

Home HVAC Energy Management and Optimization with Model Predictive Control

Radu Godina¹, Eduardo M. G. Rodrigues¹, Edris Pouresmaeil², João P. S. Catalão^{1,2,3}

¹C-MAST/UBI, Covilha; ²INESC-ID/IST-UL, Lisbon; ³INESC TEC and FEUP, Porto

PORTUGAL
catalao@fe.up.pt

Abstract—The general energy demand of the residential sector and the ensuing option for fossil fuels produce adverse results by both CO₂, greenhouse gases (GHG) and extra air pollutant emissions. As domestic energy demand consists mostly of energy necessities for space and water heating alongside the energy dedicated for appliances, distinct strategies that target to foment a practical consumption of energy have to be reinforced at all levels of human activity. In this paper the aim is to make a comparison between proportional-integral-derivative (PID), thermostat (ON/OFF) control and Model Predictive Control (MPC) models of a domestic heating, ventilation and air conditioning (HVAC) system controlling the temperature of a room. The model of the household with local solar microgeneration is implicit to be located in a Portuguese city. The house of the case study is at the mercy to the local solar temperature, irradiance and 5 Time-of-Use (ToU) electricity rates applied on a complete week of August, 2016. The second purpose of this study is to assess which is the best electricity ToU rate option provided by the local electricity retailer for the residential sector.

Keywords— Energy optimization; Model predictive control; Energy management controller; Photovoltaic microgeneration; Residential building.

NOMENCLATURE

A_w	The wall area.
A	The state (or system) matrix.
B	The input matrix.
C	The output matrix.
C_{in}	The thermal capacitance of the indoor air.
C_{wl}	The thermal capacitance of the wall.
h_o	The combined convection and radiation heat transfer coefficient.
I	The identity matrix.
$J(k)$	Infinite horizon performance cost.
k	Sampling instant and the current control interval.
$k+i$	The time instant associated to the future state prediction for $i=1 \dots N$.
N	The prediction horizon.
M	Positive definite matrix.
P	The control horizon.

Q_{ac}	The cooling power input to the room.
Q_s	The heat flow into an exterior surface of the house subjected to solar radiation
R_{wd}	The thermal resistance of the windows.
R_{wl}	The thermal resistance of the wall.
$S(t)$	A binary variable that emulates the turn-on and turn-off of the thermostat.
T_{in}	The temperature of the room.
T_{out}	The ambient temperature.
T_s	The wall surface temperature.
T_{wl}	The wall temperature.
U	The non-empty set described with linear inequalities.
$u(\cdot k)$	Future control sequence.
$u(k+i k)$	Future control signals for $i=0 \dots P-1$.
W	Positive definite matrix of the performance weights.
$x(k)$	The state vector.
$x(k k)$	The current state.
$y(k)$	The system output.
$y(k+i k)$	The estimated outputs.

Table of abbreviations

GHG	Greenhouse Gases
HVAC	Heating, ventilation and air-conditioning
LQ	Linear Quadratic
MIMO	Multiple Input-Multiple Output
MPC	Model Predictive Control
ON/OFF	Thermostat
SISO	Single-Input and Single-Output
ToU	Time of Use Tariff

I. INTRODUCTION

As a result of concerning alterations in climate, oscillations in costs of energy, uncertainty regarding energy sources and the growth in energy demand, a rising interest has been widely witnessed in alternative methods of generating, distributing and consuming energy while keeping it under control [1].

This work was supported by FEDER funds through COMPETE 2020 and by Portuguese funds through FCT, under Projects SAICT-PAC/0004/2015 - POCI-01-0145-FEDER-016434, POCI-01-0145-FEDER-006961, UID/EEA/50014/2013, UID/CEC/50021/2013, and UID/EMS/00151/2013. Also, the research leading to these results has received funding from the EU 7th Framework Programme FP7/2007-2013 under GA no. 309048.

