Mixed-integer nonlinear approach for the optimal scheduling of a head-dependent hydro chain

J.P.S. Catalão [a,*], H.M.I. Pousinho [a], V.M.F. Mendes [b]

[a] *Department of Electromechanical Engineering, University of Beira Interior, R. Fonte do Lameiro, 6201-001 Covilha, Portugal*
[b] *Department of Electrical Engineering and Automation, Instituto Superior de Engenharia de Lisboa, R. Conselheiro Emídio Navarro, 1950-062 Lisbon, Portugal*

Received 15 December 2008; received in revised form 31 October 2009

Abstract

This paper is on the problem of short-term hydro scheduling (STHS), particularly concerning a head-dependent hydro chain. We propose a novel mixed-integer nonlinear programming (MINLP) approach, considering hydroelectric power generation as a nonlinear function of water discharge and of the head. As a new contribution to earlier studies, we model the on-off behavior of the hydro plants using integer variables, in order to avoid water discharges at forbidden areas. Thus, an enhanced STHS is provided due to the more realistic modeling presented in this paper. Our approach has been applied successfully to solve a test case based on one of the Portuguese cascaded hydro systems with a negligible computational time requirement.

Keywords: Short-term hydro scheduling (STHS); Mixed-integer nonlinear programming (MINLP); Cascaded reservoirs; Variable head

---

1. Introduction

In this paper, the short-term hydro scheduling (STHS) problem of a head-dependent hydro chain is considered. Hydro plants with a small storage capacity are known as run-of-the-river. Typically, run-of-the-river hydro plants are considered to operate under stationary conditions with constant head and at the maximum water level in the reservoirs, corresponding by design to the optimum efficiency operating point. However, it is often desirable to change this policy, thus incurring into head changes. The operating efficiency is sensitive to the head — head change effect [1].

Significant loss of efficiency can occur in operating hydro plants away from their most efficient operating point. Thus, hydroelectric power generation has to be considered as a function of water discharge and also of the head in order to avoid this loss of efficiency, that is, in order to take in account the head change effect.

* Corresponding author. Tel.: +351 275 329914; fax: +351 275 329972.
E-mail address: catalao@ubi.pt (J.P.S. Catalão).

In a run-of-the-river cascaded hydraulic configuration an upstream reservoir highly influences the operation of the next downstream reservoir. The latter reservoir also influences the upstream plant by its effect on the tail water elevation and effective head [2]. Actually, the cascaded hydraulic configuration coupled with the nonlinear head change effect, augments the problem dimension and the complexity, but they should be considered because they are important for the most advantageous management of the conversion of the potential energy available in the reservoirs into electric energy.

In a competitive environment, such as the Norwegian case [3], the most advantageous management of the conversion of the potential energy available in the reservoirs into electric energy, without affecting future operation use, represents a major advantage for the hydroelectric utilities to face competition [4]. STHS models provide decision support for the operational task of bidding in the energy and system services markets [5].

Hydro plants particularly run-of-the-river hydro plants are considered to provide an environmentally friendly energy option, while fossil-fuelled plants are considered to provide an environmentally aggressive energy option, but nevertheless still in nowadays a necessary option [6]. The Portuguese fossil fuels energy dependence is among the highest in the European Union. Portugal does not have endogenous thermal resources, which has a negative influence on Portuguese economy. Moreover, the Portuguese greenhouse emissions are already out of Kyoto target and must be reduced in the near future. Hence, promoting efficiency improvements in the exploitation of the Portuguese hydro resources reduces the reliance on fossil fuels and decreases greenhouse emissions.

In the STHS optimization problem a time horizon of 1 to 7 days is considered, usually discretized in hourly periods. Hence, the STHS problem is treated as a deterministic one. Where the problem includes stochastic quantities such as inflows to reservoirs or energy prices, the corresponding forecasts are used. STHS is guided by specified hourly weighting factors, quantifying the energy price at each hour [7].

Dynamic programming (DP) is among the earliest methods applied to the STHS problem [8,9]. Although DP can handle the non-convex, nonlinear characteristics present in the hydro model, it suffers from the well-known curse of dimensionality, more difficult to avoid in short-term than in long-term optimization without losing the accuracy needed in the model [10]. For hydro systems with cascaded reservoirs, CPU-time and memory requirements expand exponentially with problem size making DP unsuitable.

Artificial intelligence techniques have also been applied to the STHS problem [11–14]. However, a significant computational effort is necessary to solve the problem for cascaded hydro systems and a time horizon of 168 intervals. Also, a fixed head is usually assumed in order to simplify the problem [11].

A natural approach to STHS is to model the system as a network flow model, because of the underlying network structure subjacent in hydro chains [15]. This network flow model is often simplified to a linear or piecewise linear one [16]. Linear programming (LP) is a well-known optimization method and standard software is available. Mixed-integer linear programming (MILP) is becoming frequently used for STHS [17–20], where integer variables allow modeling of discrete hydro unit-commitment constraints.

However, LP typically considers that power generation is linearly dependent on water discharge, thus ignoring the head change effect, leading to a solution schedule with less power generation. Also, the discretization of the nonlinear dependence between power generation, water discharge and head, used in MILP to model head variations, augment the computational burden required to solve this problem. Furthermore, methods based on successive linearization in an iterative scheme depend on the expertise of the operator to properly calibrate the parameters. For instance, the selection of the best under-relaxation factor in [18] is empiric and case-dependent, rendering some ambiguity to these methods.

A nonlinear model has advantages compared with a linear one. A nonlinear model expresses hydroelectric power generation characteristics more accurately and the head change effect can be taken into account, which represents one of the main difficulties associated with the STHS problem.

In previous studies [1,4], the use of nonlinear programming (NLP) in some case studies leads to a result that exceeds by at least three percent what is obtained by LP, requiring a negligible extra computation time. However, the nonlinear model cannot avoid water discharges at forbidden areas, which may give schedules unacceptable from an operation point of view. These concerns lead to our novel mixed-integer nonlinear programming (MINLP) approach to solve the STHS problem. Indeed, MINLP is a state-of-the-art research in the subject of STHS.

As a new contribution to previous studies, we model the on-off behavior of the hydro plants using integer variables, in order to avoid water discharges at forbidden areas. Thus, an enhanced STHS is provided due to the more realistic modeling presented in this paper. Our approach has been applied successfully to solve a test case based on one of the Portuguese cascaded hydro systems with a negligible computational time requirement.

This paper is structured as follows. Section 2 provides the notation used throughout the paper along with the mathematical formulation of the STHS problem. Section 3 presents the MINLP approach for solving the STHS problem. Section 4 presents the numerical simulation results for the MINLP approach applied on one of the Portuguese cascaded hydro systems. Section 5 outlines the conclusions.

