Modified Newton Type Algorithm-Based Frequency and Phase Estimation Technique in Harmonics-Polluted Grid

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***Abstract*—In this paper a modified Newton type algorithm (NTA) is analysed to estimate the electrical signal spectral composition. As core of this study, it is given attention to the accuracy and sensitivity of the algorithm for base frequency and harmonic content characterization under noisy measurements. Convergence speed, stability issues and computational complexity of the implementation are detailed and compared using clean and noisy test signals. Conclusions are duly drawn.**

Keywords—Signal processing; Power grid; Signal-to-Noise; Newton type algorithm; Phasor Estimation.

I. INTRODUCTION

In the most recent years the academia has been showing an increasing interest in the research topic of array signal processing due to its importance in sonar, radar, healthcare, seismology, electronics and communication systems [1]. The estimation of the frequency is considered to be an important problem in estimation theory and is currently highly employed and utilized in communication systems, power systems, radar, measurement and instrumentation. During the last few decades, a substantial attention in this research topic has been shown by the research community and a wide range of frequency estimation approaches have been proposed [2], [3].

The topic of assessing the frequency of complex sinusoidal signals subject to additive white Gaussian noise (AWGN) has received increasing attention in the research community. The wide range of applications vary from carrier retrieval in a given communication system, impurity doped quantum dots [4], measurement of a position of an object in sonar and radar systems [5], influence on neuronal firings of a heterogeneous neuronal network [6], and carrier synchronization in a distributed beamforming system [7], [8].

At the present time, a wide range of methodologies applied to the phase estimation and frequency issues, mainly notable by processing latency, computational complexity, and estimation accuracy have been created. A category of methods relies on the FFT due to its relation with the maximum likelihood estimation (MLE) of the frequency [9]. In this paper the analysis is made of the performance of a recursion numerical technique having as a basis the Newton method [10]–[12] thus by comparing a wide range of Signal-to-Noise (SNR) values to Fast Fourier Transform (FFT)-based signal spectrum estimation method.

This paper is organized as follows: in Section II the Newton type algorithm formulation is presented. In Section III the simulations are shown and finally, in Section IV the conclusions are drawn.

II. NEWTON TYPE ALGORITHM FORMULATION

Let's suppose that the signal *v(t)* that is needs to be identified is approximated by:

$$v(t) = h(\hat{x}, t) + \varepsilon(t) \tag{1}$$

where *ε(t)* is a noise signal of instant time *t* with null average, $\hat{x}$ a parameter vector and *h(.)* a nonlinear function represented by:

$$h(x,t) = V_0 + \sum_{k=1} V_k \sin(kwt + \varphi_k) \tag{2}$$

Thus, $\hat{x}$ being the parameter vector with the estimation based on the observations of the generic model *h(x,t)*. Therefore, is given by:

$$\hat{x} = \left[V_{o,} w, V_1, \ldots, V_k, \varphi_1, .., \varphi_k\right] T \tag{3}$$

where V_0 is a DC term, *w* is the fundamental angular velocity (2πf), V_k is the harmonic phasor amplitude of order *k* and φ_k is the phase angle of the harmonic phasor order *k*.

The NTA belongs to the recursive class of numerical techniques. This algorithm utilises the vales obtained in instant *i* in order to obtain the estimations in the instant *i+1*. In each time instant *i* the new estimated set of parameters will generate through function $\hat{h}(\hat{x}_i, t)$ a term of comparison with the real signal *v(t)*. This signal, *v(t)*, can be interpreted as the estimation error of the vector $\hat{x}_i$. Based on this technique the correction adjusts the estimated vector parameters to the new value $\hat{x}_{i+1}$. The NTA algorithm is given by:

$$\hat{x}_{i+1} = \hat{x}_i + J_i^I\left[\hat{v} - \hat{h}(\hat{x}_i, t)\right] \tag{4}$$

where *i* is the iteration index, $\hat{x}_i$ represents the estimated vector parameters, J_i^I is the pseudoinverse matrix of the Jacobian matrix j_i while $\hat{v}$ represents the observation vector (samples) of the signal with dimension *Nx1* and $\hat{h}(\hat{x}_i, t)$ is the nonlinear function vector determined by the mathematical model $h(\hat{x}, t)$ with *Nx1* dimension.