Thus, in order to effectively tackle such substantial challenges the planning of energy efficient buildings as one of the paths to follow in order to achieve a proper sustainable development [2]. Around the world researchers and policy makers are aiming on decreasing the GHG emissions and air pollution through the expansion of the electricity originating from renewable resources and have already started an important change in producing and supplying of electricity [3], [4].

A fundamental purpose of housing is to safeguard its inhabitants from hostile weather, mainly during periods of severe ambient temperatures which undoubtedly influence the thermal comfort. In order to keep a satisfying comfort inside the residence it becomes essential for indoor air temperature and the temperature of internal surfaces of the house enclosure to be kept within a specific desired interval. Combined they generally depend on fluxes of heat emerging through internal surfaces of the residence [5].

According to researchers, the buildings sector is accountable for circa 30% of the overall energy consumption in the world, of which the housing sector is responsible for 74% of the entire energy consumption [6], [7]. This sector is also responsible for circa 8% of direct CO₂ emissions from the end user. It is estimated that if deprived of any energy efficient solutions in this field, the overall energy demand will rise by 50% in 2050 [8].

All in all, two distinct methods at this time strive to achieve energy efficiency gains: the presence of more energy saving equipment in the dwelling, or the efficient control of the energy consumption by employing an EMS [9]. HVAC systems uphold the most significant part of building energy consumption [10] with around 20–50% and are responsible for a main share of energy consumption [11].

Quite a few methods based on the MPC have been proposed and tested with the intention to optimize the operation of HVAC systems. The MPC is, mostly, an optimization based approach in which a clear model is utilized to predict the performance of the controlled plants over a receding horizon. The recognition by the research community of the MPC occurred ever since its first use in the process industry in 1970s. At the moment, the MPC method is largely utilized in several applications that range from railway traffic management [12] to supply chain systems [13]. In the literature, the research of MPC is mainly aimed towards a centralized implementation. In contrast, with the quick progress of energy efficiency technologies and the mandatory increase in the economic performance, large scale systems, such as EMS, tend to be more complex [14]. Consequently, new and original methods are required to involve and control numerous utility-owned and third-party assets in a sustainable and reliable way.

The MPC method is thought to have better qualities when compared to the classical control techniques such as ON/OFF and PID. PID controllers are inferior since they display low accuracy in processes which are either non-linear or have a large time delay. For example, PID controllers only succeed in efficiently single input single output (SISO) systems while the MPC is capable to manage multiple input-multiple output (MIMO) systems, to display a greater accuracy, to operate with constraints, is robust when fronting disturbances and has the ability to predict the performance of the controlled plants over a

receding horizon. Nevertheless, such an upper hand is counterweighted with greater computational requirements [15].

The goal in this paper is to compare the MPC behavior with the PID and ON/OFF control of a domestic HVAC system regulating the temperature of a room subject to five different electricity summer ToU rates which are currently in force by the Portuguese electricity retailer [16]. A household model of with local solar microgeneration is considered for this study and the setting is considered to be in the city of Covilhã, Portugal. The focus is on an entire summer week of August of 2016 in which excessive levels of ambient temperature and solar irradiance were witnessed. As stated by weather researchers it was the second hottest August in Portugal since 1931 [17]. Hence, the aim is to compare the aforementioned control methods of the HVAC system under high temperatures witnessed during this period by applying the available ToU rates for summer seasons.

The remainder of the paper is organized as follows: in Section II the general overview of the MPC methodology is presented. Then, in Section III the developed test case is shown while in Section IV obtained results are discussed. Finally, conclusions are drawn in Section IV.

II. GENERAL OVERVIEW OF THE MPC

This control method which is intended to optimize a series of manipulated variable adjustments bound by a prediction horizon functions as such through the use of a process model in order for the optimization predictions of process performance based on a linear or quadratic objective, restrained by equality or inequality constraints. In a control technique of such a nature the optimization is executed repeatedly on-line – the receding horizon which is the inherent contrast between MPC and other control methods. In ideal conditions only the suboptimal result for the total solution can be accomplished, such is the limitation of such finite-horizon optimization. Nonetheless, the optimization of the receding horizon can adequately include the uncertainties suffered by the model, also the time-varying disturbances and behavior [18]. The forecasts of the MPC are formed by utilizing a dynamic model, normally a linear model. The basic concept of the behavior of the MPC can be observed in Fig. 1.