2. Problem formulation

The notation used throughout the paper is described as follows.

i Index of reservoirs.

k Index of periods in the time horizon.

I Total number of reservoirs.

K Total number of periods in the time horizon.

l_{ik} Water level in reservoir i during the period k.

$l_i^{\max}$ Maximum water level in reservoir i.

$l_i^{\min}$ Minimum water level in reservoir i.

h_{ik} Head of plant i during the period k.

$h_i^{\max}$ Maximum head of plant i.

$h_i^{\min}$ Minimum head of plant i.

v_{ik} Water storage of reservoir i at end of period k.

$v_i^{\max}$ Maximum storage of reservoir i.

$v_i^{\min}$ Minimum storage of reservoir i.

q_{ik} Water discharge of plant i during the period k.

$q_i^{\max}$ Maximum discharge of plant i.

$q_i^{\min}$ Minimum discharge of plant i.

s_{ik} Water spillage by reservoir i during the period k.

a_{ik} Inflow to reservoir *i* during the period *k*.

p_{ik} Power generation of plant *i* during the period *k*.

u_{ik} Commitment decision of plant *i* during the period *k*.

η_{ik} Efficiency of plant *i* during the period *k*.

$\eta_i^{\max}$ Maximum efficiency of plant *i*.

$\eta_i^{\min}$ Minimum efficiency of plant *i*.

π_k Forecasted energy price during the period *k*.

Ψ_i Future value of water stored in reservoir *i*.

$\boldsymbol{A}$ Constraint matrix.

$\boldsymbol{x}$ Vector of decision variables.

$\boldsymbol{b}^{\max}$ Upper bound vectors on constraints.

$\boldsymbol{b}^{\min}$ Lower bound vectors on constraints.

$\boldsymbol{x}^{\max}$ Upper bound vector on variables.

$\boldsymbol{x}^{\min}$ Lower bound vector on variables.

The STHS problem can be stated as to find out the water discharges, q_{ik}, the water storages, v_{ik}, and the water spillages, s_{ik}, for each reservoir, $i \in I$, at all scheduling time periods, $k \in K$, over the time horizon considered, that optimize a performance criterion subject to the operational and physical constraints. The water storages at the end of the time horizon, v_{iK}, are valued for future operation use. Additionally, the commitment decision, u_{ik}, is ascertained.

In the STHS problem under consideration, the objective function is a measure of the benefit obtained by the conversion of potential energy of the water available in the reservoirs into electric energy, without affecting future operation use.

In mathematical programming the STHS problem can be formulated as to maximize

$$J = \sum_{i=1}^{I} \Psi_i (v_{iK}) + \sum_{i=1}^{I} \sum_{k=1}^{K} \pi_k \, p_{ik} \tag{1}$$

Subject to:

- Water conservation equation for each reservoir.

$$v_{ik} = v_{i,k-1} + a_{ik} - q_{ik} - s_{ik} + q_{i-1,k} + s_{i-1,k}, \quad i \in I, \quad k \in K \tag{2}$$

- Power generation equation.

$$p_{ik} = P_{ik} (q_{ik}, \eta_{ik}), \quad i \in I, \quad k \in K \tag{3}$$

- Head equation.

$$h_{ik} = H_{ik} (l_{ik}, l_{i+1,k}), \quad i \in I, \quad k \in K \tag{4}$$

- Water storage constraints.

$$v_i^{\min} \le v_{ik} \le v_i^{\max}, \quad i \in I, \quad k \in K \tag{5}$$

- Water discharge constraints.

$$u_{ik} \, q_i^{\min} \le q_{ik} \le u_{ik} \, q_i^{\max}, \quad i \in I, \quad k \in K \tag{6}$$

- Water spillage constraints.

$$s_{ik} \ge 0, \quad i \in I, \quad k \in K \tag{7}$$

Eq. (1) shows our objective function. The objective function is composed of two terms. The first term expresses the economic value of the future use of the water stored in the reservoirs at the last period, Ψ_i. This term is considered if no final water storage requirement is specified as a constraint. The last term represents the profit with the hydro chain during the short-term time horizon, where π_k is the forecasted energy price during the period k and p_{ik} is the power generation of plant i during the period k.

The optimal value of the objective function is determined subject to constraints of two kinds: equality constraints and inequality constraints or simple bounds on the variables. Eq. (2) corresponds to the water conservation equation for each reservoir, assuming that the time required for water to travel from a reservoir to a reservoir directly downstream is less than the one hour period, independently of water discharge, due to the small distance between consecutive reservoirs. In Eq. (2) v_{ik} is the water storage of reservoir i at end of period k, a_{ik} is the inflow to reservoir i during the period k, q_{ik} is the water discharge of plant i during the period k, and s_{ik} is the water spillage by reservoir i during the period k. Time-delay is a difficult issue, depending on the distance between the reservoirs and on the water discharge, deserving particular attention and research. Time-delay can be accounted for by considering a different model structure for different flow levels in an iterative procedure, which is outside the scope of this paper. In Eq. (3) power generation, p_{ik}, is considered a function of water discharge and efficiency, η_{ik}, depending on the head, h_{ik}. Hence, the electrical output of a hydro plant depends on the water discharge, the head, and the efficiency. The operating points are restricted by minimal and maximal water discharges [21]. In Eq. (4) the head is considered a function of the water levels in the upstream reservoir, l_{ik}, and of the downstream reservoir, $l_{i+1,k}$, depending on the water storages in the respectively reservoirs. Typically for a powerhouse with a reaction turbine, where the tail water elevation is not constant, the head is modeled as in Eq. (4), and for a powerhouse with an impulse turbine, where the tail water elevation remains constant, the head depends only on the water level in the upstream reservoir as in [18]. Hence, tailrace effects can be considered by including a correction in the data regarding reservoir water levels. In Eq. (5) water storage has lower and upper bounds. Here for each reservoir i, $v_i^{\max}$ is the maximum storage, and $v_i^{\min}$ is the minimum storage. In Eq. (6) water discharge has lower and upper bounds. Here for each reservoir i, $q_i^{\max}$ is the maximum discharge, and $q_i^{\min}$ is the minimum discharge. The maximum discharge may be considered a function of the head, as in [4]. As a new contribution to earlier studies, we consider the commitment decision of each hydro plant. Hence, the binary variable, u_{ik}, is equal to 1 if plant i is on-line in hour k, otherwise is equal to 0. In Eq. (7) a null lower bound is considered for water spillage. Normally, water spillage by the reservoirs occurs when without it the water storage exceeds its upper bound, so spilling is necessary to avoid damage. The initial water storages, v_{i0}, and the inflows to reservoirs are assumed known.

