text{sp}}_n(t)$. In particular, according to the setpoint-price characteristic introduced in (6), slopes $K_{1,n}$ and $K_{2,n}$ determine the temperature setpoint, and thus the energy cost. We obtain the optimal values of $K_{1,n}$ and $K_{2,n}$ by solving the HVAC scheduling problem. The setpoint temperature deviation from its desirable value, $|T^{\text{sp}}_n(t)-T^{\text{des}}_n|$, reflects the thermal discomfort due to setpoint changes at time slot $t$. Hence, variables $K_{1,n}$ and $K_{2,n}$ can be used to evaluate the thermal discomfort of prosumer $n$.

Based on (6b), variable $T^{\text{sp}}_n(t)$ can take values greater than the linear function $T^{\text{des}}_n + K_{1,n}(\lambda(t) - \lambda^{\text{avg}})$, when $\lambda(t) \in [\lambda^{\text{sat}}_{1,n},\, \lambda^{\text{avg}})$. The same applies to (6c). To ensure that the values of $T^{\text{sp}}_n(t)$ are determined based on the line segments of the setpoint-price characteristic when $\lambda(t) \in [\lambda^{\text{sat}}_{1,n}, \lambda^{\text{sat}}_{2,n}]$, we consider the penalty term $c^{\text{pen}}_{1,n}(t)$ for the HVAC scheduling problem:

$$\begin{aligned} c^{\text{pen}}_{1,n}(t) =\, & w_{1,n}(T^{\text{sp}}_n(t)-(T^{\text{des}}_n+K_{1,n}(\lambda(t)-\lambda^{\text{avg}}))(1-\delta(t))) \\ & + w_{2,n}((T^{\text{des}}_n+K_{2,n}(\lambda(t)-\lambda^{\text{avg}})-T^{\text{sp}}_n(t))\,\delta(t)), \end{aligned}$$

where $w_{1,n}$ and $w_{2,n}$ are positive constants. Let $\boldsymbol{\Gamma}^1_n = (q_n(t), p_{\text{hvac},n}(t),\, T^{\text{in}}_n(t),\, T^{\text{ac}}_n(t),\, T^{\text{sp}}_n(t),\, \Delta T_n(t),\, K_{1,n},\, K_{2,n},\, t \in \mathcal{T})$ and $\boldsymbol{\Lambda}^1_n = (v_{\text{hvac},n}(t),\, t \in \mathcal{T})$ denote the decision vectors corresponding to the continuous and binary variables of the HVAC scheduling problem, respectively. ECC $n$ solves the mixed integer linear programming (MILP) optimization problem $\boldsymbol{\mathcal{P}}_{1,n}$ to operate its HVAC:

$$\begin{aligned} \boldsymbol{\mathcal{P}}_{1,n}: \quad & \underset{\boldsymbol{\Gamma}^1_n,\boldsymbol{\Lambda}^1_n}{\text{minimize}} \quad \textstyle\sum_{t\in\mathcal{T}}(c^{\text{hvac}}_n(t) + c^{\text{pen}}_{1,n}(t)) \\ & \text{subject to constraints (1)–(6).} \end{aligned}$$

### *B. Generation-Load Balancing Scheme*

In real-time, each ECC $n\in\mathcal{N}$ receives information about uncertain parameters and accordingly schedules its awake appliances to minimize its real-time generation-load mismatch, where the RHO method is deployed to manage the uncertainty regarding future time slots. Subsequently, ECC $n$ informs the MGO of its real-time net load. If the microgrid faces power shortage in real-time, MGO would launch a pay-as-MCP TE market, where willing prosumers compete to win the market by decreasing power consumption of their HVACs. A non-cooperative game is considered to determine the strategy of each market participating prosumer in the TE market.

*1)* ***Online Scheduling Problem****:* ECC $n$ schedules its awake appliances at each time slot $t$ in order to minimize the trading cost of its *real-time* net load, $L^{\text{rt}}_n(t)$. We assume that ECC $n$ observes the actual amount of renewable generation $g_n(t)$ at the *beginning* of the current time slot $t$. We define the real-time net load of prosumer $n$ at time slot $t$ as

$$L^{\text{rt}}_n(t) = p_{\text{hvac},n}(t) + \sum_{a\in A^{\text{awk}}_n(t)} p_{a,n}(t) - \big(L^{\text{da}}_n(t) + g_n(t)\big), \tag{8}$$

where $L^{\text{da}}_n(t)$ denotes the day-ahead purchased power of household $n$ for time slot $t$. Prosumer $n$ primarily exploits its day-ahead purchased power $L^{\text{da}}_n(t)$ and renewable generation $g_n(t)$ to supply its load at time slot $t$. We obtain the power shortage of prosumer $n$ at current time slot $t$ as

$$L^{\text{stg}}_n(t) = \max\{0,\, L^{\text{rt}}_n(t)\},$$

which can be expressed by the following constraints:

$$L^{\text{rt}}_n(t) \le L^{\text{stg}}_n(t), \tag{9a}$$

$$0 \le L^{\text{stg}}_n(t). \tag{9b}$$

Moreover, the excess power of prosumer $n$ at the current time slot $t$, can be expressed as

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3078655, IEEE Transactions on Smart Grid



$$L_n^{\text{exs}}(t) = L_n^{\text{stg}}(t) - L_n^{\text{rt}}(t). \tag{10}$$

At time slot $t$, prosumer $n$ would purchase the power shortage $L_n^{\text{stg}}(t)$ or sell the excess power $L_n^{\text{exs}}(t)$ to the DNO with the real-time prices $\gamma^{\text{b}}(t)$ and $\gamma^{\text{s}}(t)$, respectively. We define the real-time energy cost of prosumer $n$ at time slot $t$ as

$$c_n^{\text{rt}}(t) = \big(\gamma^{\text{b}}(t)L_n^{\text{stg}}(t) - \gamma^{\text{s}}(t)L_n^{\text{exs}}(t)\big)\Delta t.$$

The real-time energy cost, $c_n^{\text{rt}}(t)$, reflects the trading cost with the DNO and accordingly, comprises the procurement cost of power shortage, $L_n^{\text{stg}}(t)$, from real-time market and the profit that prosumer $n$ gains through selling excess power, $L_n^{\text{exs}}(t)$.

