

Energy Optimization Strategy with Model Predictive Control and Demand Response

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Abstract—The overall price of energy is gradually increasing as a result of a constant escalating demand and limited supply. Consequently, the idea of demand response is being entertained by researchers and policy makers as a viable solution to the challenges ahead. Thus, new methods that aim to reduce the energy consumption in the residential sector are required to face such challenges. However, in order to optimize the consumption of energy while guaranteeing a certain level of comfort in the interior of the building could generate several control challenges. The goal of this paper is to compare the performance of control methods such as the Model Predictive Control (MPC), ON/OFF, and proportional-integral-derivative (PID) of a domestic heating, ventilation and air conditioning (HVAC) system controlling the temperature of a room. The house with local solar microgeneration is modelled approximating a location in a Portuguese city – Évora – pilot in a demand response project. The residence of the case study is subject to the local solar irradiance, temperature and six Time-of-Use (ToU) electricity rates applied on an entire week of July 2016. The aim of this paper is to accomplish the best compromise between temperature comfort levels and energy costs given by the performance of the fittest control method under different ToU rate options.

Keywords—Model predictive control; Energy management controller; Photovoltaic microgeneration; Residential building; Energy optimization.

I. INTRODUCTION

Concerns regarding climate change tend to grow when confronted with the damaging consequences of rapid and uncontrolled urbanisation. To cope with the current energy consumption growing rate, several efforts are necessary to oppose environmental threats [1].

Results published by the Intergovernmental Panel on Climate Change (IPCC) emphasised the requirement to preserve the GHG below 450 ppm CO₂ equivalence by 2050 in order to maintain the increase of the temperature of the planet under 2°C [2].

Countries gradually concentrate their initiatives and environmental policies on reducing the negative environmental repercussions of careless energy consumption [3]. The energy sector is experiencing substantial transformation driven by legislation with the purpose to reduce energy consumption and the consequently related environmental impacts [4]. Presently, the consumption of energy in buildings is responsible for circa 33% of the final energy consumption on the planet [5].

In the case of primary energy consumption, the building sector embody about 40% in most of the IEA (International Energy Agency) nations are accountable for 36% of the European Union (EU) CO₂ emissions [6]. Also, the same sector absorbs 40% of EU final consumption and 60% of electricity consumption [7]. As part of the building sector, the residential sector is accountable for 60% of the final energy consumption and presents the highest prospective to decrease the peak demand which is described by the volatility of energy utilisation [8]. Consequently, new methods that aim to reduce the energy consumptions in the residential sector are required to face such challenges. The research community and policy makers are constantly attempting for better and improved ways of control with the purpose of increasing the energy efficiency of domestic appliances. Several forms of control have been contemplated such as fuzzy logic [9], particle swarm optimization [10], PID control [11], artificial neural networks [12], MPC [13], among others [14].

Academics around the globe have been researching a vast range of control techniques with the goal to achieve an efficient energy utilisation of HVAC systems and other domestic appliances [15]. A model predictive and genetic algorithm-based optimisation of residential temperature control of an HVAC unit and an electric heater in the presence of time-varying electricity prices was presented in [16]. In [17] the authors develop an MPC model with the purpose to minimise the energy consumption of the air temperature and flow rate of an air-handling-unit for multi-zone variable air volumes. In [18] the MPC was combined with conventional local loop PID controllers in a hierarchical structure in order to minimise the energy consumption using current energy sources and minimal retrofitting and using weather predictions. An MPC architecture design for the optimal temperature control of a real commercial building was presented in [19]. In [20] was built a Berkeley retrofitted and inexpensive HVAC testbed with the purpose to reduce the transient and steady state electricity consumption in HVAC systems using learning-based MPC. An economic MPC to a commercial building HVAC system in Milwaukee WI that described the effectiveness of the method in closed-loop load shifting and demand reduction was applied in [21]. In [22] was proposed an economic MPC operated building aggregator that can combine a significant amount of the flexible power consumption of a group of commercial building HVAC fans as fast regulation reserve to the grid. A hierarchical distributed MPC algorithm for HVAC Systems in order to regulate the temperature of buildings is shown in [23].

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The goal of this paper is to make a comparison of the MPC performance with the ON/OFF and PID control of a domestic HVAC system controlling the temperature of a room under six different demand response electricity ToU rates (Options A to F) in which five of these options are the ones currently applied by the Portuguese electricity retailer [24]. The sixth one (Option F) is an hourly price signal during 24h as can be seen in Fig. 1. For this study, the home was modelled with local solar microgeneration, and the location was chosen to be in the city of Évora, Portugal since this city is the first pilot city in a demand response project- InovGrid [25]. In this study, a complete week of July of 2016 was covered in which very high levels of ambient temperature and solar irradiance were observed. Hence, a comparison is made of the HVAC performance with the purpose in preserving the temperature of a room controlled by ON/OFF, PID and MPC steady while subject to 6 types of electricity ToU rates. These rates are represented in this paper as Options A to F. The entirety of the results, detailed analysis and further conclusions will be published in the final version of the paper.