3. MINLP approach for the STHS problem

MINLP can be stated as to maximize

$$\boldsymbol{J}(\boldsymbol{x}) \tag{8}$$

Subject to

$$\boldsymbol{x}^{\min} \le \boldsymbol{x} \le \boldsymbol{x}^{\max} \tag{9}$$

$$\boldsymbol{b}^{\min} \le \boldsymbol{A}\,\boldsymbol{x} \le \boldsymbol{b}^{\max} \tag{10}$$

$$\boldsymbol{x}_j \text{ integer}, \quad j \in J \tag{11}$$

where $\boldsymbol{J}(\cdot)$ is a nonlinear function of the vector $\boldsymbol{x}$ of decision variables, $\boldsymbol{x}^{\min}$ and $\boldsymbol{x}^{\max}$ are the lower and upper bound vectors on variables, $\boldsymbol{A}$ is the constraint matrix, $\boldsymbol{b}^{\min}$ and $\boldsymbol{b}^{\max}$ are the lower and upper bound vectors on constraints. Equality constraints are defined by setting the lower bound equal to the upper bound, i.e. $\boldsymbol{b}^{\min} = \boldsymbol{b}^{\max}$. The variables $\boldsymbol{x}_j$ are restricted to be integers. The lower and upper bounds for water discharge imply new inequality constraints that will be rewritten into Eq. (10).

Our nonlinear objective function is achieved by means of two linearizations: the first of them, efficiency as a function of head, is acceptable; the second one, water level as a function of water storage, implies reservoirs with vertical walls, which however is a good approximation for the run-of-the-river reservoirs, due to its small storage capacity, as our data have shown for our case study.

In Eq. (3) the efficiency depends on the head. We consider it given by

$$\eta_{ik} = \alpha_i\, h_{ik} + \eta_{i0}, \quad i \in I, \quad k \in K \tag{12}$$

where the parameters α_i and η_{i0} are given by

$$\alpha_i = (\eta_i^{\max} - \eta_i^{\min}) \,/\, (h_i^{\max} - h_i^{\min}), \quad i \in I \tag{13}$$

$$\eta_{i0} = \eta_i^{\max} - \alpha_i\, h_i^{\max}, \quad i \in I \tag{14}$$

In Eq. (13) parameter α_i of each plant i depends on the extreme values for efficiency and head, where $\eta_i^{\max}$ is the maximum efficiency, $\eta_i^{\min}$ is the minimum efficiency, $h_i^{\max}$ is the maximum head and $h_i^{\min}$ is the minimum head.

In Eq. (4) the water level depends on the water storage. We consider it given by

$$l_{ik} = \beta_i \, v_{ik} + l_{i0}, \quad i \in I, \quad k \in K \tag{15}$$

where the parameters β_i and l_{i0} are given by

$$\beta_i = (l_i^{\max} - l_i^{\min}) \,/\, (v_i^{\max} - v_i^{\min}), \quad i \in I \tag{16}$$

$$l_{i0} = l_i^{\max} - \beta_i \, v_i^{\max}, \quad i \in I \tag{17}$$

In Eq. (16) parameter β_i of each reservoir *i* depends on the extreme values for water level and storage, where $l_i^{\max}$ is the maximum water level, $l_i^{\min}$ is the minimum water level, $v_i^{\max}$ is the maximum water storage and $v_i^{\min}$ is the minimum water storage.

Substituting Eq. (12) into Eq. (3) we have

$$p_{ik} = q_{ik} \, (\alpha_i \, h_{ik} + \eta_{i0}), \quad i \in I, \quad k \in K \tag{18}$$

Therefore, by substituting Eqs. (4) and (15) into Eq. (18), power generation becomes a nonlinear function of water discharge and water storage, given by

$$p_{ik} = q_{ik} \, (\alpha_i \, \beta_i \, v_{ik} + \alpha_i \, l_{i0} - \alpha_i \, \beta_{i+1} \, v_{i+1,k} - \alpha_i \, l_{i+1,0} + \eta_{i0}), \quad i \in I, \quad k \in K \tag{19}$$

The parameters given by the product of α_i by β_i are of crucial importance for the behavior of head-dependent reservoirs in a hydro chain, setting optimal reservoirs storage trajectories in accordance to their relative position in the cascade. It should be noted that these parameters are not related to the solution procedure. Instead, they are determined only by physical data defining the hydro system [1].

A major advantage of our novel MINLP approach is to consider the head change effect in a single function of water discharge and water storage, Eq. (19), which can be used in a straightforward way, instead of deriving several curves for different heads.

As a new contribution to previous studies, we model the on-off behavior of the hydro plants using integer variables. Thus, the commitment decision is considered in Eq. (6) in order to obtain more realistic and feasible results, allowing for multiple operating regions.

MINLP is still a research topic among specialists in optimization. If a solution is found, there is no guarantee that it is the global optimal. Instead, a local optimal is usually obtained. This could represent a limitation of our approach. Therefore, we use a MILP approach to find a starting point for the MINLP approach. Afterwards, we check for an enhanced objective function value using the proposed MINLP approach. In our case study we always arrive at convergence to a superior solution.

4. Practical example

The proposed MINLP approach, which considers not only the head change effect but also discontinuous operating regions, avoiding water discharges at forbidden areas, has been applied on one of the Portuguese cascaded hydro systems. A thorough comparison with other approaches is carried out, to clearly demonstrate the advantages of the proposed MINLP approach comparatively to a NLP approach and a MILP approach.

Our novel approach has been developed and implemented in MATLAB and solved using the optimization solver package Xpress-MP. The numerical simulation has been performed on a 600-MHz-based processor with 256 MB of RAM.

The deregulation of the electricity markets brings uncertainty to energy prices. A good forecasting tool provides a risk hedging mechanism for generating companies against price volatility. In addition, a generating company can develop an appropriate bidding strategy to maximize its own profit with an accurate price forecast, which represents an advantage facing competition. Several forecasting procedures are available for predicting energy prices [22–24], but for the STHS problem the prices are considered as deterministic input data.

Final water storage in reservoirs is constrained so the water storage in the reservoirs at the last period is fixed. The final water storage in reservoirs is considered equal to the value at the beginning of the time horizon. Consequently, the future values of water stored in reservoirs are not considered in this example. A representation when the first term in Eq. (1) is explicitly taken into account can be seen in [25,26]. The storage targets for the short-term time horizon, which are established by medium-term planning studies, may be represented either by a penalty on water storage or by a previously determined ‘future cost function’.

The hydro system considered in this paper is shown in Fig. 1. Only the first reservoir has inflow. This inflow is due to an upstream watershed belonging to a different company.

