# A stochastic programming approach for the development of offering strategies for a wind power producer

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**Abstract**

A stochastic programming approach is proposed in this paper for the development of offering strategies for a wind power producer. The optimization model is characterized by making the analysis of several scenarios and treating simultaneously two kinds of uncertainty: wind power and electricity market prices. The approach developed allows evaluating alternative production and offer strategies to submit to the electricity market with the ultimate goal of maximizing profits. An innovative comparative study is provided, where the imbalances are treated differently. Also, an application to two new realistic case studies is presented. Finally, conclusions are duly drawn.


*Keywords:* Stochastic programming; Offering strategies; Wind power; Uncertainty

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## 1. Introduction

Nowadays, renewable energy sources play an increasingly important role in electricity production [1–2], since they produce clean energy, respecting the compromise established by the Kyoto protocol. These renewable energy sources can partly replace carbon emitting fossil based electricity generation, and thereby reduce $CO_2$ emissions [3]. Wind, as a renewable energy source, has been generally applied as a means to reach emission reduction goals as a result of increasing concern regarding environmental protection [4]. Actually, wind power is the world's fastest growing renewable energy source [5].

Portugal is a country of the European Union that has highlighted this area of energy innovation, giving a strong stimulus to the national economy by creating new jobs and scientific development based on the area of electrical power systems. In Portugal, the wind power goal foreseen for 2010 was established by the government as 3750 MW, representing about 25% of the total installed capacity in 2010 [6]. This value has recently been raised to 5100 MW, by the most recent governmental goals for the wind sector.

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The Azores islands are transforming into one of Europe's most ecologically innovative regions, by using the region's ample geothermal, hydro and wind resources to make the islands energy independent. The regional government aims to increase the electricity produced using only renewable energies to 75% in the next eight years. According to the regional power company, Electricidade dos Açores (EDA) [7], the current total wind power capacity is 11.6 MW, but significant investments are envisaged to reach 30 MW, thus nearly tripling current wind power generation. The goal is to allow annual savings of fuel oil and reduce emissions of carbon dioxide.

In deregulated markets, wind power producers are entities owning generation resources and participating in the market with the ultimate goal of maximizing profits [8]. The challenges for wind power producers are related to two kinds of uncertainty: wind power and electricity market prices. The uncertain behavior of wind power [9–10], and the large variability of electricity market prices, means a large variability in profit [11]. Thus, the decision makers have to consider these two kinds of uncertainty, as well as the several technical constraints associated to the operation of wind farms.

The offer decisions to submit for the electricity market have to be done in each hour, without knowing exactly what will be the value of power generation. The differences between the produced energy and supplied energy constitute the energy imbalances. The imbalances should be penalized by the market balance [12–13]. A wind power producer needs to know how much to produce in order to make realistic bids, because in case of excessive or moderate bids, other producers must reduce or increase production to fill the so-called deviation, causing economic losses. These economic losses are reflected in so-called costs for deviation or costs of the imbalances.

Bidding and trading of wind power in short-term electricity markets is still a relatively new problem. There are some approaches that allow dealing with the proposed problem applied to electricity markets in Europe. For instance, in [14–18] power producers seek to minimize their deviation losses using a portfolio approach, where the producers have the possibility of combining their wind production resources with energy storage technologies in order to submit a proposal for combined bids. Another approach is presented in [19], proposing an optimal bidding strategy by assuming that the wind production time series is a Markov process. In [20] the Dutch market is simulated, comparing the performances of using point predictions with probabilistic predictions.

In addition to the approaches discussed above, the stochastic models have been intended to produce optimal offering strategies for a wind power producer, introducing significant advances related with improvements in the mathematical formulation of the problem. Stochastic optimization tools can be used to avoid an excessively penalizing treatment for wind energy in order to allow optimal wind energy trading in the market.

The innovative contribution in this paper is to present and compare two different stochastic optimization approaches for trading wind power in short-term electricity markets, where the imbalances are treated differently. On the one hand, the first approach considers a cost factor over the market price for energy imbalances. On the other hand, the second approach considers the ratio between imbalance price and day-ahead market price. Our study will show not only the impact that a realistic modeling of the imbalances' penalization has in the total profit of the wind power producer, but also will allow defining the most appropriate bidding strategies. Moreover, an application to two new realistic case studies is presented: a wind farm in Portugal located in the Vila Real region, and an isolated power grid as occurs in the Azores islands.

To take into account the uncertainty of wind power and electricity market prices, multiple scenarios can be built using wind power forecasting [21–23] and electricity price forecasting [24–26] tools. A scenario tree represents the different stages that can take the random parameters, i.e., different realizations of uncertainty. The tree is a natural and explicit way of representing nonanticipativity decisions.

The stochastic nature of wind power and electricity market prices can be modeled through a stochastic programming approach [27–29]. In this approach, the set of decisions inherent to the problem can be divided into two distinct stages: first-stage decisions, which must be taken before resolving the uncertainty; second-stage decisions, which are made after the uncertainty occurs and are influenced by decisions taken in first stage. The first-stage decisions correspond to the hourly bids to be submitted to the day-ahead market, while the second-stage decisions correspond to the operation of the wind farm for each possible realization of the random variables (the electricity market price, the wind power generation, and the price for imbalance). According with [14], the random parameters can be merged into single scenarios because there are no decision variables between the market-clearing (i.e., when price uncertainty is unveiled) and the moment in which a better wind production forecast becomes available.

Fig. 1 shows the scenario tree that will be used to represent the decisions to be taken in the two stages mentioned.

"See Fig. 1 at the end of the manuscript".

The root of the tree represents the first-stage decisions where the hourly bids are shared, i.e., the bids are the same for all scenarios, since they are independent on them. Therefore, they represent a robust solution to face uncertainty. Still, this solution does not have to be optimal in any particular scenario if it is considered alone, but flexible for all scenarios [14]. In second-stage the decisions are made based on scenarios of prices and of wind power, spanning the whole market horizon. This stage is also defined by the materialization of the imbalance prices and the wind power generated in the time periods spanning the whole market horizon. So, the deviation incurred by the wind power producer in each one of these periods is known and the consequent cost for imbalance can be computed [8].

