

# Stochastic Security Constrained Unit Commitment with High Penetration of Wind Farms

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**Abstract**— Secure and reliable operation is one of the main challenges in restructured power systems. Wind energy has been gaining increasing global attention as a clean and economic energy source, despite the operational challenges its intermittency brings. In this study, we present a formulation for electricity and reserve market clearance in the presence of wind farms. Uncertainties associated with generation and line outages are modeled as different system scenarios. The formulation incorporates the cost of different scenarios in a two-stage short-term (24-hours) clearing process, also considering different types of reserve. The model is then linearized in order to be compatible with standard mixed-integer linear programming solvers, aiming at solving the security constrained unit-commitment problem using as few variables and optimization constraints as possible. As shown, this will expedite the solution of the optimization problem. The model is validated by testing it on a case study based on the IEEE RTS1, for which results are presented and discussed.

**Keywords**— Security-constrained unit commitment, market clearing, reserves, wind energy.

## NOMENCLATURE

### Indices and variables:

$i$	Index for units
$ILS$	Involuntarily load shedding
$j$	Index of bus number
$lg$	Loss of generation
$ll$	Loss of load
$msf$	Index for segment of piecewise linear cost function
$p_d$	Load demand of bus
$p_g$	Real power generation of each unit
$p_{non-wind}$	Total output power of non-wind units
$p_{wp}$	Wind farm output power
$R_g^{up}$	Up-going reserve (generation)
$R_g^{dn}$	Down-going reserve (generation)
$R_d^{up}$	Up-going reserve (demand)
$R_d^{dn}$	Down-going reserve (demand)
$s$	Index of scenario number
$sl$	Slopes' segment for piecewise linear cost function
$t$	Index of time
$u$	Unit state indicator (1 denotes unit is on)
$y$	On state indicator
$z$	Off state indicator
$\pi$	Probability of each scenario

### Matrices and vectors:

$B$	DC load flow matrix
$BG$	Bus-generator incidence matrix
$f^{max}$	Vector of lines maximum capacity
$GSF$	Matrix of generation shift factors
$LMP$	Vector of locational marginal prices
$outg$	Units outage vector
$PG$	Generation power vector
$PD$	Demand power vector
$RG$	Vector of generation side reserves
$RD$	Vector of demand side reserves

### Parameters:

$N_d$	Number of buses
$N_g$	Number of units
$N_l$	Number of lines
$N_s$	Number of scenarios
$Q_g$	Generation side offered reserve price
$Q_d$	Demand side offered reserve price
$Q_{wp}$	Wind power price
$SDC$	Shut-down cost of each unit
$SUC$	Start-up cost of each unit

## I. INTRODUCTION

Economic aspects of power systems have become an important issue in power engineering research after power systems restructuring. Various strategies have been proposed to minimize energy production costs, among which are Unit Commitment (UC) and economic load dispatch, which aim to provide customers with high quality electrical power in economic and secure conditions [1]. Economic performance of power systems is also an important concern of independent system operators (ISO). Various up and down reserves on generator side and probably demand side should be considered to protect system reliability in case of loss of units or other equipment. Secure operation is among the biggest challenges in modern power systems for independent system operators (ISO) [2], which face various burdens regarding security and reliability such as providing system reserve [3].

Various solution approaches for the UC problem exist. Those are generally categorized into three types: numerical, heuristic, and combinatorial approaches. Heuristic methods always reach sub-optimal solutions [4] while Numerical ones such as dynamic programming and Lagrange relaxation obtain final solutions closer to global optima [5].

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It is standard practice for ISOs to resolve the UC problem taking into account day-ahead security constraints [6], such as limits on: generation, reserves, up and down times (MUT/MDT), ramp-ups and ramp-downs, line flows, and power balance [7]. The nature of this security-constrained unit-commitment (SCUC) problem is a non-linear, non-convex, and mixed-integer one [8]. A comparison between Lagrange relaxation and MIP-based methods was performed in [9]. In [10], a Benders decomposition framework was proposed in order to clear both energy and ancillary service markets.

To deal with rising concerns over environmental preservation and energy security, a widespread renewable energy sources is wind [11]. European countries have already raised the penetration level of wind power to make provide between 5% and 20% of their annual demand. Meanwhile, the United States, aims at reaching 20% by the 2030 [12]. Higher penetration levels of wind power plants increase the system uncertainty which increases risks in the system and decreases system reliability [13].

