



Impact of the Future Water Value on Wind-Reversible Hydro Offering Strategies in Electricity Markets

A. A. Sánchez de la Nieta^{a,b}, J. Contreras^c, J. P. S. Catalão^{a,b}

^a*University of Beira Interior, Covilhã, Portugal*

^b*INESC-ID and IST, University of Lisbon, Lisbon, Portugal*

^c*University of Castilla-La Mancha, Ciudad Real, Spain*

Abstract

A coordinated offering strategy between a wind farm and a reversible hydro plant can reduce wind power imbalances, improving the system efficiency whilst decreasing the total imbalances. A stochastic mixed integer linear model is proposed to maximize the profit and the future water value (*FWV*) of the system using Conditional Value at Risk (*CVaR*) for risk-hedging. The offer strategies analyzed are: i) single wind-reversible hydro offer with a physical connection between wind and hydro units to store spare wind energy, and ii) separate wind and reversible hydro offers without a physical connection between them. The effect of considering the *FWV* of the reservoirs is studied for several time horizons: one week (168 h) and one month (720 h) using an illustrative case study. Conclusions are duly drawn from the case study to show the impact of (*FWV*) in the results.

Keywords: offering strategy, wind farm, reversible hydro plant, risk-hedging, future water value, single strategy, separate strategy

1. Introduction

Renewable energy sources are increasing their penetration in electricity markets since the international agreement known as the Kyoto protocol in 1997. The evolution of the wind power installed in the world

Email addresses: agustinsnl@gmail.com (A. A. Sánchez de la Nieta), Javier.Contreras@uclm.es (J. Contreras), catalao@ubi.pt (J. P. S. Catalão)

1 has ranged 10,200 MW in 1998 to 369,553 MW in 2014. The high penetration of wind power remains a
2 challenge for the current electricity markets, where the main problem is the rising uncertainty of generation
3 as a consequence of the dependence on weather to produce energy.

4 On the other hand, a mature technology such as hydro power has an installed capacity of roughly 1.3
5 TW in the world. Furthermore, the regulation of hydro power is changing rapidly, where reversible hydro
6 or hydro-pumping technology is part of the solution in case of a high penetration of wind power.

7 The water reserve of hydro power is known or predictable in the short-term and reversible hydro power
8 can be used to reduce or absorb part of the uncertain wind production.

9 This paper analyzes wind and reversible hydro energies in the mid-term within daily pool-based elec-
10 tricity markets. Specifically, the coordination between wind power and reversible hydro power is addressed,
11 incorporating the impact of the future water value in the offers, in the imbalances and in the profits. The
12 importance of reversible hydro technology is due to the fact that it can discharge or pump water between
13 different reservoirs [1], [2] and [3].

14 Usually, wind and hydro connections have been studied in the short-term. In electricity markets, the
15 typical wind and hydro problem tries to reduce uncertainty of wind power through a good regulation of hydro
16 power, storing the excess of wind energy as water in the reservoir. As a consequence of wind uncertainty,
17 wind trading always occurs in the short-term, but hydro power generation has been traded in the short-,
18 mid- and long-terms [2], [3] and [4].

19 The incorporation of FWV influences the behavior of the wind-reversible hydro offering strategies. The
20 behavior of these two kinds of offers is analyzed by the parametrization of the future water price included
21 in the FWV , where the latter is part of the objective function. FWV provides a monetary value to the
22 water stored. Due to this, the profit of a wind and reversible hydro unit comes from the power offered to
23 the electricity market and the water stored. This water will be used to generate electricity in the future.
24 In addition, the main problem of FWV is to determine a price for the water stored, because this price will
25 affect the energy that will be sold through the offer. Therefore, the income from the offer and the water
26 stored are in conflict.

27 1.1. Literature review

28 Wind energy has an intermittent production, with high uncertainty [5] and [6]. This uncertainty causes
29 imbalances in the system. The imbalance is equal to the generation minus the offer: if the generation is
30 higher than the offer, the imbalance is positive, while if the generation is lower than the offer, the imbal-
31 ance is negative, called excess or lack of energy, respectively. These imbalances are compensated through

1 sources that can regulate their productions and have reserve capacities. On the other hand, reversible hydro
 2 generation has a low uncertainty of production, depending on the water reservoir levels [7].

3 Another issue to take into account is the incorporation of renewable energy in electricity markets. Ref-
 4 erences [5] and [8] introduce electricity markets and describe several of their foundations. Risk, uncertainty
 5 and flexibility in electricity markets are shown in [5] and [9]. The main decomposition techniques based on
 6 mathematical programming applicable to electricity problems are described in [10].

7 Regarding the offers in electricity markets, the best offering strategy for a wind generator is studied
 8 in [11] and [12]. A literature survey about optimal bidding of hydro-electric producers is shown in [13].
 9 Therefore, as a consequence of the uncertainty in electricity markets due to renewable energy sources, risk-
 10 hedging measures are used, such as the *VaR* and the *CVaR* [14]. In [15] the *CVaR* with a confidence level
 11 $\alpha \in]0, 1[$ is defined as the mean of the generalized α -tail distribution, as shown in Fig. 1.