A stochastic programming approach has been used because it has the advantage of finding a near optimal solution considering all possible scenarios. The stochastic solution may not be a global optimal solution to the individual scenarios, but it is a robust solution over all possible realizations of the uncertainties.

In particular, a stochastic programming approach is proposed to generate the optimal offers under uncertainty in wind generation and electricity prices. The optimal offers generated should be submitted to the day-ahead market by a wind power producer, in order to maximize its expected profit. Hence, the main advantage of using the proposed approach is dealing in the same optimization model with:

(i) Offers submitted to the day-ahead market;

(ii) Wind power production;

(iii) Profit maximization;

(iv) Imbalance costs.

This paper is structured as follows. In Section 2, the mathematical formulation of the problem is provided. Section 3 presents the proposed stochastic programming approach. In Section 4, the proposed stochastic programming approach is applied on two new realistic case studies, to demonstrate its effectiveness. Finally, Section 5 outlines the conclusions.

## 2. Problem formulation

The notation used throughout the paper is stated as follows.

| | |
|---|---|
| $S, s$ | Set and index of scenarios. |
| $H, h$ | Set and index of hours in the time horizon. |
| $\rho_s$ | Probability of occurrence of scenario $s$. |
| $\lambda_{sh}$ | Expected market price in scenario $s$ in period $h$. |
| $r^+_{sh}$ | Ratio between positive imbalance price and day-ahead market price in scenario $s$ in period $h$. |
| $r^-_{sh}$ | Ratio between negative imbalance price and day-ahead market price in scenario $s$ in period $h$. |
| $p_{sh}$ | Power output of the wind farm in scenario $s$ in period $h$. |
| $v$ | Cost factor over the market price for energy imbalances. |
| $x_h$ | Offer by the wind power producer in the day-ahead market for time period $h$. |
| $dev_{sh}$ | Deviation for wind production in scenario $s$ in period $h$. |
| $Pdev_{sh}$ | Cost for deviation of the wind farm in scenario $s$ in period $h$. |
| $W_{sh}$ | Wind generation forecast in scenario $s$ in period $h$. |
| $P^{\max}$ | Maximum power of the wind farm. |
| $L_{sh}$ | Revenue in scenario $s$ in period $h$. |
| $\boldsymbol{c}$ | Vector of coefficients for the linear term for first-stage. |
| $\boldsymbol{x}$ | Vector of decision variables. |
| $\boldsymbol{A}$ | Matrix of coefficients for the first-stage constraints. |
| $\boldsymbol{b}^{\min}, \boldsymbol{b}^{\max}$ | Lower and upper bound vectors for the first-stage constraints. |
| $\boldsymbol{x}^{\min}, \boldsymbol{x}^{\max}$ | Lower and upper bound vectors on variables. |
| $\boldsymbol{h}_{\boldsymbol{\varpi}}^{\min}, \boldsymbol{h}_{\boldsymbol{\varpi}}^{\max}$ | Lower and upper bound vectors for the second-stage constraints. |
| $\boldsymbol{T}_{\boldsymbol{\varpi}}$ | Technology matrix. |
| $\boldsymbol{W}_{\boldsymbol{\varpi}}$ | Recourse matrix. |

$\boldsymbol{q_\omega}$ Vector of coefficients for the linear term for the second-stage variables.

$\boldsymbol{y_\omega}$ Second-stage variables that represent decisions to be made after part of the uncertainty is revealed.

*2.1 Imbalance prices*

The inclusion of wind power into the power system has a significant impact on the need for real-time balancing power due to the stochastic nature of the wind power production [30]. In scheduling, the main goal is to ensure that sufficient generation is available for days ahead of the real-time operation. Hence, the optimal bidding strategies taken into the day-ahead and real-time markets will influence the revenues, imbalance costs, and affect the overall profit for the wind power producer [31].

The forecasting errors, which are the mismatch between what is used in the unit commitment stage and the real-time dispatch, may cause great difficulty for system operators to balance the unexpected surplus or deficit of wind power. Once the treatment of the imbalances assume an important role to maintain the electrical system safe and efficient, the imbalance prices are represented by several equally probable scenarios in the planning stage in order to ensure a realistic model for the wind power producer.

Any difference (imbalance) between the actual and scheduled wind power output during the operating hour will be settled at the hourly real-time price. Imbalance prices result from the system balancing market, in which the price of the energy replacing the deviation is determined [8].

The imbalance price can be different depending on if the balance responsible player is in positive or negative imbalance and if upward or downward regulation was undertaken by the system operator during the hour in question. The following imbalance price model is used.

The settlement mechanism for imbalance prices can vary from one country to the other; however the general principles remain the same:

- $\lambda_h^+ = \lambda_h$, where $\lambda_h$ is a day-ahead market price at hour *h*, if the player is in positive imbalance in hour *h* and no downward regulation is undertaken;
- $\lambda_h^+ = \lambda_h^{DN}$, where $\lambda_h^{DN}$ is a price for downward regulation at hour *h*, if the player is in positive imbalance in hour *h* and downward regulation is undertaken with $\lambda_h^{DN} < \lambda_h$;

- $\lambda_h^- = (-)\,\lambda_h^{UP}$, where $\lambda_h^{UP}$ is a price for upward regulation at hour *h*, if the player is in negative imbalance in hour *h* and upward regulation is undertaken with $\lambda_h^{UP} > \lambda_h$;
- $\lambda_h^- = (-)\,\lambda_h$, if the player is in negative imbalance in hour *h* and no upward regulation is undertaken or $\lambda_h^{UP} < \lambda_h$.

The negative sign means that the balance responsible player is paying the price for imbalance, and the positive sign means that the balance responsible player is getting paid [32].

The following equations define the parameters $r_h^+$ and $r_h^-$ that correspond to the ratios between the positive and negative imbalance prices and the day-ahead market price [8]:

$$r_h^+ = \frac{\lambda_{\text{h}}^+}{\lambda_{\text{h}}}, \qquad 0 \le r_h^+ \le 1 \tag{1}$$

$$r_h^- = \frac{\lambda_{\text{h}}^-}{\lambda_{\text{h}}}, \qquad r_h^- \ge 1 \tag{2}$$

Note that definitions (1) and (2) are valid under the assumption that the hourly electricity prices in the day-ahead market are positive.