"See Fig. 1 at the end of the manuscript".

This example is divided into two cases to illustrate the proficiency of the proposed MINLP approach. The cases are defined by

Case 1. Time horizon of 24 hours; dry scenario of low inflows

Case 2. Time horizon of 168 hours; wet scenario of high inflows

Case 1.

The inflow on the first reservoir is shown in Fig. 2.

"See Fig. 2 at the end of the manuscript".

The forecasted energy price considered over the 24-hours time horizon is shown in Fig. 3 ($ is a symbolic economic quantity).

"See Fig. 3 at the end of the manuscript".

Firstly, we compare the proposed MINLP approach with a NLP approach. The optimal reservoir storage trajectories are shown in Fig. 4. The optimal plant discharge trajectories are shown in Fig. 5. The solid line denotes the results obtained using the MINLP approach, while the dashed line denotes the results obtained using a NLP approach.

"See Fig. 4 at the end of the manuscript".
"See Fig. 5 at the end of the manuscript".

Fig. 4 shows almost identical storage trajectories for both approaches. Nevertheless, some different behavior is possible to be observed for the first reservoir, at a neighborhood of 12h, and a more observable behavior is noticeable for the third reservoir, at a neighborhood of 15h. Fig. 5 clearly shows the advantage of the proposed MINLP approach over the NLP approach. Although both approaches consider the head change effect, the NLP approach cannot avoid water discharges at forbidden areas, clearly observable for the third reservoir at a neighborhood of 15h, and may give schedules unacceptable from an operation point of view. Hence, the results obtained using the MINLP approach are more realistic and feasible.

Secondly, we compare the proposed MINLP approach with a MILP approach. The optimal reservoir storage trajectories are shown in Fig. 6. The optimal plant discharge trajectories are shown in Fig. 7. The solid line denotes again the results obtained using the MINLP approach, while the dash-dot line denotes the results obtained using a MILP approach.

"See Fig. 6 at the end of the manuscript".
"See Fig. 7 at the end of the manuscript".

Figs. 6 and 7 clearly show the advantage of the proposed MINLP approach over the MILP approach. Although both approaches model the on-off behavior of the hydro plants using integer variables, the MILP approach ignores the head change effect, leading to a solution schedule with less power generation. Hence, the total profit for the cascaded hydro system is higher using the MINLP approach.

Case 2.

The inflow on the first reservoir is shown in Fig. 8.

"See Fig. 8 at the end of the manuscript".

The forecasted energy price considered over the 168-hours time horizon is shown in Fig. 9 ($ is a symbolic economic quantity).

"See Fig. 9 at the end of the manuscript".

The same procedure used previously is carried out in case 2. Firstly, we compare the proposed MINLP approach with a NLP approach. The optimal reservoir storage trajectories are shown in Fig. 10. The optimal plant discharge trajectories are shown in Fig. 11. The solid line denotes the results obtained using the MINLP approach, while the dashed line denotes the results obtained using a NLP approach.

"See Fig. 10 at the end of the manuscript".
"See Fig. 11 at the end of the manuscript".

Figs. 10 and 11 show almost identical storage and discharge trajectories for both approaches, since water discharges rarely occur at forbidden areas with the NLP approach, due to the consideration of a wet scenario.

Secondly, we compare the proposed MINLP approach with a MILP approach. The optimal reservoir storage trajectories are shown in Fig. 12. The optimal plant discharge trajectories are shown in Fig. 13. The solid line denotes again the results obtained using the MINLP approach, while the dash-dot line denotes the results obtained using a MILP approach.

"See Fig. 12 at the end of the manuscript".
"See Fig. 13 at the end of the manuscript".

Figs. 12 and 13 clearly show the advantage of the proposed MINLP approach over the MILP approach, due the consideration of the head change effect.

Table 1 summarizes an overall comparison between the results obtained by each approach.

"See Table 1 at the end of the manuscript".

On the one hand, the additional computational effort required by the proposed MINLP approach is negligible. The MINLP approach achieved fast convergence for all cases tested. On the other hand, an increase on the total profit is obtained using the proposed MINLP approach, comparatively to the MILP approach, about 4.5% on average. Although the profit may be slightly higher with the NLP approach, it should be noted that the results obtained using the MINLP approach are more realistic and feasible, since water discharges at forbidden areas are avoided, i.e., the water discharges at forbidden areas are scheduled at slightly less profitable hours using our MINLP approach. The NLP approach may give schedules unacceptable from an operation point of view. Hence, the proficiency of the proposed MINLP approach has been clearly demonstrated.

5. Conclusion

A novel MINLP approach is proposed for the STHS problem, considering not only head-dependency but also discontinuous operating regions. A major advantage of our approach is to consider the head change effect in a single function of water discharge and water storage that can be used in a straightforward way, instead of deriving several curves for different heads. Also, as a new contribution to earlier studies, we consider the commitment decision of each hydro plant. A thorough comparison with other approaches is carried out in this paper, namely a comparison between NLP and MINLP approaches, and between MILP and MINLP approaches, clearly demonstrating the advantages of the proposed MINLP approach. Due to the more realistic modeling presented in this paper, a better STHS is provided, assuring simultaneously a negligible computation time.

Acknowledgements

The authors gratefully acknowledge the contributions of Professor L.A.F.M. Ferreira and Dr. S.J.P.S. Mariano.

References

[1] J.P.S. Catalão, S.J.P.S. Mariano, V.M.F. Mendes, L.A.F.M. Ferreira, Parameterisation effect on the behaviour of a head-dependent hydro chain using a nonlinear model, Electr. Power Syst. Res. 76 (6–7) (2006) 404–412.

[2] A.H. Mantawy, S.A. Soliman, M.E. El-Hawary, The long term hydro-scheduling problem—a new algorithm, Electr. Power Syst. Res. 64 (1) (2003) 67–72.

[3] O.B. Fosso, A. Gjelsvik, A. Haugstad, B. Mo, I. Wangensteen, Generation scheduling in a deregulated system. The Norwegian case, IEEE Trans. Power Syst. 14 (1) (1999) 75–80.

[4] S.J.P.S. Mariano, J.P.S. Catalão, V.M.F. Mendes, L.A.F.M. Ferreira, Optimising power generation efficiency for head-sensitive cascaded reservoirs in a competitive electricity market, Elect. Power Energy Syst. 30 (2) (2008) 125–133.

[5] M.M. Belsnes, O.B. Fosso, Competitive optimal hydropower scheduling from strategy to operation, in: Proc. 8th IASTED Int. Conf. Power and Energy Systems, Corfu, Greece, 2008.