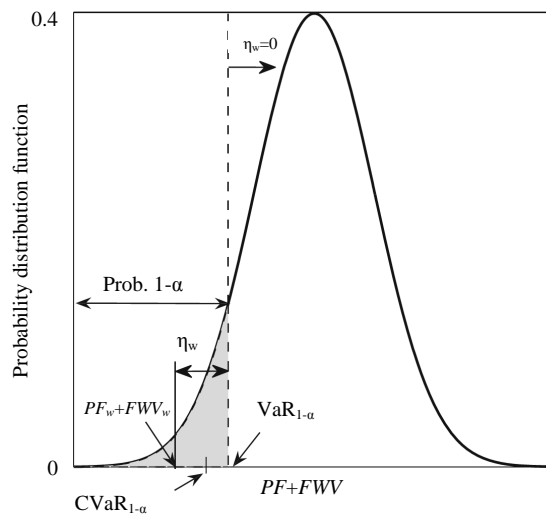


Figure 1. VaR & CVaR concepts.

12 Some electricity markets, such as the Iberian, French, German and Italian, consider the energy imbalance,
 13 so there is an imbalance market. In [16] wind imbalances using hydro production without pumping are
 14 minimized. Similarly, [17] studies the costs that come from wind imbalances, where the imbalance is the
 15 difference between generation and offer. However, a new coordinated mode of operation of wind farms
 16 and pumped-hydro-storage plants based on day-ahead wind power output forecasts through a deterministic
 17 mixed integer problem is introduced in [18]. Other related energy management models are presented in
 18 [19, 20, 21, 22] for islanded microgrids.

19 Three optimization models for the coordination between wind power and reversible hydro power to offer

1 energy in the day-ahead market are formulated in [23] where they show the offering strategies in a day-ahead
 2 market for the short-term. These strategies are divided into three types: i) separate wind and reversible
 3 hydro offers without a physical connection between them, ii) separate wind and reversible hydro offers with
 4 a physical connection, and iii) a single wind and reversible hydro offer with a physical connection.

5 The effects of wind power and hydro pumping power coordination are studied in [24]. Reference [25]
 6 studies a similar problem for 48 h. Reference [26] analyzes a portfolio composed of wind power and hydro
 7 power with some risk measures. On this topic, [27] introduces state-of-the-art research on the operation of
 8 wind and pumped storage plants in a deregulated market. These research lines do not consider the *FWV*
 9 and its effects in the management of the generators, including the effects of the time frame in the results.

10 In summary, these papers describe how wind generators could reduce their imbalances and increase
 11 their expected profits. Additionally, [28] studies a similar problem for unit commitment called stochastic
 12 price-based unit commitment (Stochastic PBUC).

13 In [29] a series of price-taker hydroelectric plants are modeled through mixed integer non linear program-
 14 ming for a time frame of one week introducing *FWV* in the formulation.

15 From a practical viewpoint, there are some projects to study and analyze the joint coordination between
 16 wind power and reversible hydro power, for example in the Canary Islands [30] and [31]. A state of the art
 17 review for the operation of wind power and pumped-storage plant is presented in [32].

18 1.2. Aims and contributions

19 This paper analyzes two types of offers in the mid-term within an electricity market context where the
 20 power offered comes from a wind power producer and a reversible hydro power producer. Thus, the time
 21 frame can reflect important differences for wind power and reversible hydro power. The periods of study
 22 are: i) one month (720 h) and ii) one week (168 h).

23 The *FWV* can be very relevant in the objective function depending on the time frame, because the *FWV*
 24 is fixed in the last period of the time frame, in the hours 720 and 168 for the two time frames.

25 The main contributions of this work are threefold:

- 26 1. Two offering models which include the *FWV* are shown, namely, i) a single wind-hydro offer with a
 27 physical connection between them and ii) a separate wind-hydro offer without a physical connection
 28 between them, are presented for several time frames including risk-hedging through the *CVaR*.
- 29 2. The *FWV* is included in the two types of offers to show its effect in an illustrative case study.
- 30 3. The coordination of wind and reversible hydro units in case of having risk-hedging strategies using
 31 *CVaR* with *FWV* is implemented.