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Once the structure of the model that is capable to reconstruct the behaviour of the signal is known the maximum number of estimated parameters will be as follows:

$$n = 2M + 2 \tag{5}$$

where M is the harmonic index of the highest order defined in the model. For each estimation, it is necessary to find a new set of parameters. Thus, in order to be resolvable, the system has an obligatory condition for being constituted by N equations equal or superior to the number of unknown values n. The pseudoinverse matrix of the Jacobian matrix is assessed by:

$$J_i^I = \left(J_i^T J_i\right)^{-1} J_i^T \tag{6}$$

where the J_i^I is Nxn matrix whose elements are partial derivatives from $h(\hat{x}_i, t)$:

$$j_1 = \frac{\partial h(x)}{\partial V_o} = 1 \tag{7}$$

$$j_2 = \frac{\partial h(x)}{\partial w} = \sum_{k=1}^{M} V_k ktcos\left(kwt + \varphi_k\right) \tag{8}$$

$$j_{2+k} = \frac{\partial h(x)}{\partial V_k} = \sin\left(kwt + \varphi_k\right) \tag{9}$$

$$j_{2+M+k} = \frac{\partial h(x)}{\partial \varphi_k} = V_k \cos\left(kwt + \varphi_k\right) \tag{10}$$

In the form given by:

$$J(x_i) = \begin{pmatrix} \frac{\partial h(x)_{t1}}{\partial V_o} & \frac{\partial h(x)_{t1}}{\partial w} & \frac{\partial h(x)_{t1}}{\partial V_k} & \cdots & \frac{\partial h(x)_{t1}}{\partial \varphi_k} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial h(x)_{tN}}{\partial V_o} & \frac{\partial h(x)_{tN}}{\partial w} & \frac{\partial h(x)_{tN}}{\partial V_k} & \cdots & \frac{\partial h(x)_{tN}}{\partial \varphi_k} \end{pmatrix} \tag{11}$$

In order to move to the implementation four design parameters have to be set (sampling frequency, data time window T_{WD}, initialization vector $\hat{x}_i$ and iterations number vs error range. With regard to the choice of the sampling frequency, the Nyquist sampling theorem states that an analog signal waveform to be reproduced correctly needs to be sampled at least twice the highest frequency present in the signal. For instance, if a complex signal has a maximum harmonic index of 10, then the minimum sampling frequency is $2\times10\times F_{fund}$ where F_{fund} is the fundamental waveform frequency is. The T_{WD} is a time specification whose purpose is to ensure that the algorithm process for each iteration a set of samples equivalent to the period of the fundamental waveform [10]. Selecting a higher T_{WD} is not recommended because it extends the convergence time. Since the NTA is a nonlinear estimation technique absolute convergence is not guaranteed [11]. It means the iteration process may end up outputting minimum local solutions instead of absolute minimum solutions. This peculiarity requires the NTA algorithm should be started with a set of initial guesses close to the real solutions using for that the vector $\hat{x}_i$.

Excepting signal frequency the remaining phasor variables should be pre-estimated, for example by using Discrete Fourier Transformation processing technique [10] or by means of the non-recursive self-tuning least error squares method [11]. Error range is a control criterion to limit the number of iterations. By establishing a range the hardware computation capacity can be released for other tasks or eventually to lower significantly the hardware power consumption.

III. Simulation

In this section the simulations regarding the performance of the modified Newton algorithm is presented. It was carried out two tests. In the first case for evaluating the algorithm a voltage signal with a fixed harmonic composition with noise is used. For the second case the same voltage signal is corrupted with white noise.