Considering the inter-temporal operational constraint (7), ECC $n$ should schedule awake appliances of time slot $t$ for the upcoming time slots $\{t+1, ..., T\}$ as well, while it has incomplete information about the future amounts of renewable generation, electricity prices, and awake appliances. We formulate the appliance scheduling problem using the RHO method which considers the decision-making horizon, $\tau(t) = \{t, ..., T\}$, for time slot $t$ to decide the operation schedule of the awake appliances [26]. Accordingly, the appliance scheduling problem is repeatedly solved in each time slot $t \in \mathcal{T}$, for the entire decision-making horizon of $\tau(t)$. To ensure that the priorly purchased power $L_n^{\text{da}}(t)$ is initially exploited in the future time slots as well as the current time slot $t$, we consider the penalty term $c_{2,n}^{\text{pen}}(t)$ in the objective function of the appliance scheduling problem of prosumer $n$ as

$$c_{2,n}^{\text{pen}}(t) = \textstyle\sum_{t'=t+1}^{T} \big(\sum_{a\in A_n^{\text{awk}}(t')} p_{a,n}(t') - L_n^{\text{da}}(t')\big)\lambda(t').$$

We define vectors $\boldsymbol{\Lambda}_n^2(t) = (v_{a,n}(t), a \in A_n^{\text{awk}}(t))$ and $\boldsymbol{\Gamma}_n^2(t) = (p_{a,n}(t), e_{a,n}(t), d_{a,n}(t), L_n^{\text{rt}}(t), L_n^{\text{stg}}(t), L_n^{\text{exs}}(t),\ a \in A_n^{\text{awk}}(t))$ for the binary and continuous variables, respectively. ECC $n$ solves the following MILP problem to schedule the awake appliances of the current time slot $t$:

$$\boldsymbol{\mathcal{P}}_{2,n}(t):\ \underset{\boldsymbol{\Gamma}_n^2(t),\boldsymbol{\Lambda}_n^2(t)}{\text{minimize}}\quad c_n^{\text{rt}}(t) + c_{2,n}^{\text{pen}}(t)$$
$$\text{subject to constraints (7)}-\text{(10)}.$$

By solving $\boldsymbol{\mathcal{P}}_{2,n}(t)$, ECC $n$ carries out the operational schedule of awake appliances for the current and future time slots. Considering the uncertainties regarding future time slots, it *only* implements the decisions made for the current time slot.

*2) **TE Market Clearing Problem**:* MGO initially broadcasts the amount of power shortage, $L_{\text{MG}}^{\text{stg}}(t) = [L_{\text{MG}}^{\text{rt}}(t)]^+$ where $L_{\text{MG}}^{\text{rt}}(t) = \sum_{n\in\mathcal{N}} L_n^{\text{rt}}(t)$, and the real-time electricity price $\gamma^{\text{b}}(t)$ to all ECCs $n \in \mathcal{N}$. Based on $\gamma^{\text{b}}(t)$, ECC $n$ decides whether to participate in the TE market or not. Let $I(t) \subseteq \mathcal{N}$ denote the set of $N_{\text{P}}(t)$ participating prosumers in the TE market at time slot $t$ as $I(t) = \{1, ..., N_{\text{P}}(t)\}$. ECCs of participating prosumers $n \in I(t)$ send their offers $\boldsymbol{o}_n(t) = (\pi_n(t), \rho_n(t))$ to the MGO, where $\pi_n(t)$ is the offered price for curtailing one unit of its HVAC power consumption and $\rho_n(t)$ denotes the amount of power to be curtailed.

• *Market Mechanism:* We consider the pay-as-MCP market mechanism, where market winners receive payments based on the MCP [16]. We assume that DNO takes part in the TE market as well as prosumers $n \in I(t)$ to ensure that the power shortage $L_{\text{MG}}^{\text{stg}}(t)$ is addressed. Accordingly, there are $N_p(t) + 1$ participants in the TE market at time slot $t$. Moreover, participation of DNO prevents the MCP to surpass the real-time electricity price $\gamma^{\text{b}}(t)$. After receiving offers from ECCs of all participating prosumers $n \in I(t)$, MGO sorts offers in ascending order based on the price components. If $\sum_{n\in I(t)} \rho_n(t) \geq L_{\text{MG}}^{\text{stg}}(t)$, then participants compete with each other to win the TE market and to maximize their utility. In this case, the smallest group of participants with the lowest price offers whose aggregate power offers can address the shortage are announced as winners and the MCP would be the greatest advertised price among winners. Otherwise, when $\sum_{n\in I(t)} \rho_n(t) < L_{\text{MG}}^{\text{stg}}(t)$, a part of $L_{\text{MG}}^{\text{stg}}(t)$ should be supplied by the DNO with the real-time price, $\gamma^{\text{b}}(t)$. In this case, all participants are announced as winners and the MCP is $\gamma^{\text{b}}(t)$.

• *Prosumer's Competition Game:* According to the market mechanism, offer of each prosumer $n \in I(t)$ affects the MCP, thereby the utility of all participants. Hence, we capture the interaction among participants as *a non-cooperative game* and refer to market participating prosumers $n \in I(t)$ as *players*. Let $\boldsymbol{O}_{-n}(t) = (\boldsymbol{o}_j(t), j \in I_{-n}(t))$ denote offers of competitors of player $n$. We obtain the *Nash equilibrium* of the game through finding the best response of each player $n$ to $\boldsymbol{O}_{-n}(t)$.

• *Player's Feasible Strategy Set:* As mentioned, the tuple $\boldsymbol{o}_n(t) = (\pi_n(t), \rho_n(t))$ is the strategy of player $n \in I(t)$. The offered price $\pi_n(t)$ is associated with the discomfort that prosumer $n$ experiences by decreasing the operation of its HVAC. We employ the setpoint-price characteristic described in (6) to analyze the resulting thermal discomfort. Hence, the discomfort that prosumer $n$ incurs at $T_n^{\text{sp}}(t)$ can be stated as

$$\underline{\pi}_n(t) = (T_n^{\text{sp}}(t) - T_n^{\text{des}})/K_{2,n}, \tag{11}$$

where $\underline{\pi}_n(t)$ is the discomfort coefficient. Recall that characteristic slopes $K_{1,n}$ and $K_{2,n}$ are the amounts of setpoint temperature deviation from its desirable amount, $T_n^{\text{des}}$, that lead to one unit of discomfort cost. Accordingly, $\underline{\pi}_n(t)$ can describe the thermal discomfort when there exists a setpoint temperature change of $T_n^{\text{sp}}(t) - T_n^{\text{des}}$.