In [14], a stochastic model was presented to solve for UC in the presence of wind resources. Reference [15] proposed a two-stage method to determine requirements for reserves, in systems with a high level of wind power penetration. Network constraints, in addition to load shedding and spillage costs, were included in the model.

In [16], artificial intelligence (namely neural network) approaches for forecasting wind power were used to integrate of wind resources into day-ahead UC. SCUC was presented in [17], with line flow limits, and solved using an enhanced “imperialist competitive algorithm”, in which a priority list was used to define the initial state. In [18], three meta-heuristic approaches were employed find the optimal UC schedule for a large-scale system.

A robust framework based on the information-gap decision theory technique is presented in [19] to solve the SCUC, which was formulated for the hourly day-ahead scheduling problem based on a mixed-integer linear programming (MILP) model. The incorporation of the transmission system security criteria (e.g., N-1) would result in a different solution for the generation scheduling solution. A SCUC formulation with an N-1 criterion was described in [20], based on the iterative methodology considering line outage distribution factors.

In [21], an analysis of the market-clearing formulation with a stochastic security model using mixed-integer linear programming techniques was presented. Economic load dispatch and UC in the presence of wind generators were discussed in [22]. The effects of wind farms on SCUC program is studied in [23]. A stochastic model was proposed in [24] for the UC program considering wind generation uncertainties.

A SCUC stochastic algorithm in [25] was demonstrated the capability of obtaining an efficient and fast solution for the case of a large-scale system. The stochastic model was employed to consider wind power uncertainties. The model is decomposed to three major sub-problems: 1) the UC problem; 2) the hourly DC optimal power flow (DC-OPF) problem, and finally 3) the constraints, which model the interaction between the UC and OPF problems. In [27], a fully adaptive two-stage UC formulation considering dispatchable wind generation was presented, using single-level MIP.

In this paper, we introduce a new formulation based on two-stage SCUC optimization, which considers a cost of occurrence for each scenario used to account for uncertainties of wind generation and loss of lines and generators. The contributions of this work are as follows:

- SCUC optimization considering uncertainties is proposed. In this framework, the SCUC with wind power is divided in: a 1) Master problem and a 2) Sub-problem.
- Considering a model of generation units and transmission lines outages. The uncertainties associated with the output of wind farms, generators and line outages are modeled in different system scenarios.
- An examination of the impacts of wind power penetration.
- An investigation of the impact of contingencies in SCUC using MIP optimization.

Section II of this manuscript discusses the objective function and problem constraints. The proposed solving algorithm is presented in Section III. Section IV demonstrated the application of the proposed method to a test system. Finally, conclusions are listed in section V.

## II. SCUC FORMULATION

This paper uses the formulation presented in [15] with some modifications. The part of the objective function where a proper reserve is calculated for each unit in different wind scenarios, is replaced with the formulation presented in [28] and additional constraints are ignored. Including the wind farm into the formulation of [28], a new formulation is also presented to consider the cost of each scenario.

### A. Objective function and constraints

The total cost of energy and reserves for certain time horizon, selected by the ISO, is considered as the objective function. Energy cost depends on the output power of units and system reserves; the ISO wants to minimize this cost. This cost function is shown in (1).

$$\begin{aligned}
 Obj = & \left\{ \sum_{t=1}^T \left[ \begin{aligned} & F^{\min}(i) \times u(i,t) + \sum_{msf=1}^{NSF} sl(i,msf) \times p(i,t,msf) \\ & + y(i,t) \times SUC(i) + z(i,t) \times SDC(i) \\ & + Q_g^{up}(i,t) \times R_g^{up}(i,t) + Q_g^{dn}(i,t) \times R_g^{dn}(i,t) \end{aligned} \right] \right. \\
 & \left. + \sum_{j=1}^{Nd} \left[ Q_d^{up}(j,t) \times R_d^{up}(j,t) + Q_d^{dn}(j,t) \times R_d^{dn}(j,t) \right] + IC(j,t) \times ILS(j,t) \right\} \\
 & + \sum_{s=1}^{Ns} \mathcal{P}_s \sum_{t=1}^T \sum_{j=1}^{Nd} \left\{ \begin{aligned} & [S_s(j,t) - W_s(j,t)] \times LMP(j,t) \times \\ & \left[ B_s(t) \times \delta_s(t) - p_d(j,t) + p_{wp,s}(t) \right] \\ & \left[ -outg_s(t) \times BG_s \times p_g(i,t) \right] \\ & + p_{wp,s}(t) \times Q_{wp}(t) \end{aligned} \right\} \quad (1)
 \end{aligned}$$

The cost function is linearized. The reserve offers are available. Also, start-up and shut-down costs of units are considered.