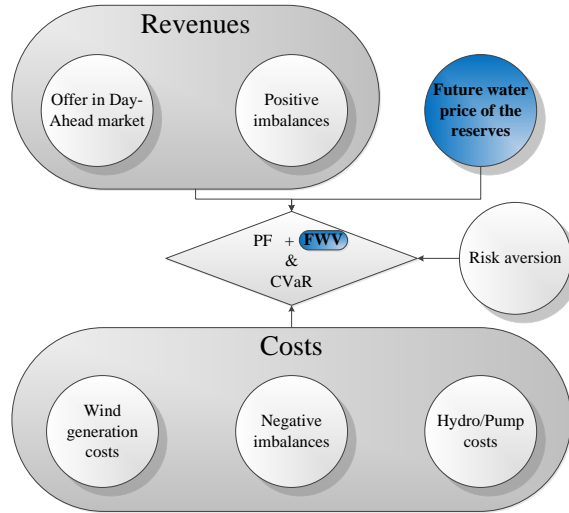


Figure 2. Optimization of the profits and FWV with risk-hedging ($CVaR$).

- 1 4. The time frame effects of the FWV in the two offering models and the differences between the offerings
 2 are shown.
 3 5. A new approach to obtain the value of the water stored in $\text{€}/\text{Hm}^3$ is obtained.

4 This paper is organized as follows: Section 2 describes the models and presents the equations and variables
 5 to model FWV , Section 3 shows the case study, Section 4 presents the results and Section 5 summarizes the
 6 main conclusions.

7 2. Mathematical formulation

8 The offering strategies are modeled using stochastic mixed integer linear programming (SMILP). SMILP
 9 is divided into 2 stages, the first stage does not depend on the index referring to scenarios such as $b_{t,i}$, bw_t
 10 and $bh_{t,i}$, while the second stage depends on the index referring to scenarios. Hydro power is modeled as in
 11 [7] and wind power is based on the model in [11], including risk-hedging, $CVaR$ being the risk measure.

12 This section presents the models using FWV . The terms that make up the profit (PF) are shown in Fig.
 13 2, where they are classified either as revenues or costs. A new term included in the objective function with
 14 respect to [23] is the FWV and the input data related to the future water price of the reserves.

15 The total revenues are composed of the selling offer to the day-ahead market and the positive imbalance,
 16 while the total costs are formed by wind marginal costs, hydro/pump costs and the negative imbalance. The
 17 FWV is equivalent to the future water price (fwp_w) based on the reservoir level, where fwp_w is measured

1 in [$\text{€}/\text{Hm}^3$].

2 The objective function maximizes the profits of the offering energy in the day-ahead electricity market,
3 *FWV* and *CVaR*.

4 The mathematical formulation below incorporates *FWV* in the objective function, which causes alter-
5 ations in the offer as a consequence of the possible future use of water resources. Furthermore, *FWV* gives
6 information on the reservoir level price from which the offer could be modified.

7 The objective functions and their constraints related to the single and separate offers are described in
8 this section. The main features of the offers are: i) the single offer can only sell or buy energy, and ii) the
9 separate offer can sell wind energy and sell/buy energy to generate/pump from the reversible hydro unit.

10 2.1. Mathematical formulation of single wind-hydro offer with a physical connection

11 The objective function maximizes the profits, *PF*, from selling or purchasing energy, the value of the
12 reservoirs (in monetary units), *FWV*, and the measure of risk, *CVaR*. The objective function of equation (1)
13 decides the optimal offering (selling/purchase) for each period t , depending on the reservoir levels, day-ahead
14 market prices, positive and negative imbalance market prices, and wind production, where these factors are
15 parameters of the model.

16 The objective function is defined as:

$$17 \quad \max \quad (1 - \beta) \cdot (PF + FWV) + \beta \cdot CVaR; \quad (1)$$

where

$$PF = \sum_w \rho_w \left[\sum_t \left(\lambda_{t,w} \cdot b_{t,i} - c^W \cdot gw_{t,w} \right. \right. \\ \left. \left. - c_i^H \cdot (p_{t,i,w} + np_{t,i,w}) - c_i^P \cdot pr_{t,i,w} \right. \right. \\ \left. \left. - c_i \cdot y_{t,i,w} + \lambda_{t,w}^+ \cdot \Delta_{t,i,w}^+ - \lambda_{t,w}^- \cdot \Delta_{t,i,w}^- \right) \right]; \quad (2)$$

$$FWV = \sum_w \rho_w \left[\sum_i r_{t=tp,i,w} \cdot fwp_w \right]; \quad (3)$$

$$CVaR = VaR - \frac{1}{1 - \alpha} \sum_w \rho_w \cdot \eta_w. \quad (4)$$

18 As can be seen in *PF*, there is a single joint offer, $b_{t,i}$. *PF* is the profit obtained from offering the energy
19 at a price given by the electricity market. If there is an imbalance between the generation and the offer,

the generator is sometimes penalized, where the excess of production is considered as a revenue equal to the imbalance energy multiplied by a positive imbalance market price (lower than the market price), whereas, if the generation is lower than the offer, this implies a cost equal to the imbalance energy multiplied by the negative imbalance market price (higher than the market price). Moreover, PF has production costs different from zero, including the hydro start-up and shutdown costs.