We assume that prosumer $n$ curtails its HVAC power consumption merely when $T_n^{\text{sp}}(t) > T_n^{\text{des}}$. Based on the TE market mechanism, MCP is less than or equal to $\gamma^b(t)$. Since player $n$ receives payments based on MCP, we assume the maximum price that player $n$ can offer is $\gamma^{\text{b}}(t)$. Thus, in case that the discomfort coefficient, $\underline{\pi}_n(t)$, exceeds the electricity price $\gamma^{\text{b}}(t)$, player $n$ would not take part in the TE market rationally. We describe the above-mentioned constraint for the price offer $\pi_n(t)$ of prosumer $n$ as

$$\underline{\pi}_n(t) \leq \pi_n(t) \leq \gamma^{\text{b}}(t). \tag{12}$$

Prosumers who have scheduled their HVACs to operate at time slot $t$, can participate in the TE market. Thus, we have the following constraint for power offer $\rho_n(t)$ of prosumer $n$

$$0 \leq \rho_n(t) \leq p_{\text{hvac},n}(t) - \underline{p}_{\,\text{hvac},n}, \tag{13}$$

where $\underline{p}_{\,\text{hvac},n} = \underline{q}_n/\mu_n$ corresponds to the minimum power consumption of the HVAC system for prosumer $n$.

• *Player's Utility:* Player $n$ aims to maximize its utility in the TE market. Accordingly, offer $\boldsymbol{o}_n(t)$ should primarily place player $n$ among market winners. Otherwise, player $n$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3078655, IEEE Transactions on Smart Grid



would gain zero utility. Hence, $\pi_n(t)$ should be less than or equal to the MCP. Note that the offers from all market participating prosumers $n \in I(t)$, along with the DNO can affect the MCP. Accordingly, to derive the utility of player $n$, the MCP should be determined based on the offer components of player $n$, other participating prosuemrs, $j \in I_{-n}(t)$, and the real-time electricity price announced by the DNO. Player $n$ initially examines prices advertised by others to determine its rank among players and further, decides $\pi_n(t)$ and $\rho_n(t)$ in order to be in the smallest group of players that can supply the power shortage. Player $n$ compares its price, $\pi_n(t)$, with prices of other players $j \in I_{-n}(t)$ as

$$M(z_j(t)-1) \leq \pi_n(t) - \pi_j(t) \leq M z_j(t). \quad (14)$$

where $M$ is a large positive constant and $z_j(t)$ is a binary variable that equates 1 when the offered price of competitor $j$, $\pi_j(t)$, is less than or equal to $\pi_n(t)$ and equates 0 when $\pi_j(t)$ is greater than or equal to $\pi_n(t)$. We include $\omega_1 \sum_{j \in I_{-n}(t)} z_j(t)$ with positive weight $\omega_1$ in the objective function of player $n$ to ensure the occurrence of $\pi_n(t) = \pi_j(t)$ is only indicated by $z_j(t) = 1$. To find the MCP, first we determine whether the aggregate amount of power offers with a price less than or equal to the offered price of prosumer $n \in I(t)$ can meet the shortage or not as:

$$S_n(t) = \sum_{j \in I_{-n}(t)} z_j(t)\rho_j(t) + \rho_n(t), \quad (15)$$

where $S_n(t)$ is the aggregate amount of power offers announced by players with a price offer less than or equal to $\pi_n(t)$. Considering that $\boldsymbol{O}_{-n}(t)$ is sorted in ascending order, we calculate a similar value for each competitor $j \in I_{-n}(t)$ as:

$$S_j(t) = \sum_{j'=1}^{j} \rho_{j'}(t) + (1 - z_j(t))\rho_n(t), \quad (16)$$

where $S_j(t)$ is the aggregate amount of power offers with a price less than or equal to the offered price of competitor $j \in I_{-n}(t)$. Next, we determine whether $S_n(t)$ and $S_j(t)$ can respectively meet the power shortage $L^{\text{stg}}_{\text{MG}}(t)$ through the following constraints:

$$M(x_n(t)-1) \leq S_n(t) - L^{\text{stg}}_{\text{MG}}(t) \leq M x_n(t), \quad (17a)$$

$$M(x_j(t)-1) \leq S_j(t) - L^{\text{stg}}_{\text{MG}}(t) \leq M x_j(t) \ \ \forall j \in I_{-n}(t), \quad (17b)$$

where binary variables $x_n(t)$ and $x_j(t)$ are equal to 1 when $S_n(t)$ and $S_j(t)$ can respectively meet the power shortage $L^{\text{stg}}_{\text{MG}}(t)$ and $x_n(t) = x_j(t) = 0$, otherwise. Note that $M$ is a large positive constant.The MCP is either equal to one of the price offers advertised by market participating prosumers $n \in I(t)$ or the real-time electricity price $\gamma^{\text{b}}(t)$ offered by the DNO.We describe the aforementioned through the following

$$x_n(t) + \sum_{j \in I_{-n}(t)} x_j(t) + x_0(t) = 1, \quad (18)$$

where binary variable $x_0(t)$ determines whether MCP is equal to $\gamma^{\text{b}}(t)$, $x_0(t) = 1$, or not, $x_0(t) = 0$. To ensure that the smallest group of players determine the MCP, we add penalty term $c^{\text{pen}}_{3,n}(t)$ to the objective function of player $n$'s bidding problem. We consider the rank of prices advertised by participating prosumers $n \in I(t)$ as:

$$r_n(t) = \sum_{j \in I_{-n}(t)} z_j(t), \quad (19)$$

where $r_n(t)$ indicates rank of $\pi_n(t)$ among $\boldsymbol{O}_{-n}(t)$, i.e., the number of players with a price offer less than or equal to $\pi_n(t)$. Based on (19), we describe the penalty term $c^{\text{pen}}_{3,n}(t)$ as

$$c^{\text{pen}}_{3,n}(t) = r_n(t)x_n(t) + \sum_{j \in I_{-n}(t)} j x_j(t) + (N_{\text{P}}(t)+1)x_0(t).$$

Accordingly, the rank of the real-time price $\gamma^{\text{b}}(t)$ is considered greater than all advertised price offers, i.e., $N_{\text{P}}(t)+1$, to ensure that power shortage is initially compensated by prosumers and not the DNO. Now, we define the MCP $\eta(t)$ as:

$$\eta(t) = x_n(t)\pi_n(t) + \sum_{j \in I_{-n}(t)} x_j(t)\pi_j(t) + x_0(t)\gamma^b(t). \quad (20)$$