[6] J.P.S. Catalão, S.J.P.S. Mariano, V.M.F. Mendes, L.A.F.M. Ferreira, Short-term scheduling of thermal units: emission constraints and trade-off curves, Eur. Trans. Electr. Power 18 (1) (2008) 1–14.

[7] L.A.F.M. Ferreira, T. Andersson, C.F. Imparato, T.E. Miller, C.K. Pang, A. Svoboda, A.F. Vojdani, Short-term resource scheduling in multi-area hydro-thermal power systems, Elect. Power Energy Syst. 11 (3) (1989) 200–212.

[8] S.M. Amado, C.C. Ribeiro, Short-term generation scheduling of hydraulic multi-reservoir multi-area interconnected systems, IEEE Trans. Power Sys. 2 (3) (1987) 758–763.

[9] A. Arce, T. Ohishi, S. Soares, Optimal dispatch of generating units of the Itaipú hydroelectric plant, IEEE Trans. Power Sys. 17 (1) (2002) 154–158.

[10] J.M. Pursimo, H.K. Antila, M.K. Vilkko, P.A. Lautala, A short-term scheduling for a hydropower plant chain, Elect. Power Energy Syst. 20 (8) (1998) 525–532.

[11] M. Basu, Hopfield neural networks for optimal scheduling of fixed head hydrothermal power systems, Electr. Power Syst. Res. 64 (1) (2003) 11–15.

[12] K.K. Mandal, N. Chakraborty, Differential evolution technique-based short-term economic generation scheduling of hydrothermal systems, Electr. Power Syst. Res. 78 (11) (2008) 1972–1979.

[13] X.H. Yuan, L. Wang, Y.B. Yuan, Application of enhanced PSO approach to optimal scheduling of hydro system, Energy Conv. Manag. 49 (11) (2008) 2966–2972.

[14] J.K. Wu, J.Q. Zhu, G.T. Chen, H.L. Zhang, A hybrid method for optimal scheduling of short-term electric power generation of cascaded hydroelectric plants based on particle swarm optimization and chance-constrained programming, IEEE Trans. Power Syst. 23 (4) (2008) 1570–1579.

[15] A.R.L. Oliveira, S. Soares, L. Nepomuceno, Short term hydroelectric scheduling combining network flow and interior point approaches, Elect. Power Energy Syst. 27 (2) (2005) 91–99.

[16] A.J. Wood, B.F. Wollenberg, Power Generation, Operation and Control, Willey, New York, 1996.

[17] A.J. Conejo, J.M. Arroyo, J. Contreras, F.A. Villamor, Self-scheduling of a hydro producer in a pool-based electricity market, IEEE Trans. Power Sys. 17 (4) (2002) 1265–1272.

[18] J. García-González, E. Parrilla, A. Mateo, Risk-averse profit-based optimal scheduling of a hydro-chain in the day-ahead electricity market, Eur. J. Oper. Res. 181 (3) (2007) 1354–1369.

[19] S.-E. Fleten, T.K. Kristoffersenb, Short-term hydropower production planning by stochastic programming, Comput. Oper. Res. 35 (8) (2008) 2656–2671.

[20] A. Borghetti, C. D’Ambrosio, A. Lodi, S. Martello, An MILP approach for short-term hydro scheduling and unit commitment with head-dependent reservoir, IEEE Trans. Power Syst. 23 (3) (2008) 1115–1124.

[21] A. Borghetti, A. Frangioni, F. Lacalandra, C.A. Nucci, Lagrangian heuristics based on disaggregated bundle methods for hydrothermal unit commitment, IEEE Trans. Power Syst. 18 (1) (2003) 313–323.

[22] A.J. Conejo, M.A. Plazas, R. Espínola, A.B. Molina, Day-ahead electricity price forecasting using the wavelet transform and ARIMA models, IEEE Trans. Power Syst. 20 (2) (2005) 1035–1042.

[23] J.P.S. Catalão, S.J.P.S. Mariano, V.M.F. Mendes, L.A.F.M. Ferreira, Short-term electricity prices forecasting in a competitive market: a neural network approach, Electr. Power Syst. Res. 77 (10) (2007) 1297–1304.

[24] A.T. Lora, J.M.R. Santos, A.G. Expósito, J.L.M. Ramos, J.C.R. Santos, Electricity market price forecasting based on weighted nearest neighbors techniques, IEEE Trans. Power Syst. 22 (3) (2007) 1294–1301.

[25] E. Gil, J. Bustos, H. Rudnick, Short-term hydrothermal generation scheduling model using a genetic algorithm, IEEE Trans. Power Syst. 18 (4) (2003) 1256–1264.

[26] W. Uturbey, A. Simões Costa, Dynamic optimal power flow approach to account for consumer response in short term hydrothermal coordination studies, IET Gener. Transm. Distrib. 1 (3) (2007) 414–421.

Figure captions

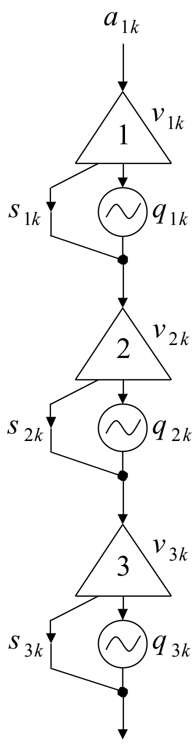

Fig. 1. Hydro system with three cascaded reservoirs.

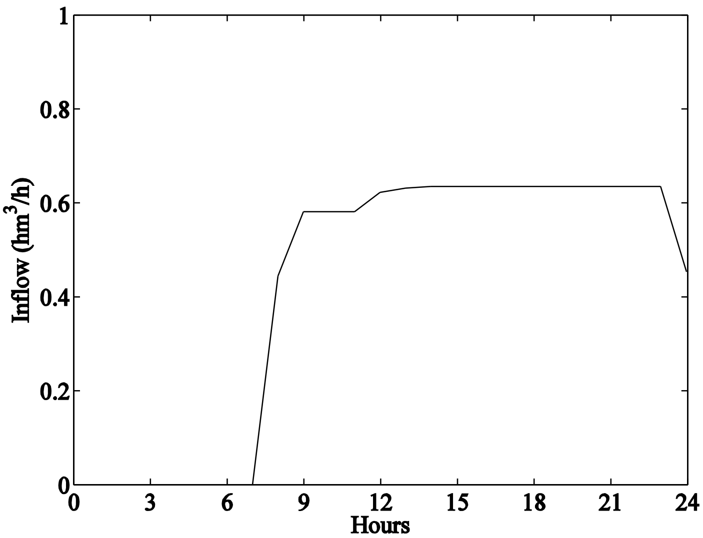

Fig. 2. Inflow on the first reservoir for case 1.