The remainder of the paper is organized as follows: in Section II the overall overview of the PID and MPC methodologies are presented. And in this chapter, the model of the room is also presented. Then, in Section IV obtained results are discussed. Finally, conclusions are drawn in Section IV.

II. METHODOLOGY

A. Model Predictive Control

The MPC is also known in the literature by many different designations, one of the most recognised being the receding-horizon control and then others such as rolling-horizon planning, dynamic matrix control, and dynamic linear programming [26]. The MPC is an optimal control technique which is intended to optimise a series of manipulated variable adjustments bound by a prediction horizon and has been demonstrated to be very effective given its capability to deal with multiobjective problems and handle hard constraints explicitly [27]. It does so through the use of a process model in order for the optimisation predictions of process performance based on a linear or quadratic objective, restrained by equality or inequality constraints. In such a control technique the optimisation is performed repeatedly on-line – the receding horizon which is the inherent contrast between MPC and other control methods. In perfect circumstances only the suboptimal result for the total solution can be achieved, this is the restriction of such finite-horizon optimisation. Yet, the optimisation of the receding horizon can efficiently include the uncertainties suffered by the model, also the time-varying disturbances and behaviour [28].

The upcoming outputs for a given horizon (N), known as the prediction horizon, by utilising the process model are projected at each instant t . The forecasted outputs $y(t+k|t)$ for $k=1\dots N$ hinge on the identified values such as the past inputs and outputs until instant t and also on the forthcoming signals of control $u(t+k|t)$ for $k=0\dots N-1$ considered to be the ones to be dispatched to the system and then calculated. The array of upcoming control signals is calculated through the optimization of a given condition of maintaining the process as near as possible to $w(t+k)$ – the reference trajectory.

The aforementioned condition normally has the representation of a quadratic function of the errors among the forecasted output signal and the forecasted reference trajectory.

The state space equations utilized in this paper are represented as follows:

$$\dot{x}(t) = Ax(t-1) + Bu(t-1) \quad (1)$$

$$y(t) = Cx(t) \quad (2)$$

where x stands for the state and the matrices of the system are given by A , input matrix B and output matrix C . In the case of the single-input single-output (SISO) model, $x(t)$ is the state vector and $y(t)$ and $u(t)$ are perceived as scalars.

The prediction equations for the state space model are represented as follows:

$$\hat{y}(t+k|t) = C\hat{x}(t+k|t) \quad (3)$$

$$\hat{y}(t+k|t) = C \left[A^k x(t) + \sum_{i=1}^k A^{i-1} B u(t+k-i|t) \right] \quad (4)$$

Such type of representation is superior over other representations since it can be utilized for multivariable processes in a simple way.

Thus, the control command is basically the response of a linear aggregation of the state vector, even though occasionally the chosen state basis does not indicate any physical signification. In the case of the states not being accessible the calculations might not be devoid of complexity due to the requirement of an inclusion of an observer.

In the case of the model input being the control increment $\Delta u(t)$ as a replacement for the control signal $u(t)$ a cumulative state space model could also be utilized.

Such model can be given in the broad state space condition by considering the following:

$$\Delta u(t) = u(t) - u(t-1) \quad (5)$$

The next representation is achieved through the combination of equation (5) with (1) and (2):

$$\begin{bmatrix} x(t+1) \\ u(t) \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} x(t) \\ u(t-1) \end{bmatrix} + \begin{bmatrix} B \\ I \end{bmatrix} \Delta u(t) \quad (6)$$

$$y(t) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ u(t-1) \end{bmatrix} \quad (7)$$

The minimization equation of cost function is as follows:

$$J = \sum_{j=1}^{n_H} [y(t+h_j) - w(t+h_j)]^2 \quad (8)$$

where generally $w(t+j)$ is represented by a first-order outline to the established reference. Since an impulse could be treated as the distinction between two steps with an interval of 1 sampling period, it may be given for a linear system as follows:

$$h_i = g_i - g_{i-1} \quad (9)$$

$$\mathbf{g}_i = \sum_{j=1}^i h_j \quad (10)$$

in which the sampled output values for the step input is represented by g_i . The basic concept of the behavior of the MPC can be seen in Fig. 1.