A. *Test signal with constant harmonic profile*

In order to estimate the signal it was chosen a $h(\hat{x}, t)$ model with a maximum of 10 harmonics. According with the aforementioned expressions for a signal with 10 harmonics the number of parameters possible to be estimated is 22. As is to be expected the algorithm does not establish a limit for the number of parameters to be identified. Therefore, the script developed in Matlab aims to estimate up the amplitude and angle of the harmonic tensions up to 10th order. For adequate application in a real situation in order to be effective the estimation, all the harmonics of higher order have to be removed with an adequate high-order analog filter if the objective is to use the lowest sampling frequency possible for a known bandwidth signal according to Nyquist sampling theorem. The signal simulated in Matlab has the following composition:

$$\begin{aligned} v(t) &= 1 + 330sin(314.16t + 1.7) \\ &+220\sin(2\times314.16t+1.6) \\ &+140\sin(3\times314.16t+1.5) \\ &+100sin(4\times314.16t+1.4) \\ &+140sin(5\times314.16t+1.3) \\ &+37sin(6\times314.16t+1.2) \\ &+33sin(7\times314.16t+1.1) \\ &+18sin(8\times314.16t+1.05) \\ &+10sin(9\times314.16t+0.9) \\ &+5sin(10\times314.16t+0.5) \end{aligned} \tag{12}$$

The waveform *v(t)* is shown in Fig.1. The signal is sampled at 5 ksps. The selected sampling frequency guarantees that 10 samples are captured with relation to the highest harmonic term in the signal which means not only satisfy the Nyquist sampling theorem as well the 10th order sinusoidal term can be accurately represented. Data window amplitude T_{WD} is set to 0.02s. With 5 ksps the T_{WD} parameter requires 100 samples. It should also be mentioned that signal acquisition was performed through a zero order hold block in Simulink in order to sample value of the signal and holding its input for a certain sampling interval. The parameters vector $\hat{x}_i$ was initialized with an identical sequence of values.

$$\hat{x}_0 = [0.5;2\times pi \times 48;290;50;45;10;60;1;50;1;10;1;1;1;1;1;1;1;1;1;1;1]^T \quad (13)$$

As can be seen in Figs. 2-12 the NTA algorithm ensures a fast convergence after a few iterations. The convergence time last a bit more than 1ms. On the other hand, phasor angles estimation is apparently faster than the remaining variables. Although the results presented here shows a good dynamic response as well as accurate, the initialization vector needs to be carefully selected in relation to the initial values. The authors of this paper did not apply any of the cited methods cited in the bibliography. It took considerable time to set adequately the vector $\hat{x}_0$. In short, to be feasible its usage on a real prototype it is mandatory to include any sort of initial estimator algorithm in the software core. Otherwise, the results may be disappointing and frustrating.

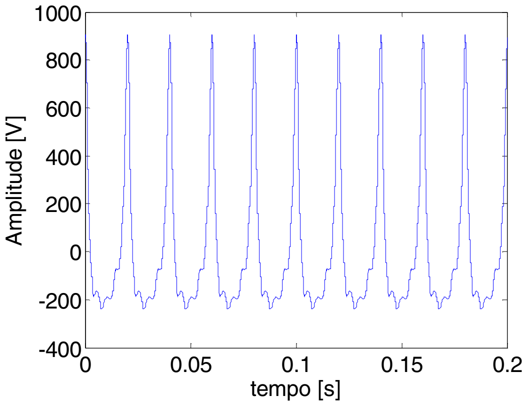

Fig. 1. Signal used for testing the algorithm.

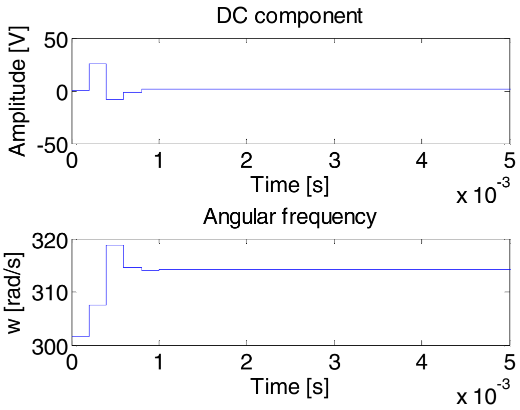

Fig. 2. Signal parameters estimation over time (V_0 and w).

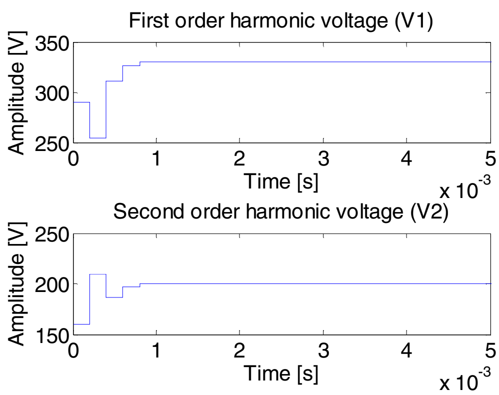

Fig. 3. Signal parameters estimation over time (V1 and V2).