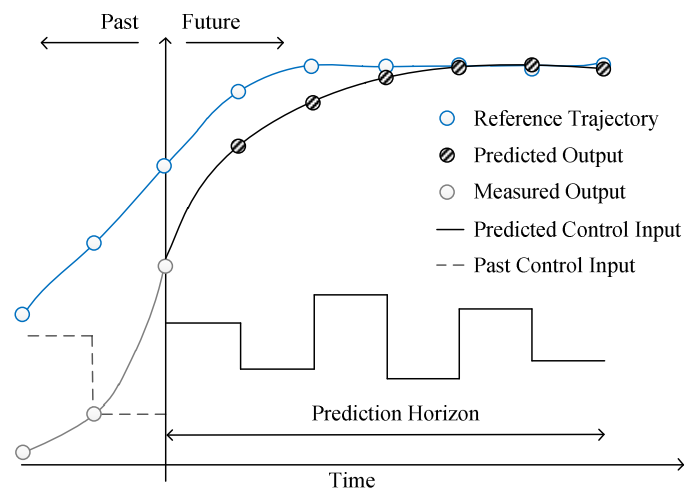


Fig. 1. Basic concept of the behaviour of the MPC.

A. Elemental MPC Controller Formulation

The most comprehensive state-space representation of a linear system with p inputs [19], q outputs and n state variables is represented as follows:

$$x(k+1) = Ax + Bu(k) \quad (1)$$

$$y(k) = Cx(k) \quad (2)$$

where

$$x(k) \in \mathbb{R}^n, u(k) \in \mathbb{R}^p, y(k) \in \mathbb{R}^q \quad (3)$$

The follow-up of such representation is that the system will be observable and controllable.

As previously mentioned, the MPC is an optimization based control law, and in an elemental MPC controller the performance measure is nearly each time a quadratic cost. Through the representation of the positive definite matrices as:

$$M = M^T \succ 0 \quad (4)$$

and the performance weights is given by:

$$W = W^T \succ 0 \quad (5)$$

The optimal control input has to be identified in order to minimize the infinite horizon performance cost:

$$J(k) = \sum_{j=k}^{\infty} x^T(j|k)Mx(j|k) + u^T(j|k)Wu(j|k) \quad (6)$$

In an unconstrained scenario, the solution to this equation is given by the linear quadratic (LQ) controller. Yet, in a constrained scenario, no analytic solution exists. As an alternative, the objective in the MPC is to establish a prediction horizon N and approximate the problem with a finite horizon cost:

$$J(k) = \sum_{j=k}^{k+N-1} x^T(j|k)Mx(j|k) + u^T(j|k)Wu(j|k) \quad (7)$$

The finite horizon is essential since it is due to it that it is possible to solve the problem, but simultaneously, other complications are brought by the finite horizon.

By utilizing the model from (1) and (2), it is possible to predict the state $x(k+j|k)$, given a future control sequence $u(\cdot|k)$ and the current state $x(k|k)$. In such a case, no state estimation is obligatory and it is assumed that $C=I$, therefore, $x(k|k) = x(k)$. Consequently, the prediction is represented as:

$$x(k+j|k) = A^j x(k|k) + \sum_{i=0}^{j-1} A^{j-i-1} Bu(k+i|k) \quad (8)$$

By utilizing such predictions, it is possible to define the following optimization equation:

$$\min_u \sum_{j=k}^{k+N-1} x^T(j|k)Mx(j|k) + u^T(j|k)Wu(j|k) \quad (9)$$

which is subject to:

$$u(k+j|k) \in U \quad (10)$$

and

$$x(k+j|k) = Ax(k+j-1|k) + Bu(k+j-1|k) \quad (11)$$

and thus, it is possible to design a basic MPC controller.