B. Proportional–Integral–Derivative Control

The most important characteristics of the PID controller can be given by the following equation:

$$u(t) = K \left(e(t) + \frac{1}{T_i} \int_0^t e(\tau) \times d\tau + T_d \frac{de(t)}{dt} \right) \quad (11)$$

where the measured process variable is represented by y , the reference variable is given by τ , the control signal is expressed by u and e gives the control error as in:

$$e = y_{sp} - y \quad (12)$$

The reference variable is frequently named the set point. Therefore, the control signal is defined by the summation of three terms: proportional to the error – P, the proportional to the integral of the error – I, and the proportional to the derivative of the error – D. The criterions of the PID controller are all non-negative and are given by the proportional gain K , integral time T_i , and derivative time T_d .

C. The model of the room

The acclimatization of the room is done with an HVAC system with a power cooling capacity of 3.516 kW. The heat exchange with the outside occurs through the outer wall of the room, and it is the main source of disturbance of the selected thermal comfort level of the room. With the aim of testing all three control strategies, the rate of heat loss/generation through external wall of the room is modelled by means of a temperature based time series with a major thermal amplitude variation upon 24 hours. The ON/OFF, the PID and the MPC are fixed with a limit of ± 1 °C and having as reference 23°C.

With the aim of creating pleasant and sustaining interior surroundings in terms of temperature in distinct rooms of the house – extra energy needs to be consumed with the purpose to remove or insert heat. As a result, the preferred comfort level is set by choosing a reference temperature and by assessing the space air temperature. The comfort level based on temperature is interrupted by the quantity of residents that inhabit the household, the thermal mass of the space itself, and by the exchange of heat with the external surroundings through the outer walls as can be observed in Fig. 2. As a result, the temperature dynamics of room in the house derives from such factors as the balance of energy of the outside environment temperatures and the HVAC equipment that inserts or extracts heat from the room in permutation with the indoor thermal mass as depicted in Fig. 2.

With the objective of calculating and comparing the behavior of the controller a thermal mass model utilizing a resistance-capacitance circuit analogy is modelled. The abovementioned model contains the heat flow balance amidst the thermal capacitance of the internal air and the outer wall and windows of the room of a house [29].

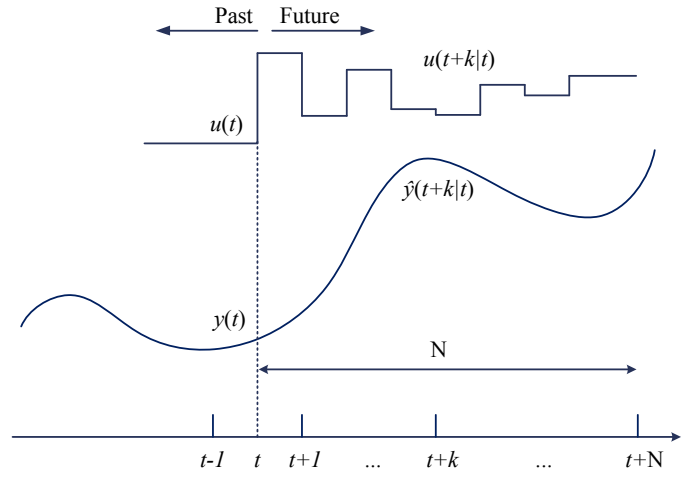


Fig. 1. Basic concept of the behaviour of the MPC.

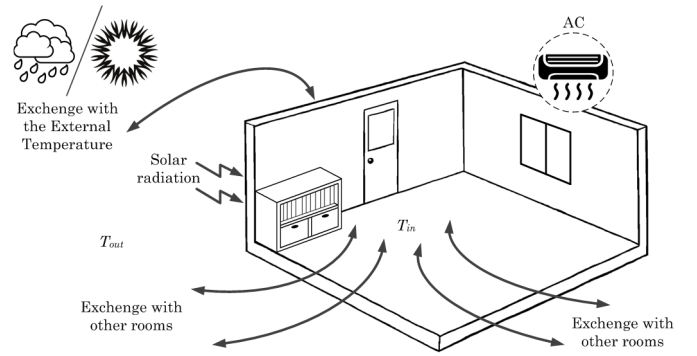


Fig. 2. Indoor environment temperature control.