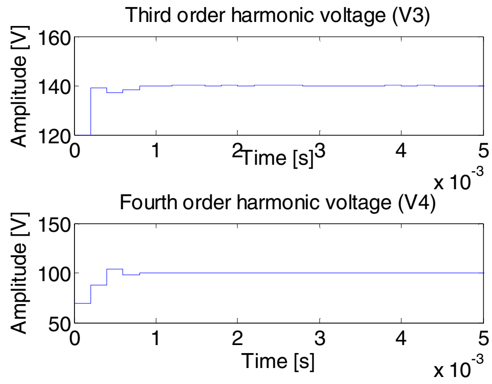

Fig. 4. Signal parameters estimation over time (V3 and V4).

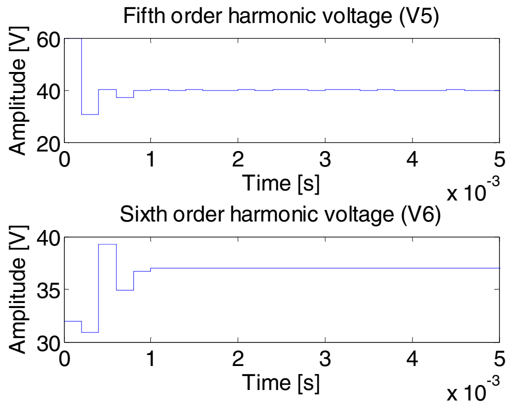

Fig. 5. Signal parameters estimation over time (V5 and V6).

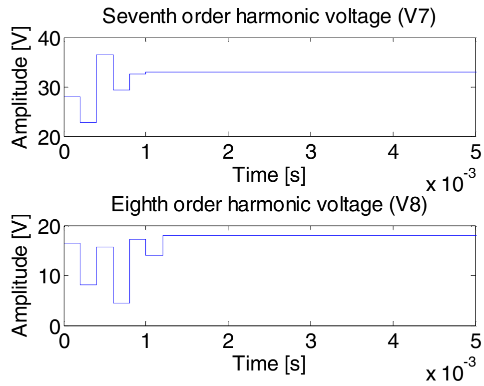

Fig. 6. Signal parameters estimation over time (V7 and V8).

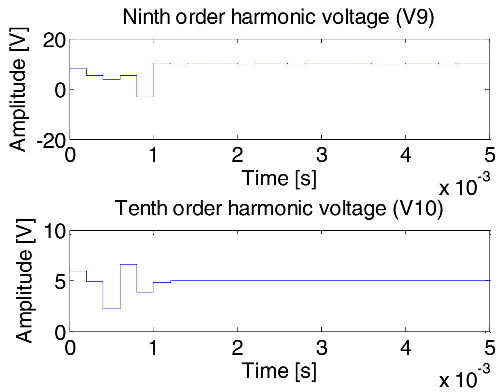

Fig. 7. Signal parameters estimation over time (V9 and V10).

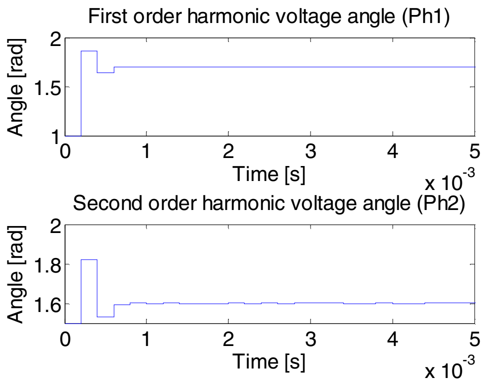

Fig. 8. Signal parameters estimation over time (φ1 and φ2).