The goal is to minimize operation cost of generating units through determining UC scheduling and units' output power, considering limitations of units, network, and wind scenarios. The first line of the objective function is the offered price of units; cost function of thermal units which is considered linear here [29]. The second line is start-up cost and shut-down cost of units. The third line represents cost of up and down going reserves on generation side. The fourth line is up and down going reserves on demand side and the fifth line represents value of loss of load. The other parts of the objective function represent the costs proposed in this paper which are the costs related to each scenario, i.e., the reserve costs applied in each scenario.  $W$  and  $S$  are determined through the bus type determination subroutine. For each bus and each scenario,  $S$  minus  $W$  can be 1 or -1, where 1 represents the need for up-going reserve and the generator must receive cost of the rise in its generation or load decrement cost must be returned to the consumer and should be added to the total cost; -1 represents need for down-going reserve which means that the generator must receive less for decrement in its generation or the consumer must pay more for the rise in its consumption and it should be subtracted from the total cost [30]. The last line is the cost of wind farm power generation, in case it is considered (in this paper, wind farm power generation cost is not considered).

It is assumed in this paper, like [15], that wind generators do not take part in energy market competition and whole output power of the wind farm is utilized.

The constraints of the SCUC problem include [28]:

- Min/max output power of each unit
- Ramping up/down constraints
- MUT/MDT of each unit
- Upper limits of various types of reserves
- DC power flow constraints
- Security constraints (critical contingencies)

### B. Bus determination subroutine after a contingency

This paper uses the bus determination subroutine proposed in [28] with some modifications. The main difference is that calculations related to wind farms are added to the formulation as shown in (2) and (3). After a contingency, in an effort to keep the system safe, system busses are divided into two types. Busses of type 1 are the ones on which power generation increment and power consumption decrement help the operator to prevent load loss. On the other hand, there are busses of type 2 on which power generation decrement and power consumption increment are useful to prevent load loss. An algorithm is presented in this section, which determines type of each bus in each scenario. As discussed before, this subroutine is used to obtain a network model with fewer constraints and solves the SCUC. The objective function in each time span is shown in (2), which has two parameters for each bus; the first one is the amount of loss of load and the second one is the loss of generation in each scenario.

$$\sum_{j=1}^n [W_s^2(j,t) + l g_s^2(j,t)] \quad (2)$$

In this step, optimization of each contingency is performed based on unit states and dispatch values which were calculated in the master problem as the system operating point, to establish the type of network busses.

The constraints considered in this step are power balance constraint (3) and maximum power transmission through network lines (4).

$$\begin{aligned} & \sum_{i=1}^{Ng} [p_g(i,t) \times \text{out}g_{s(Ng \times 1)}(t)] + p_{wp,s}(t) \\ & = \sum_{j=1}^{Nd} [p_d(j,t) - ll_s(j,t) + l g_s(j,t)] \quad \forall t, s \end{aligned} \quad (3)$$

$$GSFK(s) \times \begin{bmatrix} \left( \text{out}g_{s(Ng \times 1)}(t) \times p_g(i,t) \right) \\ + p_{wp,s}(t) - p_d(j,t) \\ + ll_s(j,t,s) - l g_s(j,t) \end{bmatrix} \leq f_s^{\max}(l) \quad (4)$$

$$\begin{aligned} W_s(j,t) &= 1 \quad \text{if } ll_s(j,t) < 0 \\ W_s(j,t) &= 0 \quad \text{if } ll_s(j,t) \geq 0 \\ S_s(j,t) &= \text{not}(W_s(j,t)) \end{aligned} \quad (5)$$

If  $W_s(j,t)=0$  then the bus belongs to type 1, otherwise it belongs to the second type of busses that determining type of busses in each iteration.