With the purpose of having a uniformed temperature in the room, it is assumed that the air inside is homogeneously mixed. The following expressions were withdrawn from [30]:

$$\frac{dT_{wl}}{dt} = \frac{Q_s}{C_{wl}} + \frac{T_{in} - T_{wl}}{R_{wl}C_{wl}} \quad (13)$$

$$\frac{dT_{in}}{dt} = \frac{Q_{ac} \times S(t)}{C_{in}} + \frac{T_{out} - T_{in}}{C_{in}R_{wd}} + \frac{T_{wl} - T_{in}}{C_{in}R_{wl}} \quad (14)$$

$$Q_s = A_w h_o (T_{out} - T_s) \quad (15)$$

where the cooling power input to the room is represented by Q_{ac} , the ambient temperature by T_{out} , T_{in} give the temperature of the room, the wall temperature is represented by T_{wl} , the thermal capacitance of the wall by C_{wl} , and the thermal resistance of the wall by R_{wl} , R_{wd} represents the thermal resistance of the windows, the thermal capacitance of the indoor air is given by C_{in} and the heat flow into an outer surface of the house subjected to solar radiation by Q_s . The combined convection and radiation heat transfer coefficient is expressed by h_o , the wall area is represented by A_w , T_s represents the wall surface temperature. Finally, $S(t)$ represents a binary variable that emulates the turn-on and turn-off of the ON/OFF.

For this study, the operation of AC is represented by a power switch block without internal losses. All the data of the physical parameters are acquired from [31].

III. RESULTS AND DISCUSSION

As soon as the energy consumption of the HVAC performance during the entire week was calculated, the consumed energy cost can be estimated by taking into account all the available ToU rates by the electricity retailer for the residential sector. The retailer announces a price signal for the 24h of the optimization horizon as displayed in Fig. 3.

The consumed energy cost in cents of ON/OFF, PID and MPC of the controlled HVAC system by employing the 6 ToU rates is shown in the Figs. 4-6. By analysing them it is possible to observe that the consumed energy cost in cents by the HVAC system controlled by MPC is lower than the remaining options, ON/OFF and PID.

A proof of this is, for instance, 27th of July, where is possible to observe that the MPC control method consumes energy with a cost in cents in the area of 200-400 for any given tariff options. This, however, is not followed by the remaining challengers.

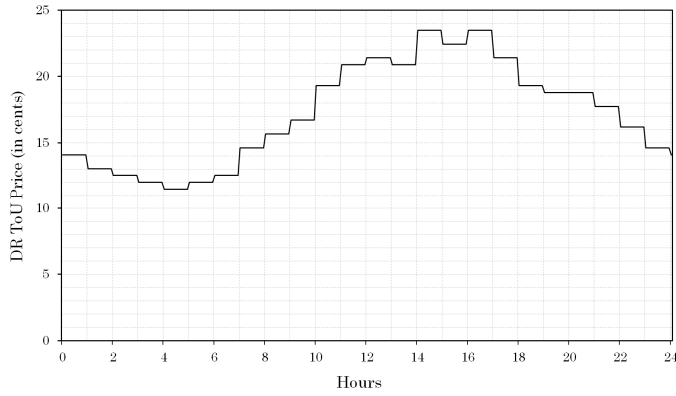


Fig. 3. Option F – A ToU price signal for the 24h of the optimization horizon.

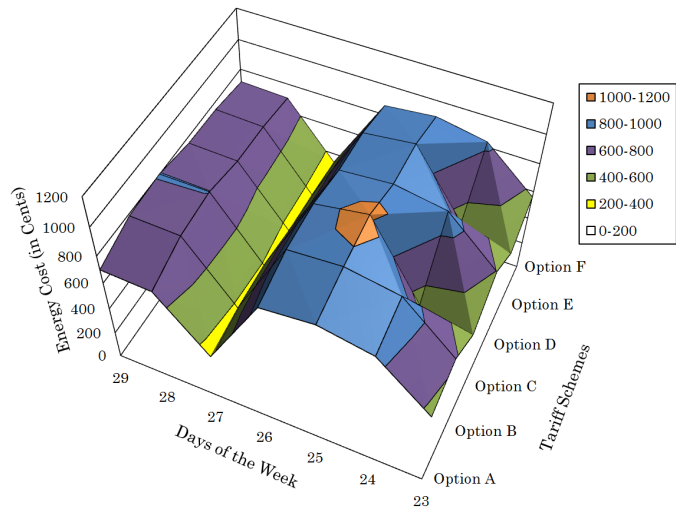


Fig. 4. The consumed energy cost in cents by the MPC of the controlled HVAC system by employing the 6 options of ToU rates.