Considering that offers are sorted in ascending order, player $n$ wins the TE market if its price offer $\pi_n(t)$ is less than or equal to the MCP $\eta(t)$.We adopt the binary variable $f_n(t)$ to describe the aforementioned as

$$-M f_n(t) \leq \pi_n(t) - \eta(t) \leq M(1 - f_n(t)), \quad (21)$$

where $f_n(t) = 1$ if player $n$ wins the TE market and $f_n(t) = 0$, otherwise. Note that $M$ is a large positive constant.To enforce $f_n(t) = 1$ when $\pi_n(t) = \eta(t)$, we add the term $\omega_2 f_n(t)$ with positive weight $\omega_2$ to the objective function of player $n$. We substitute the multiplication $\pi_n(t)\rho_n(t)$ with the auxiliary variable $y_n(t)$. Additionally, to define the utility function of player $n$ we define the auxiliary variable $u_n(t)$ to describe $\eta(t)\rho_n(t)$ as

$$u_n(t) = x_n(t)y_n(t) + \rho_n(t) \sum_{j \in I_{-n}(t)} x_j(t)\pi_j(t) + \rho_n(t)x_0(t)\gamma^{\text{b}}(t), \quad (22)$$

where the second and third terms on the right-hand side represent the impact associated with actions of competitors $j \in I_{-n}(t)$ and the DNO on utility of player $n$, respectively. Finally, we define the utility of player $n$ in the TE market as

$$U_n(t) = u_n(t) f_n(t). \quad (23)$$

• *Prosumer's Best Response:* When deciding the offer vector $\boldsymbol{o}_n(t)$, prosumer $n$ has to consider its thermal discomfort due to curtailing $\rho_n(t)$ units of HVAC power consumption. We assume that thermal discomfort is a quadratic function of $\rho_n(t)$ and approximate it as a piecewise linear function, $c^{\text{dsc}}_n(t)$, with $NL_n$ segments as

$$0 \leq \theta_{h,n}(t) \leq \overline{\theta}_{h,n} - \overline{\theta}_{h-1,n}, \quad (24a)$$

$$\theta_{1,n}(t) \leq \overline{\theta}_{1,n}, \quad (24b)$$

$$\theta_{NL_n,n}(t) \leq \overline{\rho}_n(t) - \theta_{NL_n-1,n}, \quad (24c)$$

$$c^{\text{dsc}}_n(t) = \sum_{h \in H_n} \theta_{h,n}(t) m_{h,n}, \quad (24d)$$

where $h \in H_n$ denotes each line segment with slope $m_{h,n}$. Auxiliary variable $\theta_{h,n}(t)$ determines power curtailment corresponding to line segment $h$ of prosumer $n$ at time $t$. Parameter $\overline{\theta}_{h,n}$ describes the maximum amount of power curtailment for segment $h$.

ECC $n$ solves problem $\boldsymbol{\mathcal{P}}_{3,n}(t)$ to determine its best response, $\boldsymbol{o}_n(t)$. We define $\boldsymbol{\Gamma}^3_n(t) = (\pi_n(t), \rho_n(t), y_n(t), S_n(t), S_j(t), r_n(t), \eta(t), u_n(t), U_n(t), \theta_{h,n}(t),\ h \in H_n, j \in I_{-n}(t))$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3078655, IEEE Transactions on Smart Grid



**Algorithm 1** Balancing Scheme Algorithm at Time Slot $t$.

Set $k := 0$ and $\xi := 10^{-3}$.
Each ECC $n \in \mathcal{N}$ schedules its awake appliances $a \in \mathcal{A}_n^{\mathrm{awk}}(t)$ by solving $\boldsymbol{\mathcal{P}}_{2,n}(t)$ and announces net load $L_n^{\mathrm{rt}}(t)$ to MGO.
**If** $L_{\mathrm{MG}}^{\mathrm{rt}}(t) > 0$
  MGO broadcasts $L_{\mathrm{MG}}^{\mathrm{stg}}(t)$ and $\gamma^{\mathrm{b}}(t)$ to all ECCs $n \in \mathcal{N}$.
  Each ECC $n \in I(t)$ considers a random $\boldsymbol{O}_{-n}^{0}(t)$.
  **Repeat**
    Each ECC $n \in I(t)$ determines its best response $\boldsymbol{o}_n^{k+1}(t)$ to $\boldsymbol{O}_{-n}^{k}(t)$ by solving $\boldsymbol{\mathcal{P}}_{3,n}(t)$ and sends it to MGO.
    MGO broadcasts $\boldsymbol{O}^{k+1}(t)$ to all ECCs $n \in I(t)$.
    Each ECC $n \in I(t)$ derives its $\boldsymbol{O}_{-n}^{k+1}(t)$ from $\boldsymbol{O}^{k+1}(t)$.
    $k := k + 1$.
  **Until** for all ECCs $n \in I(t)$ : $|\boldsymbol{o}_n^{k}(t) - \boldsymbol{o}_n^{k-1}(t)| \le \xi$.
**Else**
  MGO sells $L_{\mathrm{MG}}^{\mathrm{exs}}(t)$ to the DNO.
**End If**

and $\boldsymbol{\Lambda}_n^3(t) = (z_j(t), x_j(t), x_n(t),\ x_0(t), f_n(t), j \in I_{-n}(t))$ as the decision vectors for continuous and binary variables of problem $\boldsymbol{\mathcal{P}}_{3,n}(t)$:

$$\boldsymbol{\mathcal{P}}_{3,n}(t):\ \underset{\boldsymbol{\Gamma}_n^3(t), \boldsymbol{\Lambda}_n^3(t)}{\text{maximize}}\ U_n(t) + \omega_1 \textstyle\sum_{j \in I_{-n}(t)} z_j(t) + \omega_2 f_n(t) - \omega_3 c_{3,n}^{\mathrm{pen}}(t) - c_n^{\mathrm{dsc}}(t)$$
$$\text{subject to constraints (11)–(24).}$$

where $\omega_3$ is a positive constant. The non-linearity of some constrains, e.g., (20), can be addressed by the approach used in (2) to linearize the production of binary and continuous variables. Hence, problem $\boldsymbol{\mathcal{P}}_{3,n}(t)$ can be regarded as MILP.