III. CASE STUDY

The room is acclimatized with a HVAC system with a power cooling capacity of 3.516 kW. The heat exchange with the exterior occurs through the outer wall of the room and it is the most important cause of disturbance of the preferred thermal comfort level of the room. With the purpose of testing the three control strategies, the rate of heat loss/generation through the model of the external wall of the room is simulated by means of a temperature based time series with significant wide thermal amplitude variation upon 24 hours. The ON/OFF, the PID and the MPC are fixed with a limit of ± 1 °C and having as reference 23°C.

A. The model of the room

With the purpose of constructing enjoyable and satisfying interior environments in terms of temperature in distinct rooms of the house - additional energy needs to be consumed with the purpose to remove or insert heat. As a result, the preferred comfort level is established by choosing a reference temperature and by assessing the space air temperature. The comfort level based on temperature is disrupted by the amount of residents that inhabit the dwelling, the thermal mass of the space itself, and by the exchange of heat with the external environment through the external walls as can be observed in Fig. 2. Consequently, the temperature dynamics of a room in the house derives from such factors as the balance of energy of the outside environment temperatures and the HVAC equipment that inserts or extracts heat from the room in permutation with the indoor thermal mass as depicted in Fig. 2.

With the objective of calculating and comparing the behavior of the controller a thermal mass model utilizing a resistance-capacitance circuit analogy is modelled. The aforementioned model contains the heat flow balance between the thermal capacitance of the internal air and the external wall and windows of the room of a house [20]. With the purpose of having a uniformed temperature in the room it is assumed that the air inside is homogeneously mixed. The following expressions were withdrawn from [21]:

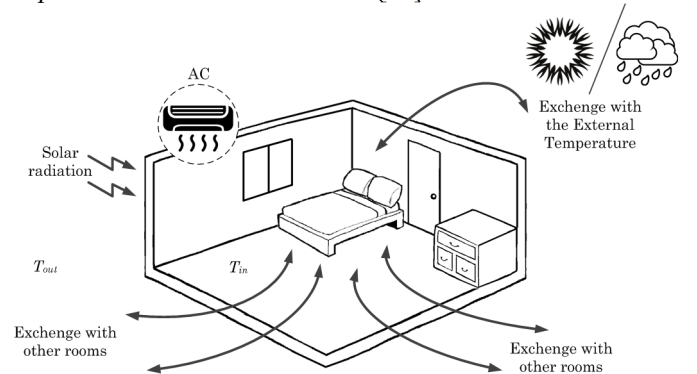


Fig. 2. Indoor environment temperature control.

$$\frac{dT_{wl}}{dt} = \frac{Q_s}{C_{wl}} + \frac{T_{in} - T_{wl}}{R_{wl}C_{wl}} \quad (12)$$

$$\frac{dT_{in}}{dt} = \frac{Q_{ac} \times S(t)}{C_{in}} + \frac{T_{out} - T_{in}}{C_{in}R_{wd}} + \frac{T_{wl} - T_{in}}{C_{in}R_{wl}} \quad (13)$$

$$Q_s = A_w h_o (T_{out} - T_s) \quad (14)$$

where Q_{ac} gives the cooling power input to the room, the ambient temperature by T_{out} , T_{in} quantifies the temperature of the room, the wall temperature is given by T_{wl} , the thermal capacitance of the wall by C_{wl} , and the thermal resistance of the wall by R_{wl} , R_{wd} represents the thermal resistance of the windows, the thermal capacitance of the indoor air is given by C_{in} and the heat flow into an exterior surface of the house subjected to solar radiation by Q_s . The combined convection and radiation heat transfer coefficient is quantified by h_o , the wall area is represented by A_w , T_s represents the wall surface temperature. Finally, $S(t)$ represents a binary variable that emulates the turn-on and turn-off of the ON/OFF. For this study, the operation of AC is represented by a power switch block without internal losses. All the data of the physical parameters are acquired from [22].