The positive and negative ratios can be related through ratio $r_h$, defined by the linear expression $r_h = r_h^+ + r_h^- - 1$, where $r_h = \{r_h^+, \text{ if } r_h^+ \ne 1;\ r_h^-, \text{ if } r_h^+ = 1\}$. Even if $r_h^+ \ne 1$, then $r_h^- = 1$, and *vice versa*. So, the ratio series $r_h^+$ and $r_h^-$ can be easily derived from $r_h$ taking into account the mechanism for imbalance prices. This modelling allows simulating the imbalance price ratio scenarios after knowing the day-ahead and balancing markets prices. Following the definition of the different components of the imbalance price ratios, the uncertainty is modeled in this paper through a scenario tree that is built as follows: i) generate $N_\lambda$ price scenarios for the day-ahead market; ii) for each scenario of the market prices, generate $N_W$ wind power realizations; iii) for each wind power realization, simulate $N_r$ imbalance price ratio scenarios. Hence, the total number of scenarios composing the tree is $S = N_\lambda N_W N_r$.

### *2.2 Objective function*

The objective function to be maximized can be expressed as:

$$F = \sum_{s=1}^{S} \rho_s \sum_{h=1}^{H} \left[ \lambda_{sh}\, p_{sh} - Pdev_{sh} \right] \tag{3}$$

The objective function (3) to be maximized includes the expected profit, where $S$ is the set of scenarios, $\rho_s$ is the probability of occurrence of scenario $s$, $H$ is the set of hours in the time horizon, $\lambda_{sh}$ is the forecasted electricity market price in scenario $s$ in period $h$, $p_{sh}$ is the power output of the wind farm in scenario $s$ in period $h$, and $Pdev_{sh}$ is the cost for deviation of the wind farm in scenario $s$ in period $h$.

The deviations can be generated by excess or deficit of energy:

$$dev_{sh} = p_{sh} - x_h \tag{4}$$

The cost for deviation corresponds to the product of the price for the shifted power in absolute value:

$$Pdev_{sh} = \begin{cases} \lambda_{\text{sh}}\, r_{sh}^{+}\, dev_{sh}, & dev_{sh} \geq 0 \\ \lambda_{\text{sh}}\, r_{sh}^{-}\, dev_{sh}, & dev_{sh} < 0 \end{cases} \tag{5}$$

The revenue is given by the product of the expected energy market price by the power output of the wind farm:

$$L_{sh} = \lambda_{sh}\, p_{sh} \tag{6}$$

The expected profit is calculated as the difference between the revenue of the wind farm and the cost for deviation.

Substituting (5) into (3) gives:

$$F = \sum_{s=1}^{S} \rho_s \sum_{h=1}^{H} \left[ \lambda_{sh}\, p_{sh} - \lambda_{sh}\, r_{sh}^{+}\, d_{sh}^{+} - \lambda_{sh}\, r_{sh}^{-}\, d_{sh}^{-} \right] \tag{7}$$

*2.3 Constraints*

The total deviation (imbalance) $dev_{sh}$ is decomposed as the sum of positive and negative imbalances, $d_{sh}^{+}$ and $d_{sh}^{-}$, respectively. For a total deviation $dev_{sh} = d_{sh}^{+} - d_{sh}^{-}$, the optimal solution is guaranteed to be achieved with one of the variables $d_{sh}^{+}$ or $d_{sh}^{-}$ equal to zero, due to the fact that $r_{sh}^{+} \leq 1$ and $r_{sh}^{-} \geq 1$:

$$p_{sh} - x_h - d_{sh}^{+} + d_{sh}^{-} = 0 \tag{8}$$

In order to make the offers to the market, it is required to satisfy the technical limitations of the wind farm. So, the optimal value of the objective function is determined subject to inequality constraints or simple bounds on the variables.

The constraints are indicated as follows:

$$0 \le p_{sh} \le W_{sh} \tag{9}$$

$$0 \le x_h \le P^{\max} \tag{10}$$

The hourly wind generation in each scenario is given by the forecast $W_{sh}$ provided for the scenario considered (9), and the offers are limited to the installed capacity $P^{\max}$ of the wind farm (10), as in [14–15]. Maximum positive deviations occur in scenarios where the wind power producer does not offer any amount of wind power in the day-ahead market for period *h*, i.e. $x_h = 0$, and the power output of the wind farm is equal to $W_{sh}$ during that period. Maximum negative deviations occur in scenarios where the wind power producer offers its full capacity in the day-ahead market for period *h*, i.e. $x_h = P^{\max}$.

*2.4 Linearization of the objective function*

The objective function, presented in the previous subsection, is characterized by nonlinearity due to the existence of the absolute value. So, it is required to use a mathematical process that allows reformulating the linear problem.

In this subsection, the problem involving absolute value terms is transformed into a standard linear programming formulation.

Initially, it is considered:

$$Max \ \boldsymbol{F} = \boldsymbol{c}^T \boldsymbol{x} - |\, \boldsymbol{x} \,| \tag{11}$$

subject to $$\boldsymbol{x}^{\min} \le \boldsymbol{x} \le \boldsymbol{x}^{\max} \tag{12}$$

$$\boldsymbol{x} \in R^n \tag{13}$$

In (11), $F(\cdot)$ is the objective function of decision variables, where $\boldsymbol{c}$ is the vector of coefficients for the linear term. In (12), $\boldsymbol{x}^{\min}$ and $\boldsymbol{x}^{\max}$ are the lower and upper bound vectors on variables. The variable $\boldsymbol{x}$ is a set of decisions variables.

Subsequently, absolute-valued variables are replaced with two strictly positive variables:

$$|\, \boldsymbol{x} \,| = \boldsymbol{x}^{+} + \boldsymbol{x}^{-} \tag{14}$$

In addition, each variable is substituted by the difference of the same two positive variables, as:

$$\boldsymbol{x} = \boldsymbol{x}^{+} - \boldsymbol{x}^{-} \tag{15}$$

The equivalent linear programming problem is given by:

$$Max\ \boldsymbol{F} = \boldsymbol{c}^{T}\boldsymbol{x} - (\boldsymbol{x}^{+} + \boldsymbol{x}^{-}) \tag{16}$$

subject to

$$\boldsymbol{x}^{\min} \leq \boldsymbol{x} \leq \boldsymbol{x}^{\max} \tag{17}$$

$$\boldsymbol{x} = \boldsymbol{x}^{+} - \boldsymbol{x}^{-} \tag{18}$$

$$\boldsymbol{x}^{+} \geq 0\ ,\ \boldsymbol{x}^{-} \geq 0 \tag{19}$$

## 3. Proposed approach

Stochastic programs are among the most challenging optimization problems. The stochasticity in the parameters appears in this approach due to the uncertainty, which is modeled via a finite set of scenarios.