• *Nash Equilibrium:* Finally, we apply an iterative algorithm based on the best response of players to find the Nash equilibrium of the competition game among TE market participants as described in Algorithm 1. In the first iteration, player $n \in I(t)$ initializes random offers of its competitors, $\boldsymbol{O}_{-n}^{0}(t)$, and determines its best response toward these random offers by solving $\boldsymbol{\mathcal{P}}_{3,n}(t)$. Each player $n \in I(t)$ informs the MGO of its offer, $\boldsymbol{o}_n^{1}(t)$. Accordingly, MGO broadcasts the offer vector $\boldsymbol{O}^1(t) = (\boldsymbol{o}_n^1(t), n \in I(t))$ to all players. In the next iteration, player $n$ determines its offer considering the actual offers of its competitors, $\boldsymbol{O}_{-n}^{1}(t)$. Eventually, TE market clears in the $k$th iteration where for all players $n \in I(t)$, we have $\boldsymbol{o}_n^k(t) = \boldsymbol{o}_n^{k-1}(t)$, i.e., no player achieves more utility by changing his offer.

Since there are a finite number of players and that constraints (12) and (13) bound the strategy profiles to finite sets, there exists at least one Nash equilibrium for such a game [27]. To prove the uniqueness of the proposed game, the best response of players has to be a concave function [28]. We obtain the best response of each player by solving problem $\boldsymbol{\mathcal{P}}_{3,n}(t)$ and do not derive a closed form best response function. However, in our simulations, the TE market clearing game converges to a single Nash equilibrium.

## IV. Performance Evaluation

In this section, we initially introduce our simulation setup. Subsequently, we evaluate performance of the proposed approaches in our TE framework.

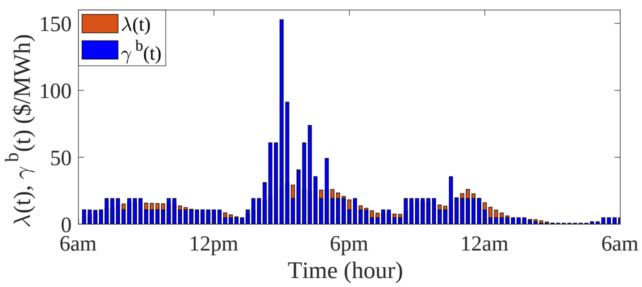


Fig. 2. Day-ahead and real-time electricity prices over time horizon [29]

TABLE I
HVAC System Specifications

| $\mu_n$ | $\underline{q}_n$ (kW) | $\overline{q}_n$ (kW) | $\dot{m}_n$ (kg/s) | $R_n$ (°C/kW) | $c_n$ (kJ/kg°C) |
|---|---|---|---|---|---|
| 2 | 1.5 | 6 | 3.352 | 5 | 1 |

### A. Simulation Setup

We consider a microgrid with $N = 150$ prosumers and a one-day time horizon which we divide into $T$=96 time slots. Each prosumer is equipped with an HVAC system and 15 appliances. For all appliances, $\underline{p}_{a,n}$ and $\overline{p}_{a,n}$ are set to a nominal value as described in [30]. We consider an interval for the wake-up time of each appliance and uniformly choose $t_{a,n}^{\mathrm{s}}$, at random from this interval. We similarly choose the deadline $t_{a,n}^{\mathrm{e}}$. To make the simulation results more practical, we consider three types of prosumers in the microgrid; flexible, semi-flexible, and inflexible, which differ in the interval for the task deadlines. A flexible prosumer has the widest time window to schedule its appliances. An inflexible prosumer operates all appliances without delay. For simplicity, we assume that there are 50 prosumers of each type. The results represented in this section are the average of multiple simulations, considering various wake-up and deadline times. We assume the RER of each prosumer to be a PV panel [29]. Fig. 2 indicates the electricity prices in day-ahead and real-time markets, $\lambda(t)$ and $\gamma^{\mathrm{b}}(t)$, over the time horizon [29]. Simulations are performed using the MOSEK solver by CVX MATLAB on a PC with Intel Core i5 3337U CPU 1.8 GHz processor.

### B. HVAC Scheduling

To assess performance of the proposed HVAC scheduling method, we assume the same ambient temperature, $T^{\mathrm{out}}(t)$, for all prosumers [31]. Table I describes the HVAC specifications considered in our simulations. The value of the indoor air temperature $T_n^{\mathrm{in}}(t)$ at the beginning of the first time slot is uniformly chosen at random from the interval [25°$C$, 28°$C$] for all prosumers $n \in \mathcal{N}$.

Fig. 3 depicts the setpoint-price characteristic of flexible prosumers where parameters $\underline{T}_n^{\mathrm{sp}}$, $T_n^{\mathrm{des}}$, and $\overline{T}_n^{\mathrm{sp}}$, are uniformly chosen at random from intervals [19.5°$C$, 20.5°$C$], [23°$C$, 25°$C$] and [27.5°$C$, 28.5°$C$], respectively. Despite having identical HVAC systems, different thermal preferences lead to different setpoint-price characteristics. As in Fig. 3, for some flexible prosumers $K_{1,n} = 0$. That is, these prosumers prefer thermal comfort over cost savings.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3078655, IEEE Transactions on Smart Grid



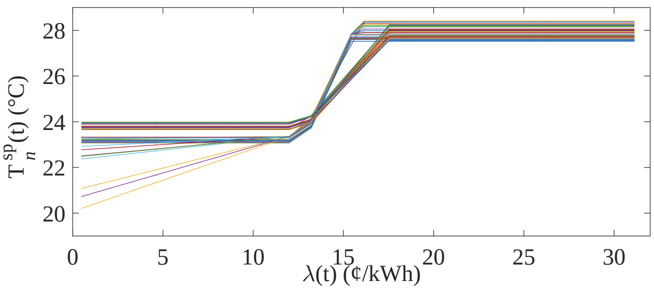


Fig. 3. Setpoint-price characteristic of flexible prosumers.

We use the scenario where the setpoint temperature, $T_n^{\mathrm{sp}}(t)$, is set to its desirable amount, $T_n^{\mathrm{des}}$, irrespective of the day-ahead price, as a benchmark for comparison. Accordingly, the proposed method cuts the energy cost of HVAC systems for flexible, semi-flexible, and inflexible prosumers by 10.42%, 16.31%, and 25.62% on average, respectively.

### C. Online Appliance Scheduling

We discuss performance of the proposed online appliance scheduling method for the set of appliances described in Table II. Considering our assumption that prosumers purchase an approximate amount of their future net load from the day-ahead market, we primarily simulate the load forecasts of prosumers using a sample-based stochastic optimization approach introduced in [32]. We adopt the conditional value-at-risk (CVaR) as an index to measure the risk associated with the trading cost in the real-time market that the forecasted value of day-ahead demand will impose. We assume all prosumers to be risk-averse with the confidence level of 0.99. We use prosumers' net loads of the previous 100 days as samples to forecast the day-ahead demand.