### C. Scenarios of generation

To generate scenarios based on Monte-Carlo simulations, the algorithm proposed in [28] is used in this paper. A large number of scenarios is generated in order to precisely rebuild the intended distributed function. First, the selected statistical distribution is divided into Nseg segments (five segments here,  $I_1$  to  $I_5$  in Fig. 1). Also, output power of the wind farm is assumed to be modeled with a normal distribution; where  $\mu$  is mean value of the predicted wind and  $\sigma$  is the standard deviation. Monte Carlo simulation method is used to generate several wind scenarios. Wind power in each scenario is determined randomly, based on the predicted value (mean value) of the wind power. The fast forward method is employed to decrease the number of scenarios by combining them to obtain a reduced number of scenarios by which the relative distance from the original ones is less than 10% [31].

## III. SOLUTION ALGORITHM

The SCUC problem is divided in two sub-problems in which the UC problem with wind farms is represented by the objective function in Eq. (1) and its constraints.

- Bus type determination subroutine: sub-problem 1.
- Constraint checking subroutine for different scenarios: sub-problem 2.

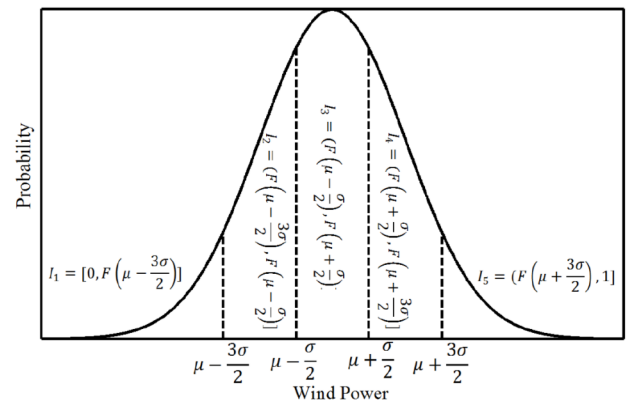


Fig. 1. Dividing normal distribution into five segments

Fig. 2 shows the algorithm employed for UC in the presence of wind farms. The type of buses at the first iteration can be found using a simple UC considering the network constraints. The UC program covers all credible contingencies including loss of generating units, line outage and different wind scenarios. Type of buses is determined afterward through results of this step and subroutine 1.

In the next step, power flow of every line is calculated through results of the main problem and bus determination problem. If the power flow exceeds the flow limit of transmission lines in any scenarios, the respective constraint is added to constraints of the main problem. In the next iteration, the UC problem includes these additional constraints, which are updated during each iteration, are also added to the list of main problem constraints. Different methods can be used as a stopping criterion for the program. In this case, optimization will continue until there is no violation in allowed criterions in all generated scenarios. The constraints related to the line flow violations which are added to the list of constraints in each iteration, will remain in the list until the last iteration of optimization.

#### IV. CASE STUDY

The system under study is the 24-bus IEEE RTS1 network. Like [14], the transmission line flow limits are different from IEEE RTS1 network [32]. Maximum power transmission for lines 11-13, 15-16, and 15-24 are reduced from the values described in [32] to 175, 60, and 175 MW, respectively. The model was implemented using the GAMS modeling package and solved using the CPLEX solver. Mean wind power generation is deemed to be 150 MW and the standard deviation is 5% of the mean value. Random parameter vector includes 64 random parameters that indicate accessibility of 26 generating units, 37 transmission lines and 1 wind farm generation. Total 3967 scenarios have been generated with Monte Carlo method. Using fast forward scenario reduction method, considering 10% relative distance between generated and reduced scenarios, 58 scenarios were selected [31].

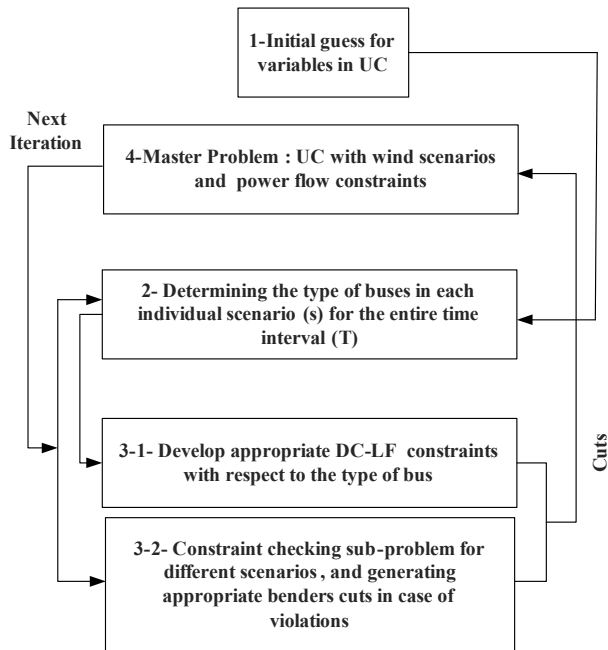


Fig. 2. Flowchart of the SCUC program in the presence of wind farms

Table I presents energy and reserve costs for installing the wind farm in different busses. The results show that different UC costs are calculated for connecting the wind farm on different busses, even with the same wind scenarios. This indicates the importance of placement of wind units, because it has a great effect on costs.