The dwelling model with local solar microgeneration is assumed to be located in Portugal, specifically in the city of Covilhã. The modelling tool employed in this study is Simulink, developed by MathWorks, Inc. The solar PV panel capacity used in this study is 0.55kW. The dwelling of the case study is subject to the local solar irradiance, temperature and electricity ToU rates of a specific week of summer – from 8th to 14th August, 2016. Five different electricity ToU rate options are used in this study and the prices of the electricity tariffs can be observed in Table 1. The aforementioned five electricity ToU rate options are the ones currently applied by the Portuguese electricity retailer and can be observed in Fig. 1. Option B is the standard flat tariff of Portugal, option A and C are three tier Tariffs, and options D and E are both two tier Tariffs. Electricity ToU rates and price information was taken from [23] which were recently made available for the Portuguese residential market by the electricity retailer – EDP - Energias de Portugal. As it can be observed in Table 1 the highest price is naturally at critical peak hours with 0.27 €/kWh for three tier ToU rates and the lowest one is at valley hours for both two and three tier ToU rates with 0.12 €/kWh.

IV. RESULTS AND DISCUSSION

Once the assessment in maintaining the temperature of a room controlled by ON/OFF, PID and MPC of the HVAC performance is effectuated, the obtained results indicate that if the system is managed by ON/OFF the utilized energy in kWh is higher when compared to other two options, except Monday, 8th August. The obtained results have also revealed that the greatest HVAC operation is by using the MPC control option. The energy consumption of all three control options of the entire week can be observed in Fig. 4. The lower energy use witnessed on 10th of August occurs as a result of being the coolest day of the week, justifying a lower use of the HVAC system. The consumed energy cost of each controller option per ToU tariff option is shown in Fig. 5. By analyzing the information given by

this figure, it can be noticed that the MPC is the best controller option in terms of energy efficiency and the tariff ToU option A – a three tier pricing scheme – is the option that allows a lower energy consumption.

TABLE I. THE PRICES OF THE ELECTRICITY TARIFFS IN €/kWh

	Without VAT (in €)	With VAT (in €)
Flat Tariff	0.1634	0.2010
Two tier ToU rate Valley	0.1002	0.1232
Two tier ToU rate Non-Valley	0.1909	0.2348
Three tier ToU rate Valley	0.1002	0.1232
Three tier ToU rate Peak	0.1716	0.2111
Three tier ToU rate Critical Peak	0.2169	0.2668

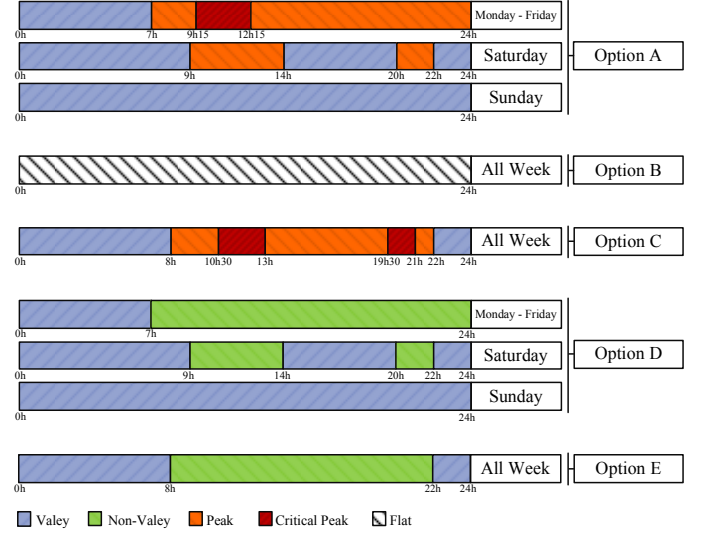


Fig. 3. The options of ToU electricity rates.