### *3.1 Stochastic programming*

The stochastic programming model can be formulated as:

$$Max\quad \boldsymbol{c}^{T}\boldsymbol{x} + \boldsymbol{E}[\max_{y_{\omega}} \boldsymbol{q}_{\boldsymbol{\omega}}^{T}\ \boldsymbol{y}_{\boldsymbol{\omega}}] \tag{20}$$

subject to

$$\boldsymbol{b}^{\min} \leq \boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{b}^{\max} \tag{21}$$

$$\boldsymbol{h}_{\boldsymbol{\omega}}^{\min} \leq \boldsymbol{T}_{\boldsymbol{\omega}}\ \boldsymbol{x} + \boldsymbol{W}_{\boldsymbol{\omega}}\ \boldsymbol{y}_{\boldsymbol{\omega}} \leq \boldsymbol{h}_{\boldsymbol{\omega}}^{\max} \tag{22}$$

$$\boldsymbol{x} \geq 0,\quad \boldsymbol{y}_{\boldsymbol{\omega}} \geq 0. \tag{23}$$

where $\boldsymbol{c}$ is a known vector of the objective function coefficients for the $x$ variables in the first-stage, $\boldsymbol{b}^{\min}, \boldsymbol{b}^{\max}$ are the lower and upper bound vectors for the first-stage constraints, and $\boldsymbol{A}$ is the known matrix of coefficients for the first-stage constraints. For each $\omega$, $\boldsymbol{h}_{\boldsymbol{\omega}}^{\min}, \boldsymbol{h}_{\boldsymbol{\omega}}^{\max}$ are the vectors for the second-stage constraints, and $\boldsymbol{q}_{\boldsymbol{\omega}}$ is the vector of the objective function coefficients for the $y$ variables, while $\boldsymbol{T}_{\boldsymbol{\omega}}$ is the technology matrix and $\boldsymbol{W}_{\boldsymbol{\omega}}$ is the recourse matrix under scenario $\omega$. Finally, $E_{y_{\omega}}$ represents the expectation with respect to $\omega$ over the set of scenarios $S$. In the second-stage, the constraints and right-hand sides are permitted to be random. The matrix that links the random parameters related to first-stage variables is called technology matrix.

In the first-stage, the “here-and-now” decision should be taken, before the uncertainties represented by $x$ are known. In the second-stage, where the information $x$ is already available, the decision is made about the value of the vector $y$.

The first-stage decision of $x$ depends only on the information available until that time; this principle is called nonanticipativity constraint. The problem of two stages means that the decision $x$ is independent of the achievements of the second-stage, and thus the vector $x$ is the same for all possible events that may occur in the second-stage of the problem.

The first-stage decisions, corresponding to $x$, are the hourly bids to be submitted to the day-ahead market and the second-stage decisions, corresponding to $y$, are related to the operation of the wind farm for each possible realization of the random variables.

*3.2 Deterministic equivalent programming*

The stochastic model is usually a difficult computational problem, so it is common to choose the deterministic model solution using the average of random variables or solving a deterministic problem for each scenario.

The problem shown in the previous subsection is equivalent to the so-called deterministic equivalent programming that in the splitting variable representation is as follows:

$$Max_{x,y_s} \quad \boldsymbol{c}^T \boldsymbol{x} + \sum_{s=1}^{S} \rho_s \boldsymbol{q}_s^T \boldsymbol{y}_s \tag{24}$$

$$\text{subject to} \qquad \boldsymbol{b}^{\min} \le \boldsymbol{A}\boldsymbol{x} \le \boldsymbol{b}^{\max} \tag{25}$$

$$\boldsymbol{h}_s^{\min} \le \boldsymbol{T}_s\, \boldsymbol{x} + \boldsymbol{W}_s\, \boldsymbol{y}_s \le \boldsymbol{h}_s^{\max} \quad \text{for} \quad s = 1,\ldots,S \tag{26}$$

$$\boldsymbol{x} \ge 0, \quad \boldsymbol{y}_s \ge 0. \qquad \text{for} \quad s = 1,\ldots,S \tag{27}$$

The matrix composed by (25) and (26), for large-scale linear problems can be generally represented according with Fig. 2.

"See Fig. 2 at the end of the manuscript".

**4. Case studies**

The proposed approach has been developed and implemented in MATLAB and solved using the optimization solver package CPLEX. The numerical simulation has been performed on a 2-GHz based processor with 2GB of RAM.

Two new realistic case studies are provided to illustrate the proficiency of the proposed approach. These cases are defined by:

- Case A. Based on a wind farm in Portugal located in the Vila Real region;
- Case B. For an isolated power grid, as occurs in the Azores islands.

### *4.1 Case A*

The proposed approach has initially been applied on a realistic case study based on a wind farm in Portugal located in the Vila Real region. The total installed wind power capacity considered is 270 MW.

Two different approaches have been implemented:

- Approach 1. Considering the cost factor over the market price for energy imbalances, i.e., $Pdev_{sh} = \nu\,\lambda_{sh}\,dev_{sh}$, such that the objective function is $F = \sum_{s=1}^{S} \rho_s \sum_{h=1}^{H} \left[\lambda_{sh}\,p_{sh} - \nu\,\lambda_{sh}\left|p_{sh} - x_h\right|\right]$ subject to constrains (8)–(10);
- Approach 2. Considering the ratio between imbalance price and day-ahead market price.

In the first approach, the deviation cost has been fixed at 30% of the daily market price, $\nu = 0.3$. This is a simplification of the deviation cost assessment defined by the Iberian market operator. In this assessment procedure, the deviations costs obey to rules described in [33]. The number of scenarios generated for the day-ahead market in the optimization problem is $\mathrm{N} = 100$.