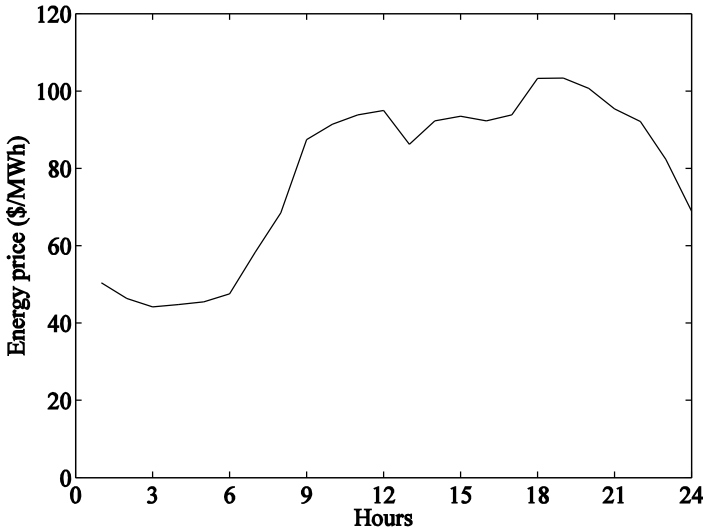

Fig. 3. Forecasted energy price over the 24-hours time horizon.

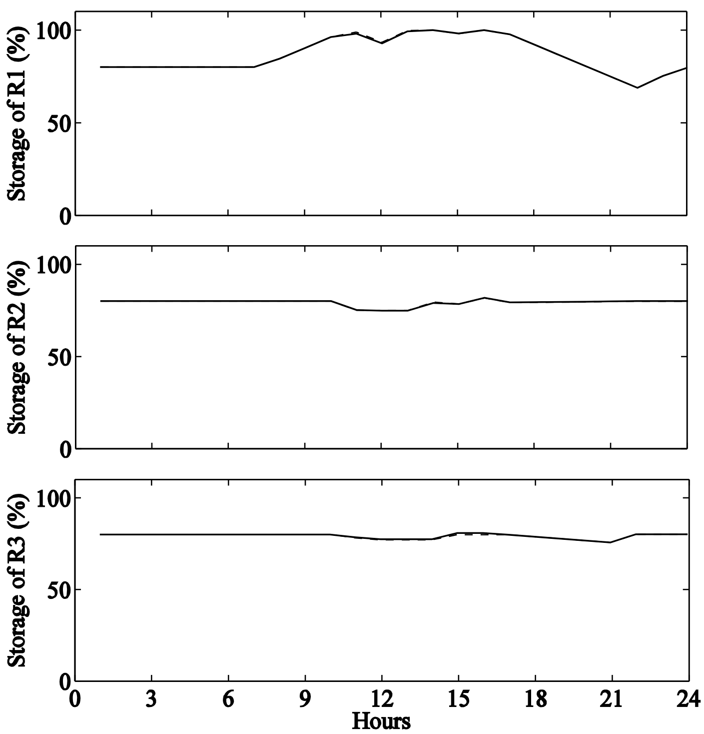

Fig. 4. Optimal reservoir storage trajectories for case 1. The solid line denotes the results obtained using the MINLP approach, while the dashed line denotes the results obtained using a NLP approach.

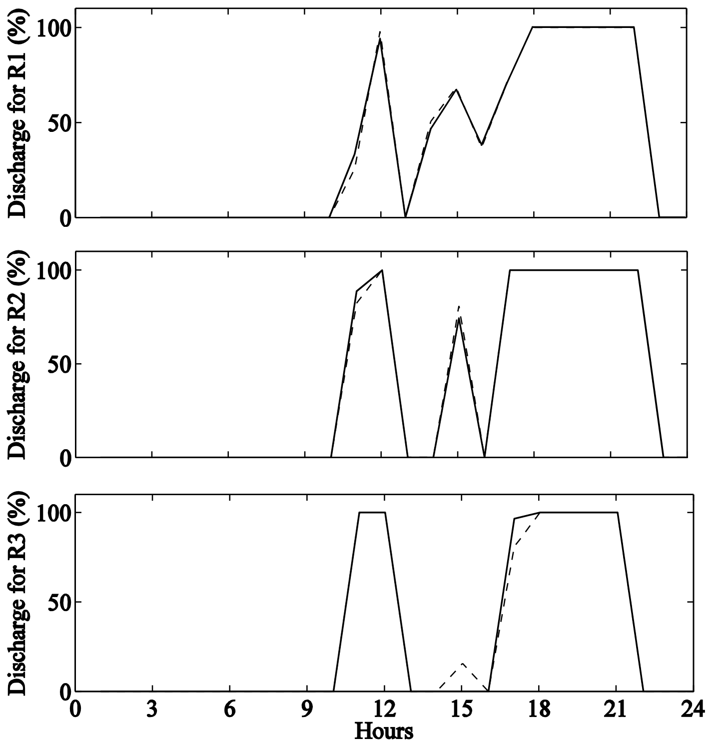

Fig. 5. Optimal plant discharge trajectories for case 1. The solid line denotes the results obtained using the MINLP approach, while the dashed line denotes the results obtained using a NLP approach.

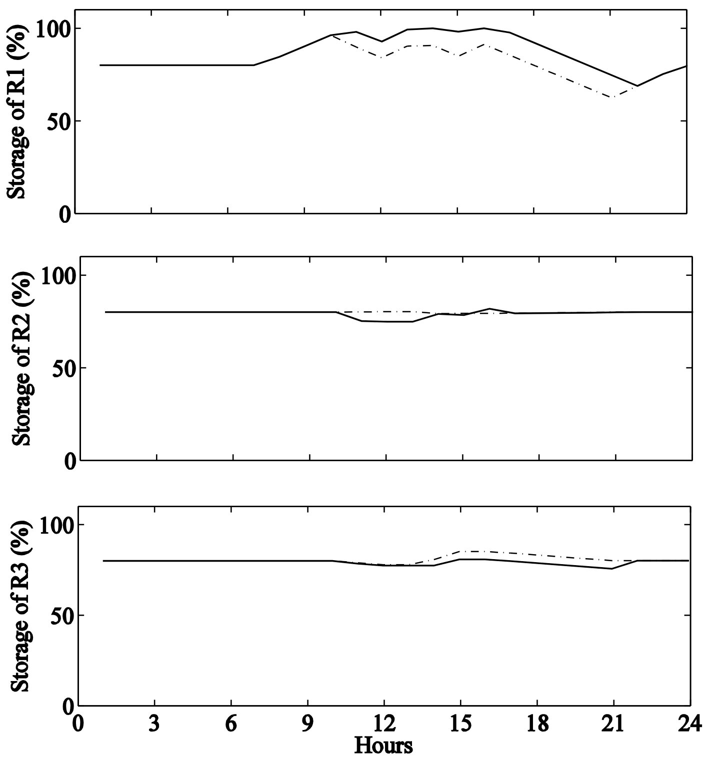

Fig. 6. Optimal reservoir storage trajectories for case 1. The solid line denotes again the results obtained using the MINLP approach, while the dash-dot line denotes the results obtained using a MILP approach.