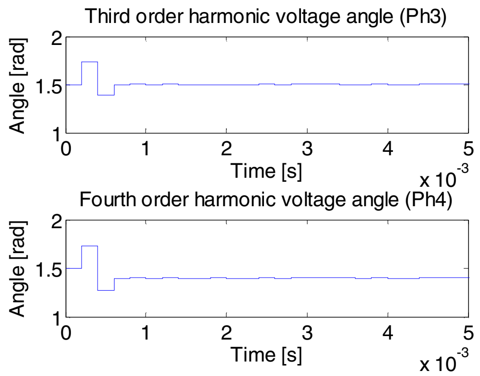

Fig. 9. Signal parameters estimation over time (φ3 and φ4).

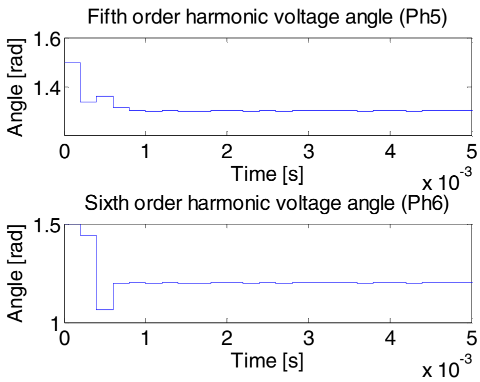

Fig. 10. Signal parameters estimation over time (φ5 and φ6).

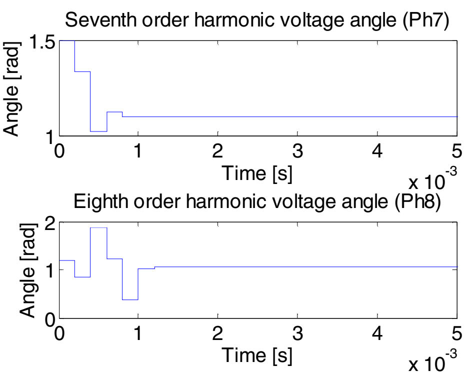

Fig. 11. Signal parameters estimation over time (φ7 and φ8).

B. *Test signal with white noise*

In contrast with the previous test the signal is now contaminated with Gaussian noise. As such the signal was contaminated with a noise power equivalent to a Signal-to-noise (SNR) ratio of 72.5 dB. This level of SNR was chosen to simulate the instrumentation chain noise targeting a digital conversion resolution of 12 bits.

Using the same initialization vector the results are shown in Figs. 13-23. Identification of sinusoidal harmonic tensions amplitude is done at the same speed around 1-1.5 ms without been affected by the presence of the noise. However, the angle estimation as can be observed is only accurate and stable for the first harmonic sinusoidal term.

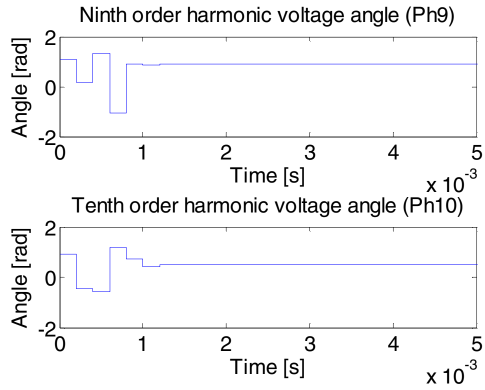

Fig. 12. Signal parameters estimation over time (φ9 and φ10).

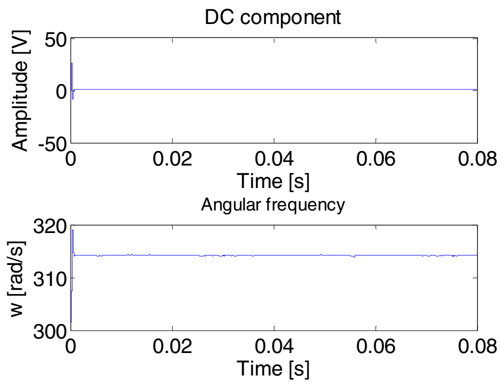

Fig. 13. Signal parameters estimation over time (V_0 and w).

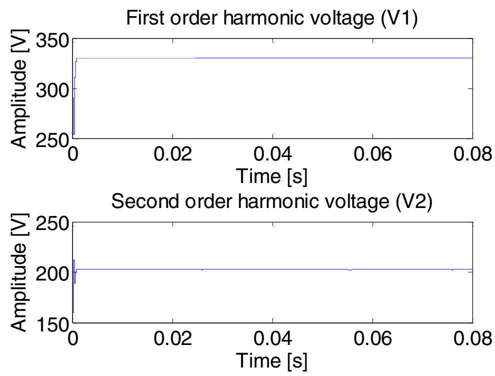

Fig. 14. Signal parameters estimation over time (V1 and V2).