The second approach takes into account the uncertainty not only regarding wind power and electricity market prices, but also regarding the imbalance price ratio scenarios. Thus, the total number of scenarios generated in the second approach is $\mathrm{N} = 1000$. Imbalance pricing provides an incentive for market participants to execute their programs as planned, while at the same time it provides an incentive for market participants to respond to the secondary control signal received from the system operator.

The wind power scenarios are modeled through the method proposed in [23], where the historical wind power data for year 2009, available at [34], are the main inputs to train the artificial neural network (ANN). Therefore, the ANN approach is used to predict random scenarios using real production data. Regarding to electricity prices scenarios, they are modeled using the method described in [24] for the Spanish market. The proposed approaches have been tested with the data belonging to the electricity market of the Iberian Peninsula [35], available at [36] and corresponding to February 2009, in order to obtain the spot market and imbalance prices scenarios.

Note that the aim is not to forecast imbalance prices for a particular day and to assess its accuracy, but instead to simply simulate possible imbalance price scenarios.

Initially, the time horizon chosen is one day divided into 24 hourly periods. Uncertainties in hourly wind power generation, day-ahead electricity prices are all represented in terms of scenarios. The electricity market prices scenarios are shown in Fig. 3, while the wind power scenarios are shown in Fig. 4.

"See Fig. 3 at the end of the manuscript".

"See Fig. 4 at the end of the manuscript".

Figs. 5 and 6 show the imbalance price ratios scenarios considering, respectively, the negative system imbalance ( $r_h^-$ ) and the positive system imbalance ( $r_h^+$ ).

"See Fig. 5 at the end of the manuscript".

"See Fig. 6 at the end of the manuscript".

The optimal bids for the two approaches are shown in Fig. 7. The dashed lines denote the results for the first approach, while the solid lines denote the results for the second approach.

"See Fig. 7 at the end of the manuscript".

According to Fig. 7, it can be seen that the optimal offers for the first approach are higher than for the second approach, since imbalance prices are simply modeled as proportional to the spot market price.

Choosing one scenario of the problem, it can be verified in Figs. 8 and 9 that the wind farm adjusts its production in order to reduce its own imbalance costs which may in turn lead to an increased system balancing efficiency. Nevertheless, in almost every hour there are differences between the offers and the power output of the wind farm. The consideration of uncertain information on wind power forecasts, to estimate the imbalance costs, reveals more realistic results considering the rules of the electricity market, as was presented in Section 2.

"See Fig. 8 at the end of the manuscript".

"See Fig. 9 at the end of the manuscript".

The deviations from generated power, for this scenario, are shown in Figs. 10 and 11 for the first and second approaches, respectively.

"See Fig. 10 at the end of the manuscript".

"See Fig. 11 at the end of the manuscript".

Table 1 provides the expected value of profit for both approaches. Although the profit may be higher for the first approach, it should be noted that the second approach provides more realistic results. Hence, the profit is overestimated if we consider the first approach.

"See Table 1 at the end of the manuscript".

The dispersion of profit considering both approaches is show in Fig. 12. Although the expected profit is higher for the first approach, as mentioned previously, the dispersion of profit is also much more considerable than in the second approach.

"See Fig. 12 at the end of the manuscript".

Note that there are hours when the deviation prices are so high, that the losses in these hours may be higher than the benefits of the rest of the year. And this may happen every year for a few hours. Hence, the results of Fig. 12 may change for other days.

The proposed approaches have also been applied on the case study considering a time horizon of two days divided into 48 hourly periods. These electricity market prices scenarios are shown in Fig. 13.

"See Fig. 13 at the end of the manuscript".

Both approaches have been successfully tested, as shown in Table 2. Hence, it has been demonstrated that the performed simulations can be easily applied on longer time horizons, since the computation time scales up linearly with the number of hours.

"See Table 2 at the end of the manuscript".

### *4.2 Case B*

A second realistic case study for an isolated power grid is also taken into account, as occurs in the Azores islands with ever-increasing wind energy penetration. The total installed wind power capacity considered is 11.6 MW.

The electricity market prices scenarios are computed by the approach proposed in [24], Fig. 14, while the wind power scenarios are computed by the approach proposed in [23], Fig. 15.

"See Fig. 14 at the end of the manuscript".

"See Fig. 15 at the end of the manuscript".

In this case, the optimal bids for the two approaches are shown in Fig. 16. The dashed lines denote the results for the first approach, while the solid lines denote the results for the second approach.

"See Fig. 16 at the end of the manuscript".

The expected value of profit for the first approach is 1896 Eur, and for the second approach is 1832 Eur. As mentioned previously, the profit is overestimated if we consider the first approach. More realistic results are provided for the second approach.

## 5. Conclusion

The new environment of competitive electricity markets for energy requires new computing tools to allow wind power producers to achieve better offering strategies. In this paper, the goal is to maximize the profit of a wind power producer, reducing deviations, and taking into account the uncertainty associated with wind energy production and electricity market prices, using stochastic optimization. Two new realistic case studies are provided: a wind farm in Portugal located in the Vila Real region, and an isolated power grid as occurs in the Azores islands. Also, two different approaches have been successfully implemented, considering the cost factor over the market price for energy imbalances, or considering the ratio between imbalance price and day-ahead market price. This second approach provides more realistic results, while the profit can be overestimated using the first approach. The innovative comparative results presented in this paper illustrate the importance of a correct modeling of imbalance prices for the optimal participation of wind energy in spot electricity markets. The possible participation of wind farms in the adjustment markets represents future work.

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**Figure captions**

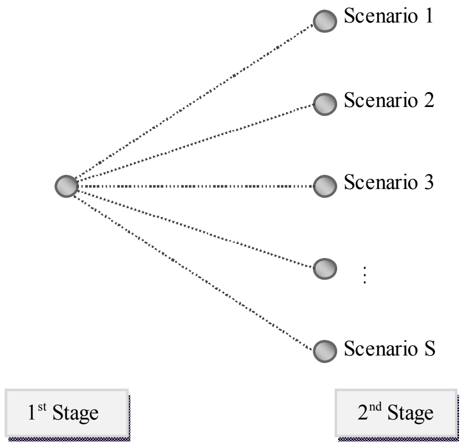


Fig. 1. Scenario tree.