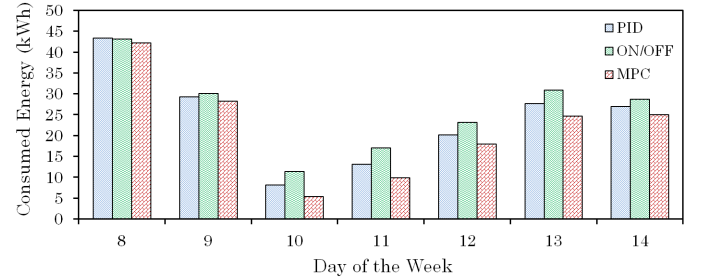


Fig. 4. The energy consumption of ON/OFF, PID and MPC of the controlled HVAC system.

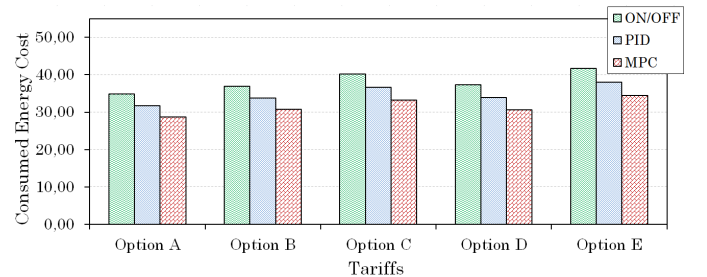


Fig. 5. The consumed energy cost of each ToU tariff.

V. CONCLUSION

A distinct MPC strategy with the goal to foment an efficient consumption of home heating energy has been presented in this paper. A comparison between the ON/OFF and PID control models and a MPC model of a domestic HVAC system managing the temperature of a room was made. The household model with local solar microgeneration was assumed to be located in Portugal, specifically in the city of Covilhã. The residential building of the case study was at the mercy of the temperature, local solar irradiance, and five ToU rates provided by the local electricity retailer for the residential sector and applied on an entire week of August, 2016. Therefore, the energy consumption of the HVAC system and its performance during the whole week was calculated and the consumed energy cost assessed. Obtained results showed that the MPC was the best controller option in terms of energy efficiency. It also showed that the tariff ToU option A, a three-tier pricing scheme, was the option that allowed a lower energy consumption degree.