In the remaining estimations, despite at first sight the convergence speed is high there are continuous oscillations over time and which are not so small in some cases. Naturally a signal that is acquired through an instrumentation chain, some sort of noise will be introduced. These plots show the estimation performance degradation due to a limited resolution in digital conversion. In addition to this, it is more significant because the harmonic voltage angles are low thus suggesting that is necessary to increase the resolution of the conversion process. The phasor angle – tested for very small values – the response it is not only quick but also accurate.

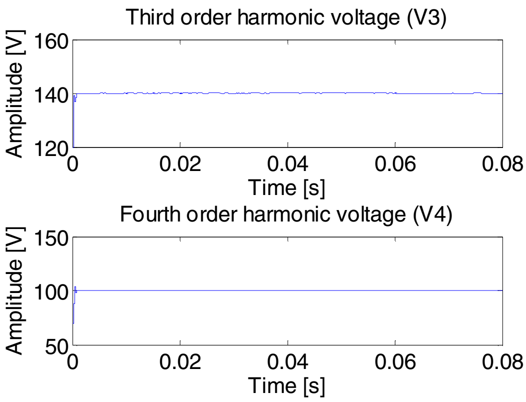

Fig. 15. Signal parameters estimation over time (V3 and V4).

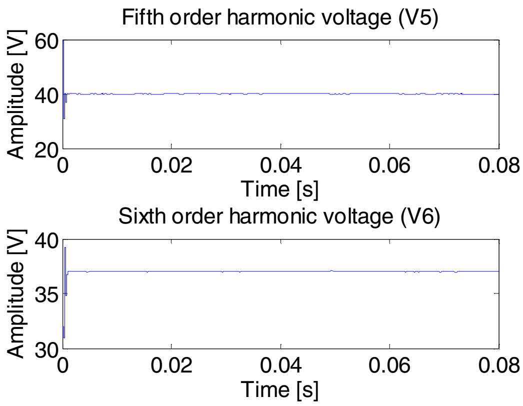

Fig. 16. Signal parameters estimation over time (V5 and V6).

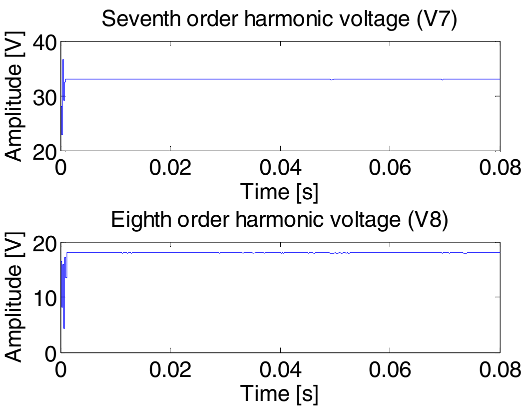

Fig. 17. Signal parameters estimation over time (V7 and V8).

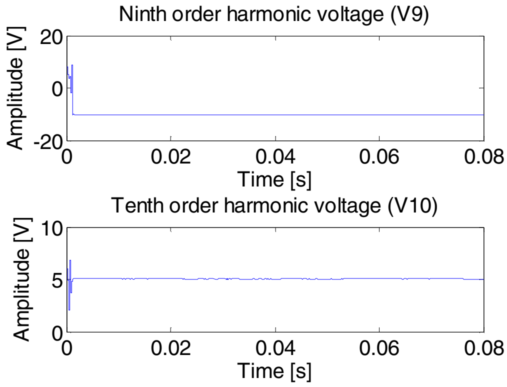

Fig. 18. Signal parameters estimation over time (V9 and V10).