Fig. 4 $(a)$ compares the day-ahead demand forecasts of a flexible prosumer when appliances were scheduled based on our online approach during the past 100 days with the case where appliances were not scheduled. Accordingly, for the period of 12 am to 6 am, prosumer forecasts relatively larger amount of demand when its appliances were scheduled in the previous days. This occurs because appliances were operated mainly in this period during the past days due to lower electricity prices. Contrarily, forecasts based on the unscheduled net loads of prosumers, are irrespective of the price as appliances were operated whenever they became awake in the previous days.

Fig. 4 $(b)$ compares the real-time net load of the flexible prosumer when awake appliances are scheduled based on our method versus the benchmark scenario where appliances are operated without delay when they become awake. The proposed scheduling method shifts the operation of awake appliances to the period of 12 am to 6 am, when the real-time price is low. The significant amount of PV generation during 6 am to 7 pm results in a negative net load, which is sold to the real-time market. Table III and Table IV compare the average amounts of power traded in day-ahead and real-time markets considering the proposed scheduling method and the benchmark scenario for flexible and semi-flexible prosumers, respectively. To comprehensively evaluate the economic efficiency of the online scheduling method, we compare the total cost, $c_n^{\mathrm{total}}(t) = \lambda(t) L_n^{\mathrm{da}}(t)\Delta t + c_n^{\mathrm{rt}}(t)$, which includes the power procurement cost from the day-ahead market and the real-time energy cost of prosumers. The proposed online scheduling method benefits prosumers by cutting the total cost by 35.64% and 27.96% on average for flexible and semi-flexible prosumers, respectively.

TABLE II
APPLIANCE SPECIFICATIONS [30]

| Appliance Name | $D_{a,n}$ (Number of time slots) | $\underline{p}_{a,n} = \overline{p}_{a,n}$ (kW) | $E_{a,n}$ (kWh) |
|---|---|---|---|
| Electric stove | 12 | 1.5 | 4.5 |
| Dryer | 8 | 0.5 | 1 |
| Vaccume cleaner | 8 | 1.5 | 3 |
| Refrigerator | 80 | 0.125 | 2.5 |
| Dish washer | 8 | 1 | 2 |
| Washing machine | 12 | 0.7 | 2.1 |
| Water heater | 8 | 1 | 2 |
| Pool pump | 8 | 2 | 4 |
| Electric vehicle | 16 | 2.5 | 10 |
| Steam iron | 8 | 1 | 2 |
| Hair dryer | 4 | 1 | 1 |
| TV | 16 | 0.25 | 1 |
| PC | 24 | 0.25 | 1.5 |
| Lighting | 24 | 0.5 | 3 |
| Other | 16 | 1.5 | 6 |

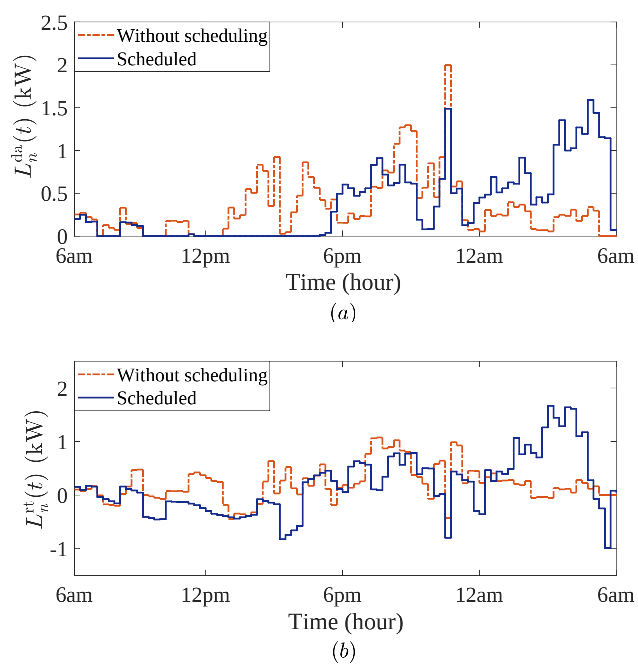


Fig. 4. $(a)$ Day ahead load forecast; $(b)$ real-time net load of a flexible prosumer using the proposed scheduling method and the benchmark scenario.

### D. TE Market

To assess participation of HVAC systems in the TE market, we study the behavior of a group of $N_{\mathrm{p}} = 15$ prosumers. Fig. 5 shows the ratio of the total amount of power cleared by market participants to the power shortage of the microgrid at each time slot. Participation of HVAC systems in the TE market at each time slot depends on parameters related to HVAC systems such

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3078655, IEEE Transactions on Smart Grid



TABLE III
AVERAGE AMOUNTS OF POWER TRADED BY FLEXIBLE PROSUMERS

| Approach | Day-Ahead Power Purchased (kW) | Real-time Power Purchased (kW) | Real-time Power Sold (kW) |
|---|---|---|---|
| Scheduled | 34.15 | 28.44 | 11.37 |
| Without Scheduling | 29.9 | 26.97 | 5.62 |

TABLE IV
AVERAGE AMOUNTS OF POWER TRADED BY SEMI-FLEXIBLE PROSUMI

| Approach | Day-Ahead Power Purchased (kW) | Real-time Power Purchased (kW) | Real-tir Powe Sold (kW) |
|---|---|---|---|
| Scheduled | 29.63 | 24.50 | 10.87 |
| Without Scheduling | 26.23 | 23.41 | 7.21 |