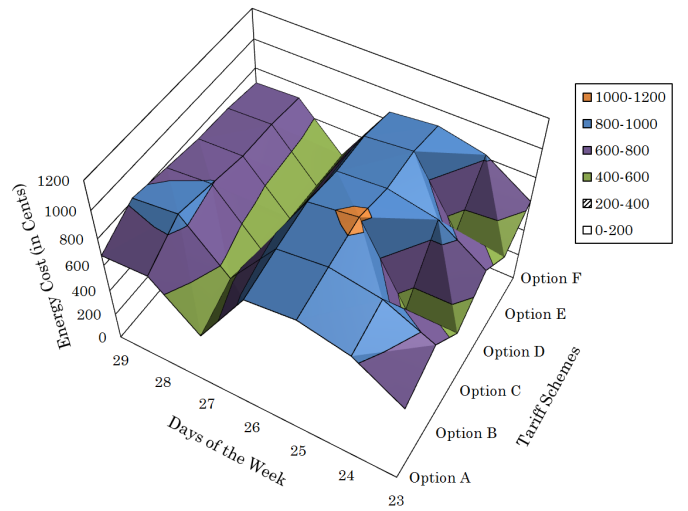


Fig. 5. The consumed energy cost in cents by the PID of the controlled HVAC system by employing the 6 options of ToU rates.

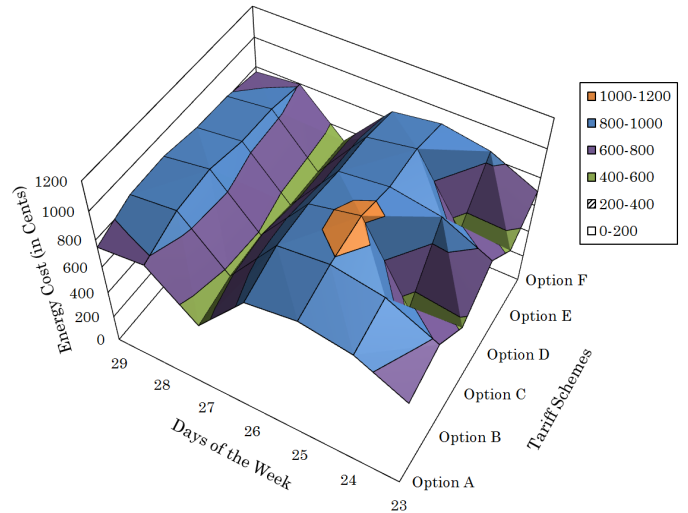


Fig. 6. The consumed energy cost in cents by the ON/OFF of the controlled HVAC system by employing the 6 options of ToU rates.

Table I shows the overall results of the simulation of all the three control methods in function of all tariff options, including the hourly price signal during 24h, option F. By carefully observing Table I, it is noticeable that the MPC option is the best among the studied methods, and the cheapest tariff option is E. This tariff is a three tier pricing scheme that offers lower prices during weekends. Also, it can be observed that ON/OFF is the most expensive control method between the three.

TABLE I. THE COST IN € FOR EACH TARIFF OPTION

Tariff Options	PID	ON/OFF	MPC
Option A	49,76 €	53,54 €	49,13 €
Option B	57,65 €	59,67 €	54,23 €
Option C	50,49 €	54,47 €	49,56 €
Option D	53,45 €	57,65 €	52,30 €
Option E	47,45 €	51,26 €	46,55 €
Option F	49,80 €	53,71 €	48,97 €

The MPC control method allows savings of 9.2% when compared to ON/OFF, meanwhile the PID allows savings of 7.4% when compared with the conventional ON/OFF.

Option F – a price signal for the 24h of the optimization horizon announced by the retailer is still the second best tariff option and is only outshined by tariff option E due to the prices practiced during weekend periods. Nevertheless, this tariff option is only 5.2% more expensive when compared with the best solution for the consumer – option E. The most expensive option is Option B – a two tier pricing scheme that is always the same during the entire period of summer with two alternating levels. The consumer saves 14.2% by choosing tariff option E instead of option B and by opting for the MPC control method.

IV. CONCLUSION

In this paper, a comparison was made of the performance of control methods such as MPC, ON/OFF and PID of a domestic heating, HVAC system controlling the temperature of a room. The house with local solar microgeneration was modelled approximating a location in a Portuguese city – Évora – pilot in a demand response project. The residence of the case study is subject to the local solar irradiance, temperature and six ToU electricity rates applied on an entire week of July 2016. The aim of this paper was to accomplish the best compromise between the temperature comfort levels and the energy costs given by the performance of the fittest control method under different ToU rate options. Thus, the energy consumption of the HVAC during the whole week was assessed as well as the final price of the consumed energy. The final results indicated that the MPC solution was the less costly between all the available options while the ON/OFF was the most expensive solution.

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