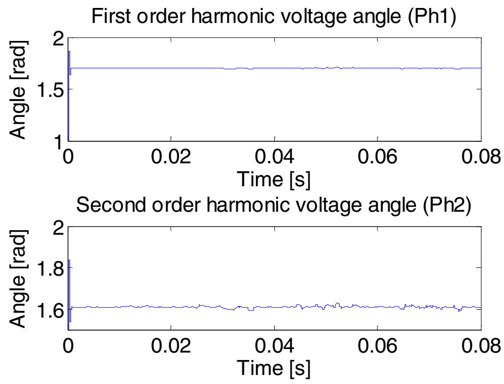

Fig. 19. Signal parameters estimation over time (φ1 and φ2).

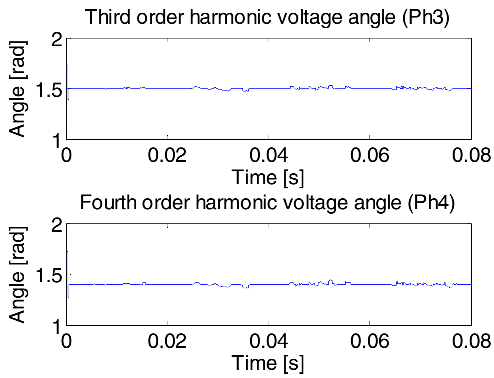

Fig. 20. Signal parameters estimation over time (φ3 and φ4).

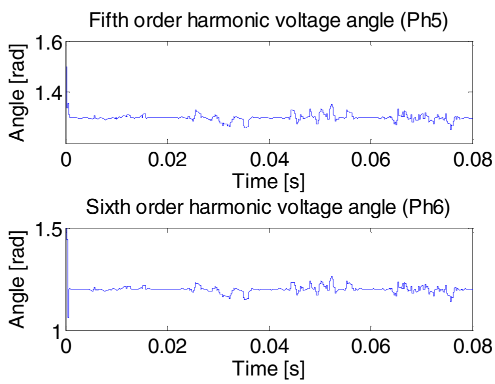

Fig. 21. Signal parameters estimation over time (φ5 and φ6).

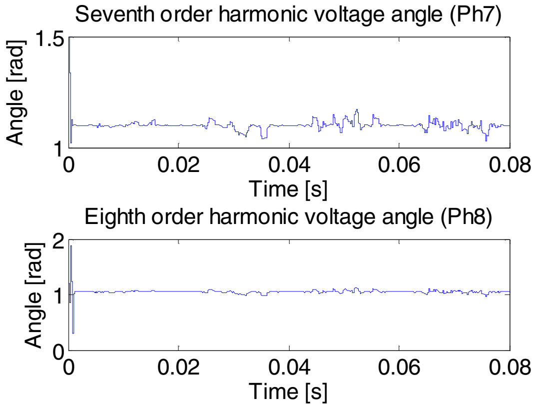

Fig. 22. Signal parameters estimation over time (φ7 and φ8).

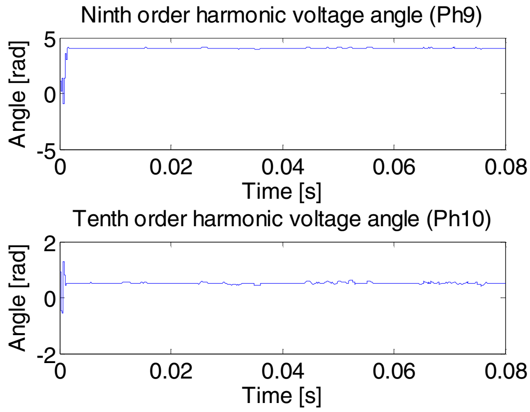

Fig. 23. Signal parameters estimation over time (φ9 and φ10).

IV. Conclusion

In this paper, a modified NTA was analysed to estimate the electrical signal spectral composition. Consequently, it was given particular attention to the accuracy and sensitivity of the algorithm for base frequency and harmonic content characterization under noisy measurements. From the point view of a clean signal it was demonstrated that the speed convergence speed is really quick since the group of estimated variables are obtained just over 1ms. In addition, the phasor angle – tested for very small values – and the response was not only quick but also accurate. As for the tests made with a signal injection with a given level of noise (SNR = 72 dB) it was observed that the amplitude of harmonic phasors are also estimated in the same interval of quickness. However, it was observed that for the phasor angles the convergence speed is apparently identical; yet, there is a tendency to show permanent oscillations around of the value that is expected for every phasor angle.

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