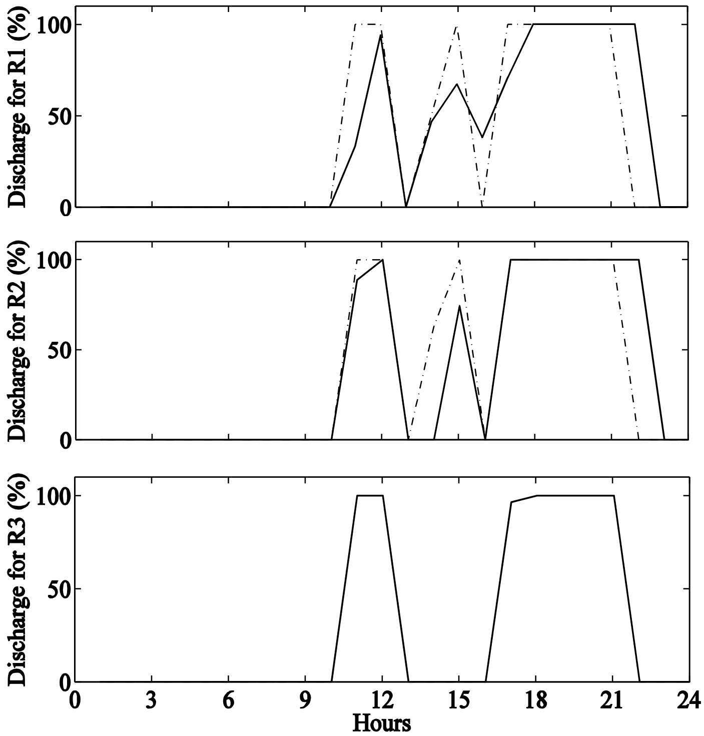

Fig. 7. Optimal plant discharge trajectories for case 1. The solid line denotes again the results obtained using the MINLP approach, while the dash-dot line denotes the results obtained using a MILP approach.

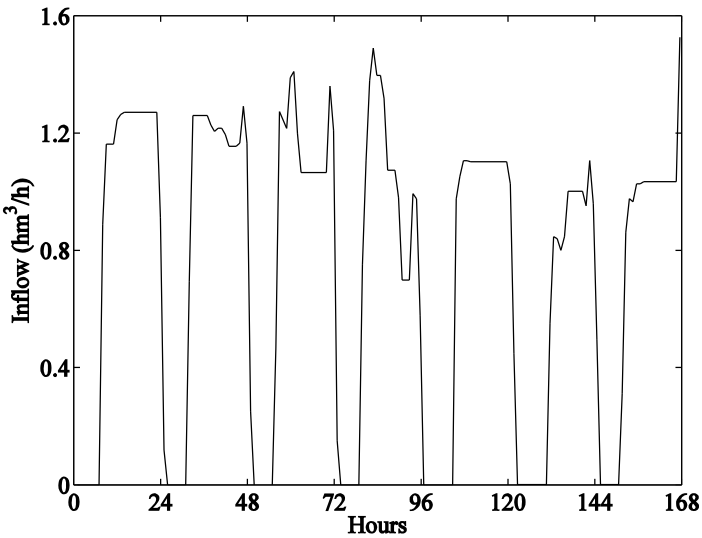

Fig. 8. Inflow on the first reservoir for case 2.

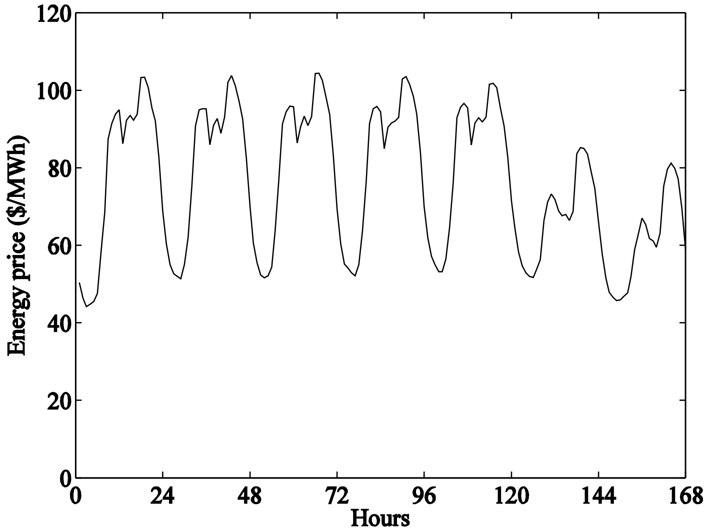

Fig. 9. Forecasted energy price over the 168-hours time horizon.

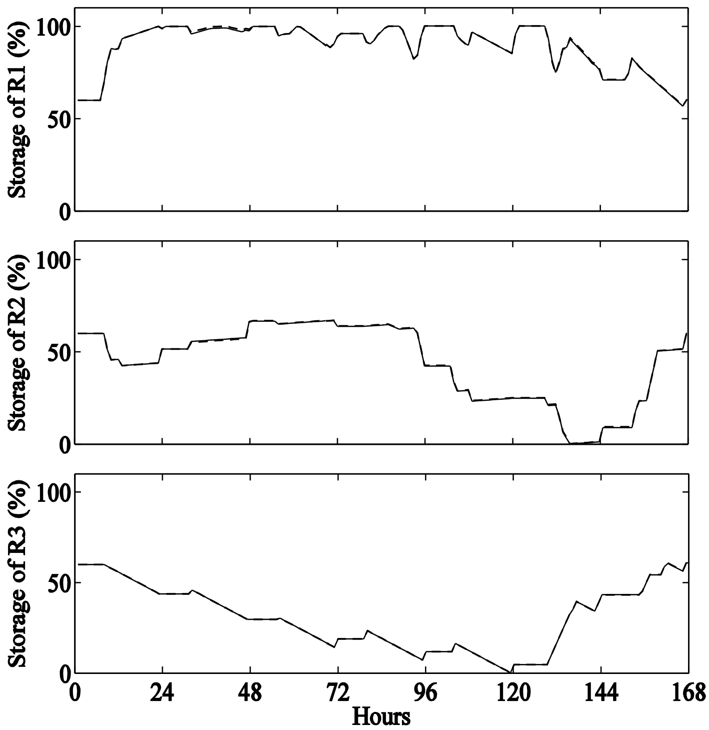

Fig. 10. Optimal reservoir storage trajectories for case 2. The solid line denotes the results obtained using the MINLP approach, while the dashed line denotes the results obtained using a NLP approach.