The second term, FWV , is the monetary value of the reserves, where all the reserves will be sold at the end of the lifetime at a specific price. FWV is calculated as the reservoir level in Hm^3 and the future water price (fwp_w) per scenario, w , has a value expressed in $\text{€}/\text{Hm}^3$. The inclusion of FWV in the objective function can modify the optimal offer, $b_{t,i}$, depending on fwp_w . This water price is parameterized by a factor called the future water price factor, fpf .

fwp_w is calculated as the average price for the entire lifetime per scenario w given an fpf factor, where $fwp_w = fpf \cdot \bar{\lambda}_w$. The fpf factor can be set to several values, where $\bar{\lambda}_w$ is calculated as $\bar{\lambda}_w = \frac{\sum_t \lambda_{t,w}}{tp}$.

The objective function is weighted using a risk aversion parameter called β . The first part considers PF and FWV , while the second part is only composed of the risk-hedging term, $CVaR$. On the other hand, the behavior of PF , FWV and $CVaR$ is evaluated for several β values, from 0 to 1. The $CVaR$ is calculated from the PF and the FWV distributions.

The objective functions are subject to several constraints. These constraints are classified into seven types: $CVaR$ constraints, hydro power curve linearization constraints, hydro reserve constraints, pumping constraints, wind-hydro interconnection constraints, power offer constraints and power imbalance constraints. The $CVaR$ constraint in this work shows some differences due to the incorporation of FWV as shown below.

2.1.1. $CVaR$ constraints

This constraint considers all the profits including the FWV , evaluating variable η_w per scenario and the value of VaR .

$$\begin{aligned}
& - \left[\sum_t (\lambda_{t,w} \cdot b_{t,i} - c^W \cdot gw_{t,w} \right. \\
& - c_i^H \cdot (p_{t,i,w} + np_{t,i,w}) - c_i^P \cdot pr_{t,i,w} \\
& - c_i \cdot y_{t,i,w} + \lambda_{t,w}^+ \cdot \Delta_{t,i,w}^+ - \lambda_{t,w}^- \cdot \Delta_{t,i,w}^-) \\
& \left. + r_{t=tp,i,w} \cdot fwp_w \right] + VaR - \eta_w \leq 0; \tag{5}
\end{aligned}$$

$$\eta_w \geq 0. \quad (6)$$

Equation (5) evaluates the profits per scenario including the *FWV* plus *VaR* minus the auxiliary variable η_w , the equation being lower than or equal to zero because this is a maximization problem. Equation (5) maximizes the lower distribution tail of *PF* and *FWV*, which, in this case, is the left distribution tail of *PF* and *FWV*, as can be observed in Fig. 1.

2.1.2. Hydro Power Curve Linearization Constraints

The hydro power curve is linearized based on [7], where (7) is the maximum capacity in MW.

$$B0_{i=1} = porhoh_{i=1} + \sum_i umax_{l,i=1} \cdot rhoh_{l,i=1}. \quad (7)$$

Equation (8) is the discharge of hydro unit i .

$$u_{t,i=1,w} = \sum_l (ul_{t,i=1,w} + umin_{i=1} \cdot v_{t,i=1,w}). \quad (8)$$

Equations (9)–(12) present the block limits for the linearized hydro power curve.

$$ul_{l=1,t,i=1,w} \leq umax_{l=1,i=1} \cdot v_{t,i=1,w}; \quad (9)$$

$$ul_{l=1,t,i=1,w} \geq umax_{l=1,i=1} \cdot w_{l=1,t,i=1,w}; \quad (10)$$

$$ul_{l,i,t} \leq umax_{l,i} \cdot w_{l-1,t,i=1,w}; \quad (11)$$

$$ul_{l,i,t} \geq umax_{l,i} \cdot w_{l,t,i=1,w}. \quad (12)$$

To determine which power curves are used: the higher, the intermediate or the lower one, see (13)–(15), depending on the reservoir level.

$$d_{t,i,w}^1 \geq d_{t,i,w}^2; \quad (13)$$

$$r_{t,i,w} \geq vc1_i \cdot (d_{t,i,w}^1 - d_{t,i,w}^2) + vc2_i \cdot d_{t,i,w}^2; \quad (14)$$

$$r_{t,i,w} \leq vc1_i \cdot (1 - d_{t,i,w}^1) + vc2_i \cdot (d_{t,i,w}^1 - d_{t,i,w}^2)$$

$$+Vmax_i \cdot d_{t,i,w}^2. \quad (15)$$

1 Equations (16)–(21) determine the power production for the several curves linearized. Variable $np_{t,i=1,w}$,
 2 is included to calculate the amount of power that a hydro unit could generate to reduce the negative wind
 3 imbalance.