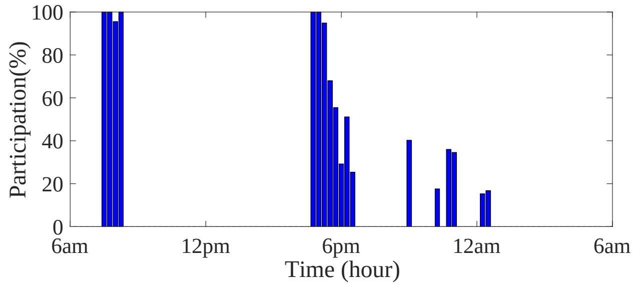


Fig. 5. Participation of HVACs in the TE market

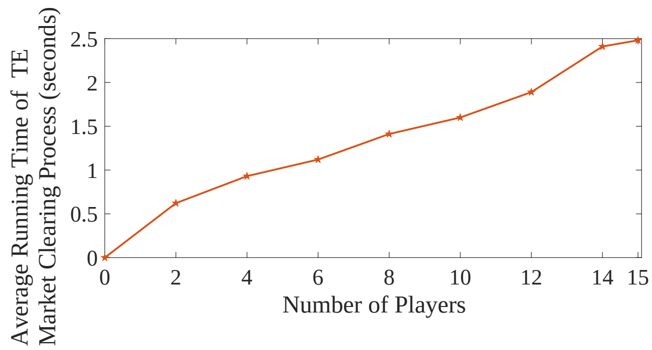


Fig. 6. Average running time of the TE market clearing process

as the operation state and prosumer's discomfort cost as w as other factors including the amount of mismatch and the re time electricity price. For time slots around 8 am, the amou of power mismatch is relatively small due to the PV generati and the fewer appliances that need to be operated. Hence, 2( to 33% of HVACs could manage more than 95% of the re time power mismatch in the TE market. From 5:30 pm to pm, 93% to 100% of HVAC systems were able to address 28% to 68% of the mismatch. For these time slots the amount of real-time mismatch is relatively larger since more appliances are scheduled to be operated. For time slots between 9 pm and 12 am where the TE market was launched, an HVAC proportion of 53% to 60% could mitigate 17% to 41% of the microgrid's mismatch. The engagement of HVAC systems in this period relies on the availability of these systems.

It is worth noting that the total capacity of available power offered by HVAC systems in the TE market relies on the thermal preferences of prosumers. Setpoint-price characteristic slopes confine the price and power strategies of market players through constraints (13) and (14). Additionally, the number of players at each time slot is determined based on the discomfort coefficient $\underline{\pi}_n(t)$. During 8:30 am to 4:30 pm, PV generation is sufficient to meet the demand; thus the TE market is not launched. In other periods when there is no participation, the discomfort cost of participants is greater than the real-time price. Thus, no prosumer is willing to alter power consumption of its HVAC and hence, does not take part in the TE market.

Our simulation results demonstrate that the MCP in the TE market is equal to the real-time electricity price, $\gamma^{\mathrm{b}}(t)$. In most time slots when the TE market is launched, the total power cleared by the market participants cannot fully meet the microgrid's net load; hence power shortage is partially supplied by the DNO. In other time slots, participants cooperate to keep the MCP close to the real-time price, thereby maximizing their utility, as the maximum power offer of each participant cannot cover the power shortage. In case that the maximum power offer of each HVAC is comparable to the microgrid's power shortage, participants compete to compensate a greater amount of power shortage and their attempt may result in an MCP less than the real-time price. Note that the TE market offers a platform to address the power mismatch of the microgrid in an independent manner and by using the flexibility of local HVAC systems. While launching the proposed market allows for a more reliable operation of the microgrid, it reduces the need for spinning reserve capacities as well. Thus, even in scenarios where the MCP is close to the electricity price, TE market offers mutual benefits to MGO and the system operator. In our simulations, the TE market clearing game converges to the Nash equilibrium in a few number of iterations. Fig. 6 shows the running time of the TE market clearing process, which is the time it takes to reach the Nash equilibrium, per iteration per player. Accordingly, the proposed TE market is suitable for implementation in practice. Note that the values of HVAC power consumption, real-time net load of prosumers $n \in \mathcal{N}$, and real-time net load of the microgrid used in our simulations are obtained by solving scheduling problems $\boldsymbol{\mathcal{P}}_{1,n}$ and $\boldsymbol{\mathcal{P}}_{2,n}(t)$. Accordingly, performance of our approach is not limited to certain predetermined conditions of the microgrid.

## V. CONCLUSION

In this paper, we proposed a TE framework to address the real-time generation-load mismatch of a residential microgrid where prosumers primarily schedule their awake appliances in an online manner and further participate in an market to compensate the remaining real-time power shortage using their HVAC systems. We proposed a decentralized approach for scheduling the awake appliances of each prosumer and tackled the incomplete information of prosumers about uncertain parameters through the RHO method. We evaluated the willingness of prosumers for participating in the TE market by proposing a price-based control method for their HVAC systems. We further captured the prosumers interaction in the TE

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSG.2021.3078655, IEEE Transactions on Smart Grid



market as a game and obtained the Nash equilibrium of such a game. Our simulations illustrate that the proposed framework has reduced the energy cost of HVACs as well as the total power trading cost in real-time electricity markets, while preserving prosumers' privacy. The TE market has shown a great potential for supplying the microgrid during power shortages as the market clearing game properly converges to its Nash equilibrium with the least information exchange. For future work, we plan to pursue research on TE frameworks in the following directions. As presence of energy storage systems might change methods of addressing the real-time power mismatch, we plan to investigate the role of these systems in balancing supply and demand. Furthermore, we plan to examine the potential of additional thermal storage methods such as ice storage, in other areas such as commercial sector, under the context of TE. Finally, we plan to study the required considerations of implementing TE frameworks, which would further facilitate their accommodation in distribution systems.

## REFERENCES


[1] S. Chen and C. C. Liu, "From demand response to transactive energy: State of the art," *Journal of Modern Power Systems and Clean Energy*, vol. 5, no. 1, pp. 10–19, Jan. 2017.

[2] Y. Huang, L. Wang, W. Guo, Q. Kang, and Q. Wu, "Chance constrained optimization in a home energy management system," *IEEE Trans. on Smart Grid*, vol. 9, no. 1, pp. 252–260, Jan. 2018.

[3] Z. Chen, L. Wu, and Y. Fu, "Real-time price-based demand response management for residential appliances via stochastic optimization and robust optimization," *IEEE Trans. on Smart Grid*, vol. 3, no. 4, pp. 1822–1831, Dec. 2012.

[4] M. Rastegar, M. Fotuhi-Firuzabad, and M. Moeini-Aghtai, "Developing a two-level framework for residential energy management," *IEEE Trans. on Smart Grid*, vol. 9, no. 3, pp. 1707–1717, May 2018.

[5] X. Yang, Y. Zhang, H. He, S. Ren, and G. Weng, "Real-time demand side management for a microgrid considering uncertainties," *IEEE Trans. on Smart Grid*, vol. 10, no. 3, pp. 3401–3414, May 2019.