Fig. 3 shows generation power of units, up and down-going reserve on generation side and up and down-going reserve on demand side when the wind farm is connected to bus 3. Results illustrate that the total generation of thermal units and the wind unit at each hour is equal to the predicted load. The total system reserve has been increased compared to the network without the wind unit and it is because of production uncertainty of wind farms.

Fig. 4 shows the results of simulations for different percentages of wind power penetration (percentage of the total generated power coming from the wind farm) in the power system (wind farm is installed on bus 3).

It is seen in Fig. 4 that when there are no wind units in the network, the operation cost is 1214888.883\$. Thereby, it is clear that connecting a wind unit to the network results in a reduction of the cost. The magnitude of this reduction depends on the percentage of the total generated power coming from the wind farm. Next step is calculating power system operation cost for different uncertainties in production of the wind unit. The wind unit is installed on bus 3.

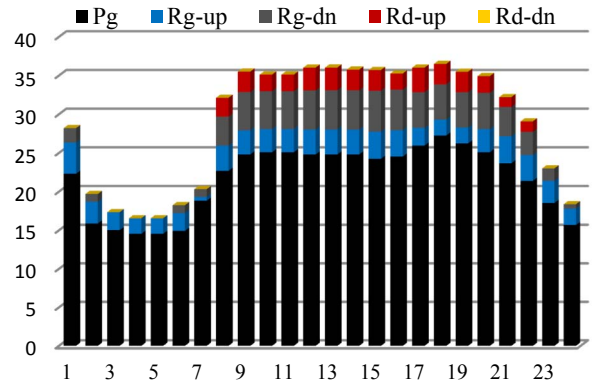


Fig. 3. Generation state of units and up and down reserves on load side

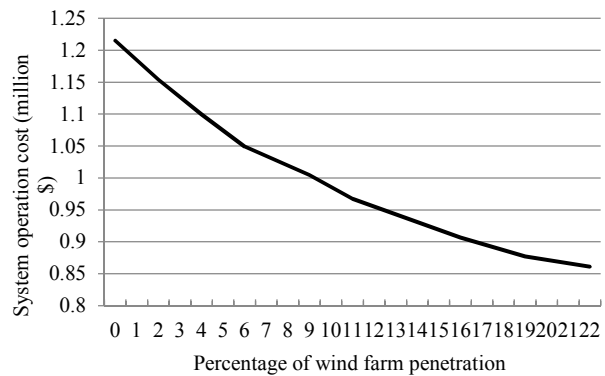


Fig. 4. Total operation cost for different values of wind farm penetration

TABLE I. COST OF INSTALLING THE WIND UNIT AT DIFFERENT BUSES

Bus	Costs (\$)	Bus	Costs (\$)
1	1053033	13	1074588
2	1053170	14	1077557
3	1049335	15	1126375
4	1054067	16	1148209
5	1054746	17	1134247
6	1056347	18	1134514
7	1110904	19	1131607
8	1055995	20	1118478
9	1055653	21	1129668
10	1058710	22	1134100
11	1064996	23	1112452
12	1081584	24	1066761

## V. CONCLUSION

Wind units are known to have high uncertainty of their output power and thus it should be considered in the market-clearing problem. A new formulation has been proposed in this paper to model this uncertainty. Some extra terms have been eliminated from the formulation, which leads to a noticeable decrease in programs run times. Simulations show that connecting a wind unit to the network decreases the total output power of conventional units and more system reserve is required because of the uncertainty in output power of the wind unit, but eventually the overall operation cost of the system would decrease. Installing the wind unit at various buses in the network leads to dissimilar costs for the UC, which indicates that optimal placement is an important factor for ISO and investors. The operation cost decreases by increasing the penetration level of the wind power. However, increasing the uncertainty in power output of the wind unit leads to a higher need of system reserves and thus higher costs.

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