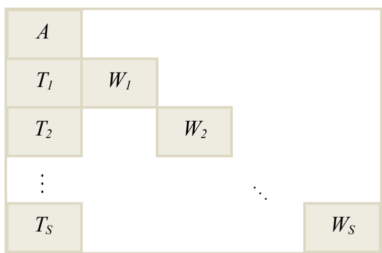


Fig. 2. Layout of the constraints associated with two stages.

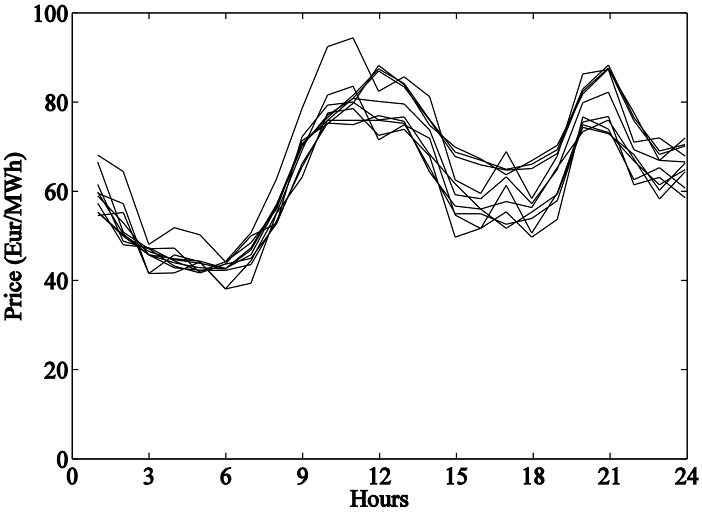


Fig. 3. Electricity market price scenarios considered in case study A.

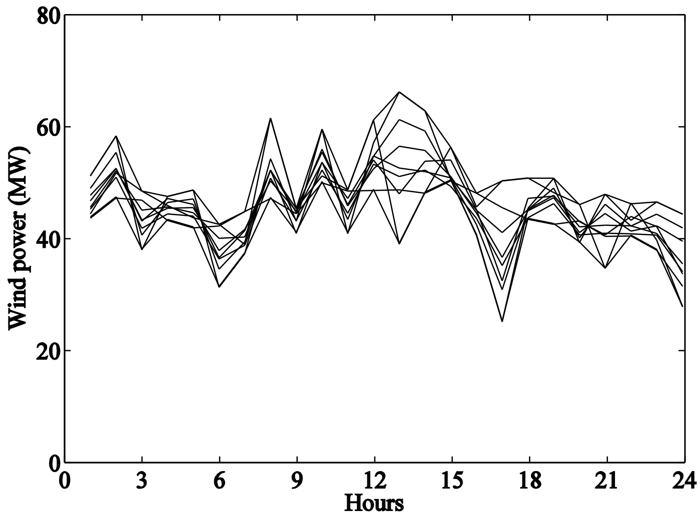


Fig. 4. Wind power scenarios considered in case study A.

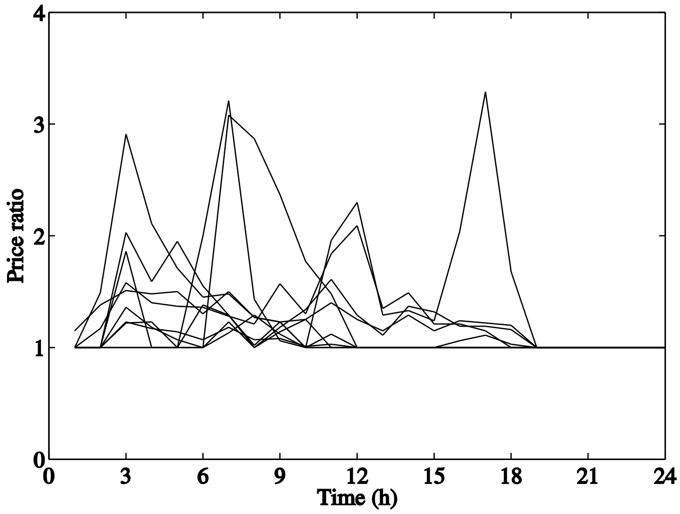


Fig. 5. Representation of imbalance price ratios (negative system imbalance).

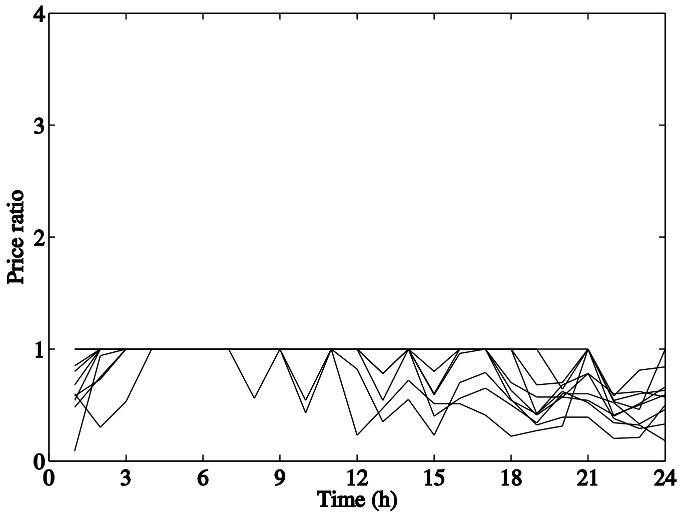


Fig. 6. Representation of imbalance price ratios (positive system imbalance).