[6] E. Mocanu, D. C. Mocanu, P. H. Nguyen, A. Liotta, M. E. Webber, M. Gibescu, and J. G. Slootweg, "On-line building energy optimization using deep reinforcement learning," *IEEE Trans. on Smart Grid*, vol. 10, no. 4, pp. 3698–3708, Jul. 2019.

[7] B. Kim, Y. Zhang, M. van der Schaar, and J. Lee, "Dynamic pricing and energy consumption scheduling with reinforcement learning," *IEEE Trans. on Smart Grid*, vol. 7, no. 5, pp. 2187–2198, Sep. 2016.

[8] Y. Liang, L. He, X. Cao, and Z. Shen, "Stochastic control for smart grid users with flexible demand," *IEEE Trans. on Smart Grid*, vol. 4, no. 4, pp. 2296–2308, Dec. 2013.

[9] J. Li, C. Zhang, Z. Xu, J. Wang, J. Zhao, and Y. A. Zhang, "Distributed transactive energy trading framework in distribution networks," *IEEE Trans. on Power Systems*, vol. 33, no. 6, pp. 7215–7227, Nov. 2018.

[10] A. K. Zarabie, S. Das, and M. N. Faqiry, "Fairness-regularized dlmp-based bilevel transactive energy mechanism in distribution systems," *IEEE Trans. on Smart Grid*, vol. 10, no. 6, pp. 6029–6040, Nov. 2019.

[11] W. Tushar, B. Chai, C. Yuen, S. Huang, D. B. Smith, H. V. Poor, and Z. Yang, "Energy storage sharing in smart grid: A modified auction-based approach," *IEEE Trans. on Smart Grid*, vol. 7, no. 3, pp. 1462–1475, May 2016.

[12] Y. Wang, W. Saad, Z. Han, H. V. Poor, and T. Başar, "A game-theoretic approach to energy trading in the smart grid," *IEEE Trans. on Smart Grid*, vol. 5, no. 3, pp. 1439–1450, May 2014.

[13] L. Xiao, N. Mandayam, and H. Vincent Poor, "Prospect theoretic analysis of energy exchange among microgrids," *IEEE Trans. on Smart Grid*, vol. 6, no. 1, pp. 63–72, Jan. 2015.

[14] H. Wang and J. Huang, "Incentivizing energy trading for interconnected microgrids," *IEEE Trans. on Smart Grid*, vol. 9, no. 4, pp. 2647–2657, Jul. 2018.

[15] S. Bahrami, M. H. Amini, M. Shafie-Khah, and J. P. S. Catalao, "A decentralized renewable generation management and demand response in power distribution networks," *IEEE Trans. on Sustainable Energy*.

[16] P. Samadi, V. W. S. Wong, and R. Schober, "Load scheduling and power trading in systems with high penetration of renewable energy resources," *IEEE Trans. on Smart Grid*, vol. 7, no. 4, pp. 1802–1812, Jul. 2016.

[17] L. Jiang and S. Low, "Multi-period optimal energy procurement and demand response in smart grid with uncertain supply," in *in Proc. of 2011 50th IEEE Conference on Decision and Control and European Control Conference*, Dec. 2011, pp. 4348–4353.

[18] L. Yu, D. Xie, C. Huang, T. Jiang, and Y. Zou, "Energy optimization of hvac systems in commercial buildings considering indoor air quality management," *IEEE Trans. on Smart Grid*, vol. 10, no. 5, pp. 5103–5113, Sep. 2019.

[19] L. Yu, D. Xie, T. Jiang, Y. Zou, and K. Wang, "Distributed real-time hvac control for cost-efficient commercial buildings under smart grid environment," *IEEE Internet of Things Journal*, vol. 5, no. 1, pp. 44–55, Feb. 2018.

[20] S. Bahrami, M. H. Amini, M. Shafie-khah, and J. P. S. Catalão, "A decentralized electricity market scheme enabling demand response deployment," *IEEE Trans. on Power Systems*, vol. 33, no. 4, pp. 4218–4227, Jul. 2018.

[21] S. Bahrami, Y. C. Chen, and V. W. S. Wong, "An autonomous demand response algorithm based on online convex optimization," in *in Proc. of 2018 IEEE International Conference on Communications, Control, and Computing Technologies for Smart Grids (SmartGridComm)*, Dec. 2018, pp. 1–7.

[22] Y. Kim, "Optimal price based demand response of hvac systems in multizone office buildings considering thermal preferences of individual occupants buildings," *IEEE Trans. on Industrial Informatics*, vol. 14, no. 11, pp. 5060–5073, Nov. 2018.

[23] J. Kazempour and B. F. Hobbs, "Value of flexible resources, virtual bidding, and self-scheduling in two-settlement electricity markets with wind generation—part i: Principles and competitive model," *IEEE Trans. on Power Systems*, vol. 33, no. 1, pp. 749–759, Jan. 2018.

[24] H. Hao, C. Corbin, K. Kalsi, and R. Pratt, "Transactive control of commercial buildings for demand response," *IEEE Trans. on Power Systems*, vol. 32, no. 1, pp. 774–783, Jan. 2017.

[25] Z. Zhao, G. Verbič, and F. Fiorito, "Investigating thermal inertia in lightweight buildings for demand response," in *in Proc. of Australasian Univ. Power Eng. Conf.*, Perth, Australia, Sep. 2014.

[26] W. Wei, F. Liu, and S. Mei, "Charging strategies of ev aggregator under renewable generation and congestion: A normalized Nash equilibrium approach," *IEEE Trans. on Smart Grid*, vol. 7, no. 3, pp. 1630–1641, May 2016.

[27] Y. Shoham and K. Leyton-Brown, *"Multiagent Systems: Algorithmic, Game-theoretic, and Logical Foundations"*. NY: Cambridge University Press, 2008.

[28] J. Rosen, *"Existence and uniqyeness of equilibrium points for concave n-person games"*. Econometrica, 1965.

[29] Power data. [Online]. Available: http://www.ieso.ca/power-data

[30] P. Samadi, H. Mohsenian-Rad, V. Wong, and R. Schober, "Real-time pricing for demand response based on stochastic approximation," *IEEE Trans. on Smart Grid*, vol. 5, no. 2, pp. 789–798, Mar. 2014.

[31] Weather data. [Online]. Available: https://www.wunderground.com /history/daily/ca /toronto/CYTZ/date/2018-9-28

[32] R. Rockafellar and S. Uryasev, *"Optimization of Conditional Value-at-Risk"*. J. of risk, Apr. 2000, vol. 2.