$$\begin{aligned} p_{t,i=1,w} + np_{t,i=1,w} - porhol_{i=1} \cdot v_{t,i=1,w} \\ - \sum_l (ul_{l,t,i=1,w} \cdot rhol_{l,i=1}) \\ - B0_{i=1} \cdot (d_{t,i,w}^1 + d_{t,i,w}^2) \leq 0; \end{aligned} \quad (16)$$

$$\begin{aligned} p_{t,i=1,w} + np_{t,i=1,w} - porhol_{i=1} \cdot v_{t,i=1,w} \\ - \sum_l (ul_{l,t,i=1,w} \cdot rhol_{l,i=1}) \\ + B0_{i=1} \cdot (d_{t,i,w}^1 + d_{t,i,w}^2) \geq 0; \end{aligned} \quad (17)$$

$$\begin{aligned} p_{t,i=1,w} + np_{t,i=1,w} - porhom_{i=1} \cdot v_{t,i=1,w} \\ - \sum_l (ul_{l,t,i=1,w} \cdot rhom_{l,i=1}) \\ - B0_{i=1} \cdot (1 - d_{t,i,w}^1 + d_{t,i,w}^2) \leq 0; \end{aligned} \quad (18)$$

$$\begin{aligned} p_{t,i=1,w} + np_{t,i=1,w} - porhom_{i=1} \cdot v_{t,i=1,w} \\ - \sum_l (ul_{l,t,i=1,w} \cdot rhom_{l,i=1}) \\ + B0_{i=1} \cdot (1 - d_{t,i,w}^1 + d_{t,i,w}^2) \geq 0; \end{aligned} \quad (19)$$

$$\begin{aligned} p_{t,i=1,w} + np_{t,i=1,w} - porhoh_{i=1} \cdot v_{t,i=1,w} \\ - \sum_l (ul_{l,t,i=1,w} \cdot rhoh_{l,i=1}) \\ - B0_{i=1} \cdot (2 - d_{t,i,w}^1 + d_{t,i,w}^2) \leq 0; \end{aligned} \quad (20)$$

$$\begin{aligned} p_{t,i=1,w} + np_{t,i=1,w} - porhoh_{i=1} \cdot v_{t,i=1,w} \\ - \sum_l (ul_{l,t,i=1,w} \cdot rhoh_{l,i=1}) \end{aligned}$$

$$+B0_{i=1} \cdot (2 - d_{t,i,w}^1 + d_{t,i,w}^2) \geq 0. \quad (21)$$

1 The up/down hydro logic is shown in (22):

$$y_{t,i=1,w} - z_{t,i=1,w} = v_{t,i=1,w} - v_{t-1,i=1,w}. \quad (22)$$

2 Equations (23) and (24) are the maximum limits of hydro production and extra discharge for reducing
 3 the negative imbalance. Moreover, binary variable $a_{t,i=1,w}$ can fix the hydro production limit between the
 4 maximum capacity and zero. Meanwhile, $np_{t,i=1,w}$ variable is limited by the maximum hydro capacity minus
 5 the hydro production, being $np_{t,i=1,w}$ the maximum value when the hydro production is zero.

$$p_{t,i=1,w} \leq B0_{i=1} \cdot a_{t,i=1,w}; \quad (23)$$

$$np_{t,i=1,w} \leq B0_{i=1} \cdot a_{t,i=1,w} - p_{t,i=1,w}. \quad (24)$$

6 2.1.3. Hydro Reservoir Constraints

7 The equation of the balance for the reservoirs is calculated through (25), (26) is the minimum reservoir
 8 needed to commence the discharge of the hydro unit and (27) fixes the reservoir level at the end of the
 9 simulations.

$$\begin{aligned} r_{t,i,w} = & r_{t-1,i,w} + if_{t,i,w} - cv \cdot u_{t,i,w} + cv \cdot u_{t-1,i-1,w} \\ & - cv \cdot s_{t,i,w} + cv \cdot s_{t-1,i-1,w} \\ & + cv \cdot wf_{t,i,w} - cv \cdot wf_{t,i-1,w}; \end{aligned} \quad (25)$$

$$r_{t,i,w} \geq Vmin_i \cdot a_{t,i,w}. \quad (26)$$

10 To limit the discharge for the time horizon, (27) is used, where the upper water reservoir value at the
 11 end of the time horizon is greater than or equal to 90% of the upper value of the water reservoir at the
 12 beginning of the time horizon.

$$r_{t=tp,i,w} \geq 0.9 \cdot r_{t=0,i,w}^{initial}. \quad (27)$$

2.1.4. Pumping Constraints

The pumping efficiency is represented in (28)–(30). Binary variable, $a_{t,i=1,w}$, is set to decide whether to generate or to pump by the reversible hydro unit.

$$pr_{t,i=1,w} \leq (1 - a_{t,i=1,w}) \cdot ppm_{i=1}; \quad (28)$$

$$pn_{t,i=1,w} = pr_{t,i=1,w} \cdot eff; \quad (29)$$

$$wf_{t,i=1,w} = \frac{pn_{t,i=1,w}}{rhopp_{i=1}}. \quad (30)$$

2.1.5. Wind-Hydro Interconnection Constraints

See (31)–(41) for the interconnection between the wind farm and the hydro unit. The interconnection can be used for reducing the wind energy imbalances when the energy is offered, or to buy less energy when there is a purchase of energy. Equation (31) is used when there is a power offer.