REFERENCES

- [1] M. Hansen and B. Hauge, "Prosumers and smart grid technologies in Denmark: developing user competences in smart grid households," *Energy Effic.*, pp. 1–20, Mar. 2017.
- [2] F. Ascione, N. Bianco, C. De Stasio, G. M. Mauro, and G. P. Vanoli, "A new comprehensive approach for cost-optimal building design integrated with the multi-objective model predictive control of HVAC systems," *Sustain. Cities Soc.*, vol. 31, pp. 136–150, May 2017.
- [3] J. Sommerfeld, L. Buys, and D. Vine, "Residential consumers' experiences in the adoption and use of solar PV," *Energy Policy*, vol. 105, pp. 10–16, Jun. 2017.
- [4] D. Oliveira, E. M. G. Rodrigues, R. Godina, T. D. P. Mendes, J. P. S. Catalão, and E. Poursmaeil, "Enhancing home appliances energy optimization with solar power integration," in *IEEE EUROCON 2015 - International Conference on Computer as a Tool (EUROCON)*, 2015, pp. 1–6.
- [5] J. Lucero-Álvarez and I. R. Martín-Domínguez, "Effects of solar reflectance and infrared emissivity of rooftops on the thermal comfort of single-family homes in Mexico," *Build. Simul.*, vol. 10, no. 3, pp. 297–308, Jun. 2017.
- [6] A. Stefanović and D. Gordić, "Modeling methodology of the heating energy consumption and the potential reductions due to thermal improvements of staggered block buildings," *Energy Build.*, vol. 125, pp. 244–253, Aug. 2016.
- [7] D. Oliveira, E. M. G. Rodrigues, R. Godina, T. D. P. Mendes, J. P. S. Catalão, and E. Poursmaeil, "MPC weights tuning role on the energy optimization in residential appliances," in *2015 Australasian Universities Power Engineering Conference (AUPEC)*, 2015, pp. 1–6.
- [8] P. Marin et al., "Energy savings due to the use of PCM for relocatable lightweight buildings passive heating and cooling in different weather conditions," *Energy Build.*, vol. 129, pp. 274–283, Oct. 2016.
- [9] R. Godina, E. M. G. Rodrigues, E. Poursmaeil, J. C. O. Matias, and J. P. S. Catalão, "Model predictive control technique for energy optimization in residential sector," in *2016 IEEE 16th International Conference on Environment and Electrical Engineering (EEEIC)*, 2016, pp. 1–6.
- [10] J. Wang, G. Huang, Y. Sun, and X. Liu, "Event-driven optimization of complex HVAC systems," *Energy Build.*, vol. 133, pp. 79–87, Dec. 2016.
- [11] A. Beghi, L. Cecchinato, M. Rampazzo, and F. Simmini, "Energy efficient control of HVAC systems with ice cold thermal energy storage," *J. Process Control*, vol. 24, no. 6, pp. 773–781, Jun. 2014.
- [12] G. Caimi, M. Fuchsberger, M. Laumanns, and M. Lüthi, "A model predictive control approach for discrete-time rescheduling in complex central railway station areas," *Comput. Oper. Res.*, vol. 39, no. 11, pp. 2578–2593, Nov. 2012.
- [13] G. Schilbach and M. Morari, "Scenario-based model predictive control for multi-echelon supply chain management," *Eur. J. Oper. Res.*, vol. 252, no. 2, pp. 540–549, Jul. 2016.
- [14] M. G. Forbes, R. S. Patwardhan, H. Hamadah, and R. B. Gopaluni, "Model Predictive Control in Industry: Challenges and Opportunities," *IFAC-Pap.*, vol. 48, no. 8, pp. 531–538, Jan. 2015.
- [15] E. M. Wanjiru, L. Zhang, and X. Xia, "Model predictive control strategy of energy-water management in urban households," *Appl. Energy*, vol. 179, pp. 821–831, Oct. 2016.
- [16] ERSE, "Preços no mercado liberalizado de energia elétrica e gás natural em Portugal continental [in Portuguese]," Lisboa, 2016.
- [17] Instituto Português do Mar e da Atmosfera, I.P., "Resumo Climatológico – verão 2016 Portugal Continental [in Portuguese]," Lisboa, 2016.
- [18] Y. Zong, L. Mihet-Popa, D. Kullmann, A. Thavlov, O. Gehrke, and H. W. Bindner, "Model Predictive Controller for Active Demand Side Management with PV self-consumption in an intelligent building," in *2012 3rd IEEE PES Innovative Smart Grid Technologies Europe (ISGT Europe)*, 2012, pp. 1–8.
- [19] L. Wang, *Model Predictive Control System Design and Implementation Using MATLAB®*. Springer-Verlag London, 2009.
- [20] E. M. G. Rodrigues, R. Godina, E. Poursmaeil, J. R. Ferreira, and J. P. S. Catalão, "Domestic appliances energy optimization with model predictive control," *Energy Convers. Manag.*, vol. 142, pp. 402–413, Jun. 2017.
- [21] Y. Lin, T. Middelkoop, and P. Barooah, "Issues in identification of control-oriented thermal models of zones in multi-zone buildings," in *2012 IEEE 51st IEEE Conference on Decision and Control (CDC)*, 2012, pp. 6932–6937.
- [22] Y. A. Cengel, *Heat Transfer: A Practical Approach*. Boston: Mcgraw-Hill, 2002.
- [23] EDP, "(2016) Opção Horária - Apoio ao Cliente - EDP." [Online]. Available: <https://energia.edp.pt/particulares/apoio-cliente/opcao-horaria/> [Accessed: 11-Dec-2016].