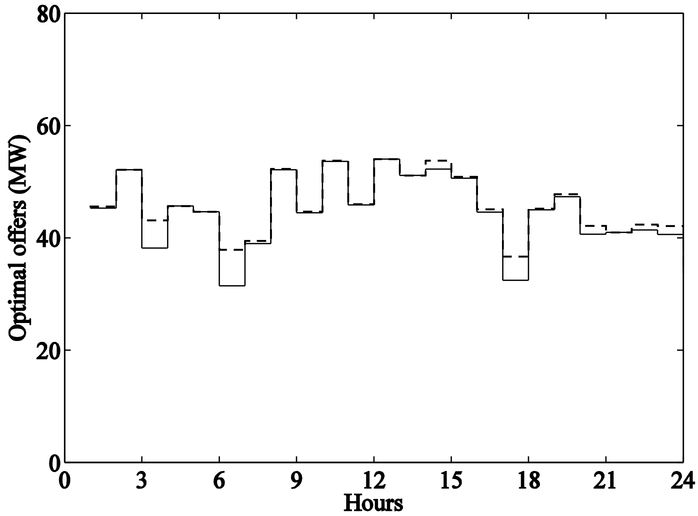


Fig. 7. Optimal hourly bids for case study A. The dashed lines denote the results for the first approach, while the solid lines denote the results for the second approach.

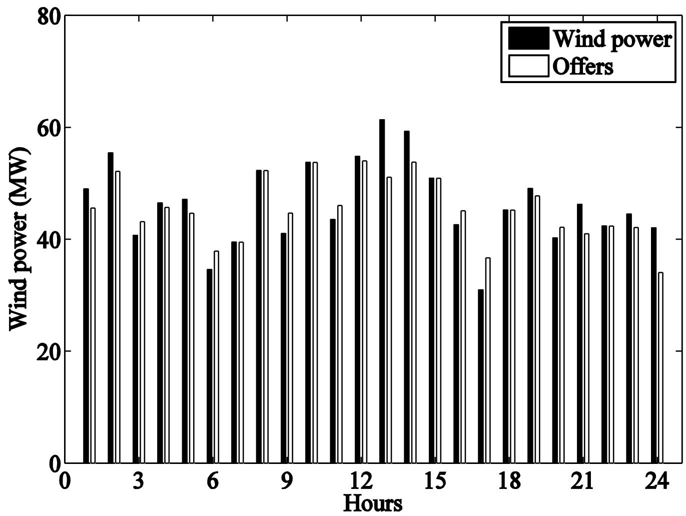


Fig. 8. Optimal offers to be submitted to the day-ahead market and power produced, for the first approach within case study A.

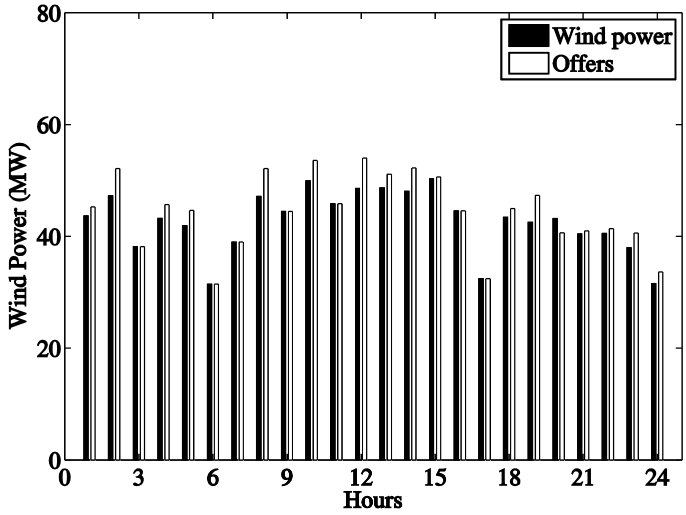


Fig. 9. Optimal offers to be submitted to the day-ahead market and power produced, for the second approach within case study A.

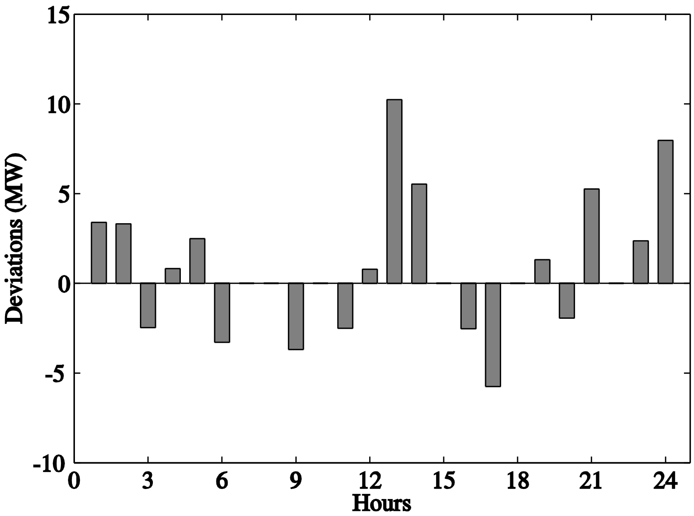


Fig. 10. Deviations resulting from the difference between the power produced and the offers, for the first approach within case study A.

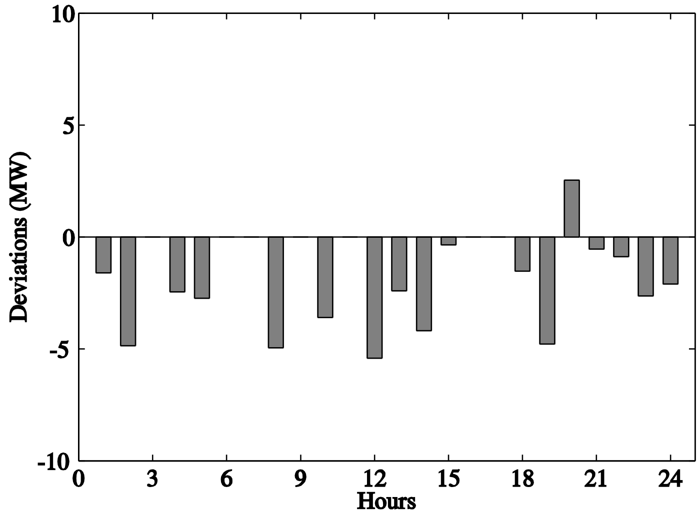


Fig. 11. Deviations resulting from the difference between the power produced and the offers, for the second approach within case study A.