$$if \quad gw_{t,w} > b_{t,w}^+$$

$$ppwm_{t,i=1,w} \leq (gw_{t,w} - b_{t,w}^+) \cdot bd_{t,i=1}$$

$$else \quad ppwm_{t,i=1,w} = 0. \quad (31)$$

Equations (32)–(40) determine the power offering and the energy that could be charged from the excess of wind power in form of water, decreasing the positive imbalance. And in the case of buying the energy, the energy that comes from wind can be used to reduce the energy bought.

$$ppw_{t,i=1,w} = -gw_{t,w} \cdot (1 - bd_{t,i=1}) + ppwm_{t,i=1,w}; \quad (32)$$

$$ppw_{t,i=1,w}^- \leq ppm_{i=1} \cdot (1 - bd_{t,i=1}); \quad (33)$$

$$ppw_{t,i=1,w}^+ \leq ppm_{i=1} \cdot bd_{t,i=1}; \quad (34)$$

$$ppw_{t,i=1,w}^\pm = ppw_{t,i=1,w}^+ - ppw_{t,i=1,w}^-; \quad (35)$$

$$ppw_{t,i=1,w}^\pm = ppw_{t,i=1,w}; \quad (36)$$

$$ppb_{t,i=1,w} \leq ppm_{i=1} \cdot (1 - bd_{t,i=1}) - ppw_{t,i=1,w}^-; \quad (37)$$

$$pp_{t,i=1,w} = ppw_{t,i=1,w}^+ + ppw_{t,i=1,w}^- + ppb_{t,i=1,w}; \quad (38)$$

$$pp_{t,i=1,w} \leq ppm_{i=1} \cdot (1 - a_{t,i=1,w}); \quad (39)$$

$$pr_{t,i=1,w} = pp_{t,i=1,w}. \quad (40)$$

1 If there is a negative imbalance (41), the reversible hydro power unit could discharge more, depending
2 on its capacity limit.

$$\text{if } gw_{t,w} < b_{t,w}^-$$

$$np_{t,i=1,w} \leq (b_{t,w}^- - gw_{t,w}) \cdot bd_{t,i=1}$$

$$\text{else } np_{t,i=1,w} = 0. \quad (41)$$

3 2.1.6. Power Offer Constraints

4 Constraints (42) and (43) are the limits for the energy that could be sold.

$$b_{t,i=1} \leq Pmax \cdot bd_{t,i=1} + p_{t,i=1,w}; \quad (42)$$

$$b_{t,i=1} \geq 0 \cdot bd_{t,i=1} + p_{t,i=1,w} - ppm_{i=1} \cdot (1 - bd_{t,i=1}). \quad (43)$$

5 2.1.7. Imbalance Constraints

6 The limit of each kind of imbalance is fixed in (44) for a negative imbalance and in (45) for a positive
7 imbalance.

$$0 \leq \Delta_{t,i=1,w}^- \leq (B0_{i=1} + Pmax) \cdot d_{t,i=1,w}; \quad (44)$$

$$0 \leq \Delta_{t,i=1,w}^+ \leq (Pmax + ppm_{i=1}) \cdot (1 - d_{t,i=1,w}). \quad (45)$$

8 The imbalance is calculated in (46).

$$\begin{aligned} \Delta_{t,i=1,w} &= gw_{t,w} \cdot bd_{t,i=1} + gw_{t,w} \cdot (1 - bd_{t,i=1}) + p_{t,i=1,w} \\ &\quad - b_{t,i} + np_{t,i=1,w} - ppb_{t,i=1,w} - ppw_{t,i=1,w}^- - ppw_{t,i=1,w}^+. \end{aligned} \quad (46)$$

9 Whether the imbalances are positive or negative is solved in (47), where $\Delta_{t,i=1,w}^+$ and $\Delta_{t,i=1,w}^-$ are positive

1 variables.

$$\Delta_{t,i=1,w} = \Delta_{t,i=1,w}^+ - \Delta_{t,i=1,w}^- \quad (47)$$

2 2.2. Mathematical formulation of separate wind and hydro offers without a physical connection

3 The main difference with respect to the previous objective function is the number of offers. In this
4 strategy there are two offers: wind and hydro/pumping. So, there are wind profits (*WPF*), reversible hydro
5 profits (*HPF*) and *FWV*.

6 The constraints needed in this model are: (7)–(30). The wind-hydro interconnection constraints are not
7 needed in the separate wind and hydro offers without a physical connection (31)–(41). However, the power
8 offer and imbalance constraints are redefined, see (55)–(65).