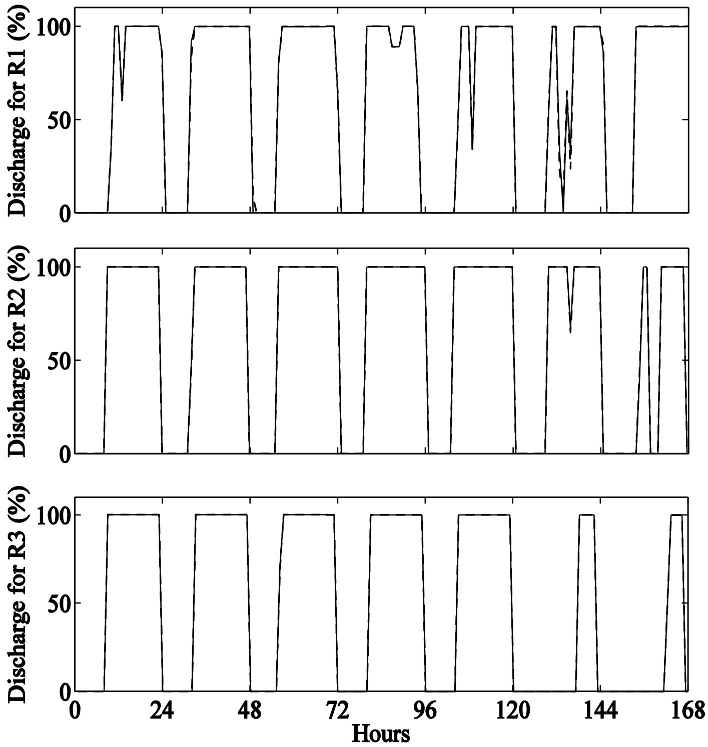

Fig. 11. Optimal plant discharge trajectories for case 2. The solid line denotes the results obtained using the MINLP approach, while the dashed line denotes the results obtained using a NLP approach.

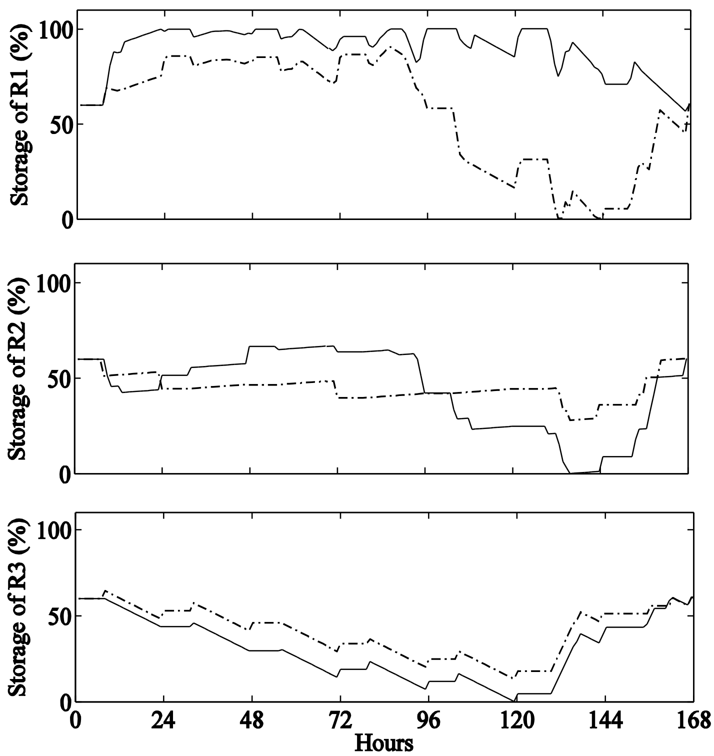

Fig. 12. Optimal reservoir storage trajectories for case 2. The solid line denotes again the results obtained using the MINLP approach, while the dash-dot line denotes the results obtained using a MILP approach.

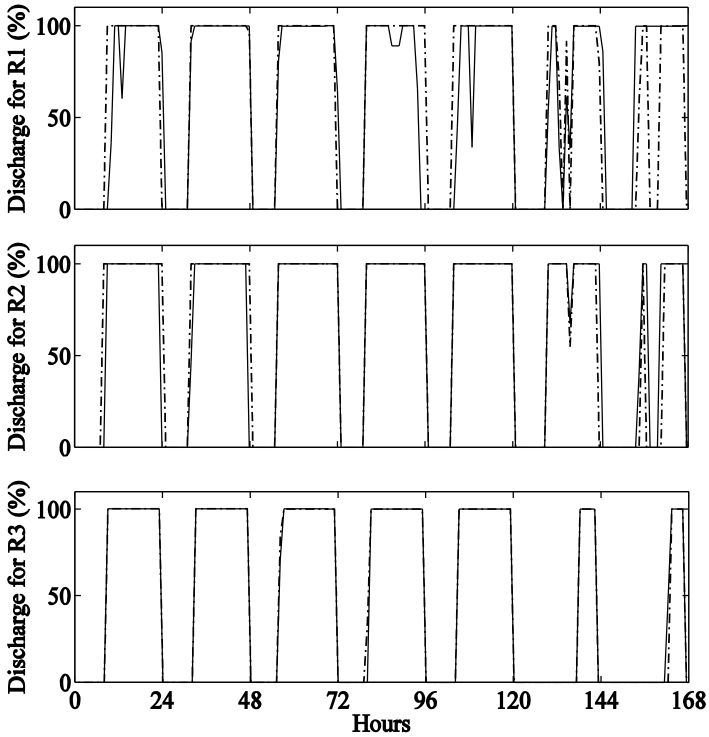

Fig. 13. Optimal plant discharge trajectories for case 2. The solid line denotes again the results obtained using the MINLP approach, while the dash-dot line denotes the results obtained using a MILP approach.

Tables

Table 1

Comparison of the proposed MINLP approach with NLP and MILP approaches

	Profit ($)	% Increase	CPU time (s)
MILP – Case 1	465,436	-	1.33
NLP – Case 1	487,025	4.64	1.34
MINLP – Case 1	487,020	4.64	1.41
MILP – Case 2	5,258,373	-	2.16
NLP – Case 2	5,490,867	4.42	2.41
MINLP – Case 2	5,490,607	4.42	3.79