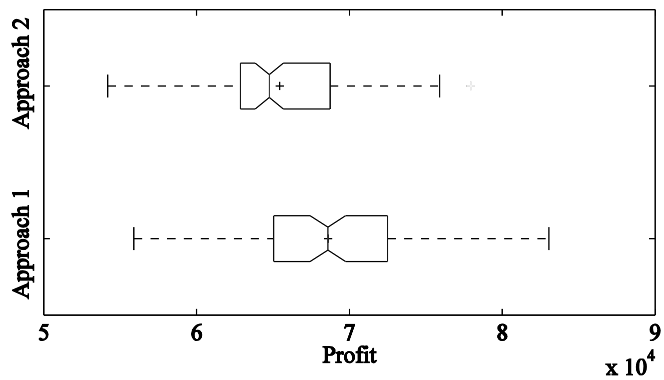


Fig. 12. Dispersion of profit considering both approaches within case study A.

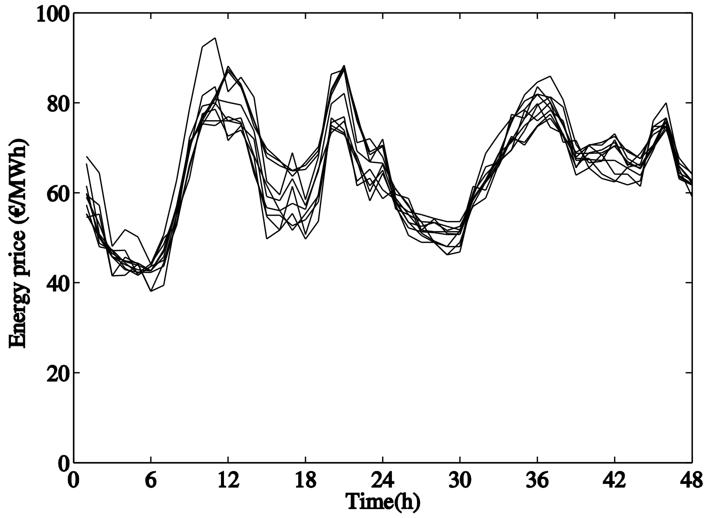


Fig. 13. Electricity market price scenarios for the two days within case study A.

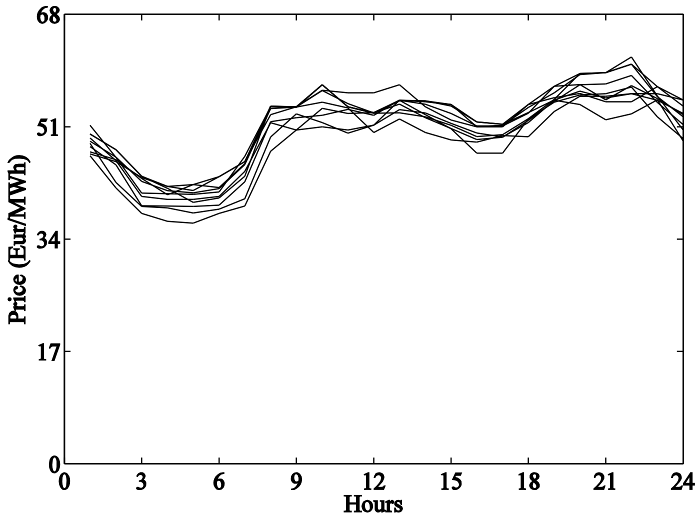


Fig. 14. Electricity market price scenarios considered in case study B.

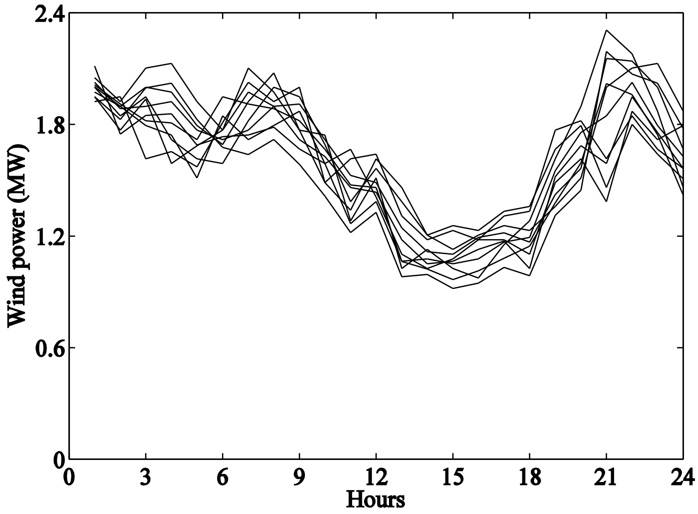


Fig. 15. Wind power scenarios considered in case study B.

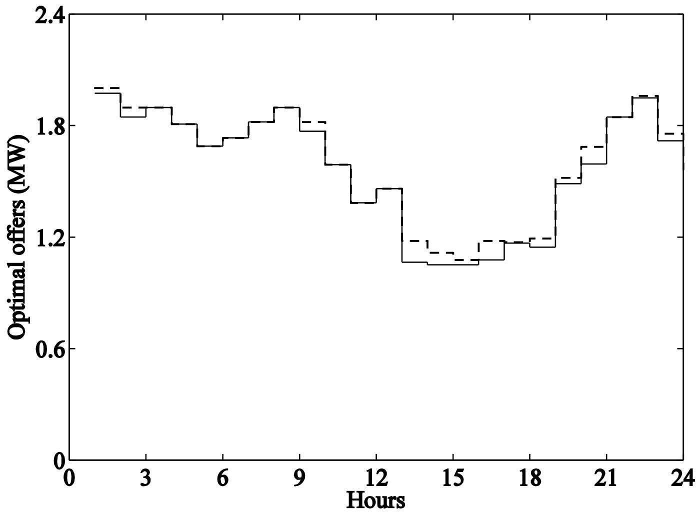


Fig. 16. Optimal hourly bids for case study B. The dashed lines denote the results for the first approach, while the solid lines denote the results for the second approach.

**Tables**

Table 1

Comparison of results for case study A with a 24 hours time horizon

| Approach | Expected profits (Eur) | CPU Time (s) |
|---|---|---|
| Approach 1 | 68598 | 0.78 |
| Approach 2 | 65438 | 1.43 |

Table 2

Comparison of results for case study A with a 48 hours time horizon

| Approach | Expected profits (Eur) | CPU Time (s) |
|---|---|---|
| Approach 1 | 141610 | 1.59 |
| Approach 2 | 136340 | 3.22 |