9 Then, the objective function is defined as:

$$\max (1 - \beta) \cdot (WPF + HPF + FWV) + \beta \cdot CVaR; \quad (48)$$

10 where

$$\begin{aligned} WPF = \sum_w \rho_w \left[\sum_t \left(\lambda_{t,w} \cdot bw_t - c^W \cdot gw_{t,w} \right. \right. \\ \left. \left. + \lambda_{t,w}^+ \cdot \Delta w_{t,w}^+ - \lambda_{t,w}^- \cdot \Delta w_{t,w}^- \right) \right]; \end{aligned} \quad (49)$$

$$\begin{aligned} HPF = \sum_w \rho_w \left[\sum_t \left(\lambda_{t,w} \cdot bh_{t,i} \right. \right. \\ \left. \left. - c_i^H \cdot p_{t,i,w} - c_i^P \cdot pr_{t,i,w} - c_i \cdot y_{t,i,w} \right. \right. \\ \left. \left. + \lambda_{t,w}^+ \cdot \Delta h_{t,i,w}^+ - \lambda_{t,w}^- \cdot \Delta h_{t,i,w}^- \right) \right]; \end{aligned} \quad (50)$$

$$FWV = \sum_w \rho_w \left[\sum_i r_{t=tp,i,w} \cdot fwp_w \right]; \quad (51)$$

$$CVaR = VaR - \frac{1}{1 - \alpha} \sum_w \rho_w \cdot \eta_w. \quad (52)$$

11 The different parameters are calculated as in the previous objective function of subsection 2.1 where the
12 main difference regarding the single offer is the existence of two variables to define the offers, bw_t and $bh_{t,i}$.

1 *2.2.1. CVaR constraints for separate wind and reversible hydro offers without a physical connection*

2 This constraint considers wind profit, *WPF*, and hydro profit *HPF*. *FWV* and *VaR* are also included.

3 With these terms variable η_w can be selected per scenario.

$$\begin{aligned}
& - \left[\sum_t (\lambda_{t,w} \cdot bw_t + \lambda_{t,w} \cdot bh_{t,i} - c^W \cdot gw_{t,w} \right. \\
& \quad - c_i^H \cdot p_{t,i,w} - c_i^p \cdot pr_{t,i,w} - c_i \cdot y_{t,i,w} \\
& \quad + \lambda_{t,w}^+ \cdot (\Delta w_{t,w}^+ + \Delta h_{t,i,w}^+) \\
& \quad - \lambda_{t,w}^- \cdot (\Delta w_{t,w}^- + \Delta h_{t,i,w}^-) \\
& \quad \left. + r_{t=tp,i,w} \cdot fwp_w \right] + VaR - \eta_w \leq 0; \tag{53}
\end{aligned}$$

$$\eta_w \geq 0. \tag{54}$$

4 *2.2.2. Power Offer Constraints*

5 The limits of the energy that can be sold are shown in (55)–(57).

$$0 \leq bw_t \leq Pmax; \tag{55}$$

$$bh_{t,i=1} \leq B0_{i=1} \cdot a_{t,i=1}; \tag{56}$$

$$bh_{t,i=1} \geq -ppm_{i=1} \cdot (1 - a_{t,i=1}). \tag{57}$$

6 *2.2.3. Imbalance Constraints*

7 The limit of each kind of imbalance is fixed in (58) for a negative imbalance and in (59) for a positive
8 imbalance. The wind imbalance is calculated in (60).

$$0 \leq \Delta w_{t,w}^- \leq Pmax \cdot j_{t,w}; \tag{58}$$

$$0 \leq \Delta w_{t,w}^+ \leq Pmax \cdot (1 - j_{t,w}); \tag{59}$$

$$\Delta w_{t,w} = gw_{t,w} - bw_t; \tag{60}$$

$$\Delta w_{t,w} = \Delta w_{t,w}^+ - \Delta w_{t,w}^-; \tag{61}$$

$$0 \leq \Delta h_{t,i=1,w}^- \leq B0_{i=1} \cdot d_{t,i=1,w}; \quad (62)$$

$$0 \leq \Delta h_{t,i=1,w}^+ \leq ppm_{i=1} \cdot (1 - d_{t,i=1,w}); \quad (63)$$

$$\Delta h_{t,w} = \Delta h_{t,w}^+ - \Delta h_{t,w}^-. \quad (64)$$

1 The imbalance is calculated through equation (65).

$$\Delta h_{t,i=1,w} = p_{t,i=1,w} - bh_{t,i} - pr_{t,i=1,w}. \quad (65)$$

2 3. Case Study

3 3.1. Description

4 The case study contains a wind farm and a reversible hydro plant. The wind farm has a capacity of
5 50 MW with 25 turbines of 2 MW of capacity, and the reversible hydro unit has 28.62 MW of generation
6 capacity and 35.77 MW of pumping capacity.

7 Wind power is calculated through the general expression $P(v) = 0,5 \cdot c_p(v) \cdot \rho \cdot A \cdot v^3$, where $c_p(v)$ is the
8 overall efficiency of the wind turbine as a function of the wind speed, ρ is the air density, A is the area swept
9 by the wind turbine rotor and v is the wind speed [33]. The wind farm is located in Navarre, Northern
10 Spain. The wind marginal cost is €16.9/MWh, calculated as in [34].

11 The hydro/pump plant is composed of two reservoirs: an upper and a lower reservoir. The initial volume
12 of the upper reservoir is 110 Hm³ and the initial volume of the lower reservoir is 80 Hm³. Moreover, the
13 water volume of the reserve at the end of the time horizon must be greater than or equal to 90% of the
14 initial volume. The hydro/pump marginal costs are €10/MWh for generation and €3/MWh for pumping
15 [34]. The hydro generation parameters and the linearized hydro power curve parameters are described in
16 [7] and [23].

17 The simulation is divided into three parts: input data, models and output data. In Fig. 3, a diagram
18 of the inputs and outputs is shown. The input data are composed of prices, wind power, water inflows,
19 hydro plants parameters, costs, risk aversion, and the future water price. The model provides total expected
20 profits, FWV , offer per hour, $CVaR$ and imbalances.

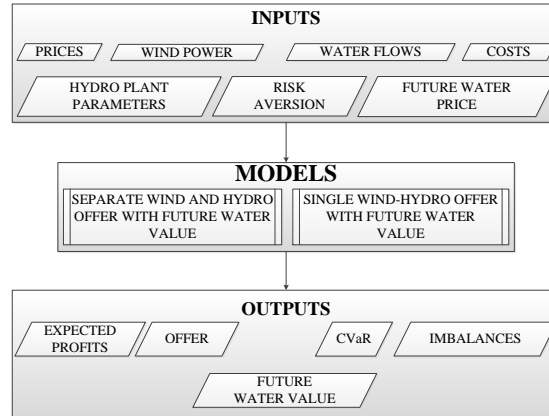


Figure 3. Inputs and outputs of the simulations.

1 These models are programmed in MATLAB [35] and GAMS [36] in a computer with Xeon E5-2687W
 2 processor at 3.10 GHz and 256 GB of RAM. The simulation procedure is shown in Fig. 4. MATLAB is used
 3 to treat the input and output data and GAMS, using the CPLEX 12 solver, is used to solve the proposed
 4 mathematical problem.

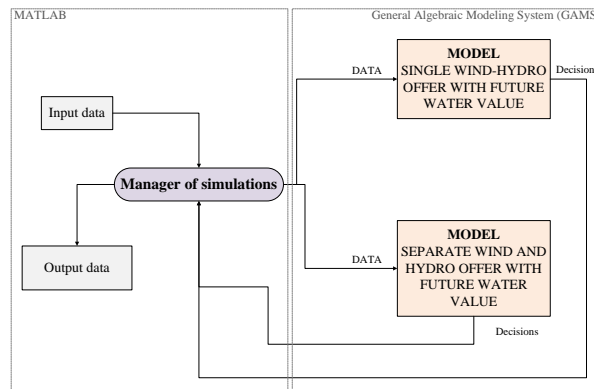


Figure 4. Diagram of the simulations procedure.

5 The simulation is focused on the behavior of the month of February 2013. The model is tested for 720 h
 6 and 168 h (for the first 168 h of the 720 h), respectively. As a consequence of using a reversible hydro unit
 7 to reduce wind imbalances, it is necessary to use an hourly optimization model.

8 The prices are adjusted hourly to a Normal distribution in a 24-h period. Data come from January and
 9 February of 2013 from the Spanish day-ahead market [37] and from the Spanish imbalance market [38]. The
 10 negative imbalance market prices, the day-ahead market prices, and the positive imbalance market prices
 11 are known.

1 Wind power is simulated through a Weibull distribution, where wind speed data from the Navarre area
 2 are adjusted hourly for a 24-h period. Also, wind power is between 0 and 50 MW per hour.

3 Inflows are adjusted hourly to a Normal distribution in a 24-h period. The inflows have very low values
 4 to reduce their effects on hydro generation, where the inflows can be positive or negative.

5 3.2. Scenarios

6 To verify the models, scenarios are created as part of a scenario tree with three branches. The first
 7 branch is the water inflow, the second branch is the price (market price, positive imbalance price and
 8 negative imbalance price), and the last branch is the wind generation and the wind generation limits required
 9 to calculate possible imbalances. Thus, the number of scenarios is equal to the number of water inflow
 10 scenarios multiplied by the number of price scenarios and by the number of wind generation scenarios. The
 11 total number of scenarios = water inflow scenarios · price scenarios · wind generation scenarios = $2 \cdot 6 \cdot 8$
 12 = 96.

13 This number of scenarios is enough to appreciate differences and changes between *CVaR* and *VaR*, where
 14 the expected profit is stable. Day-ahead prices, positive imbalance prices, negative imbalance prices and
 15 wind generation are shown in Fig. 5, Fig. 6, Fig. 7 and Fig. 8, respectively.

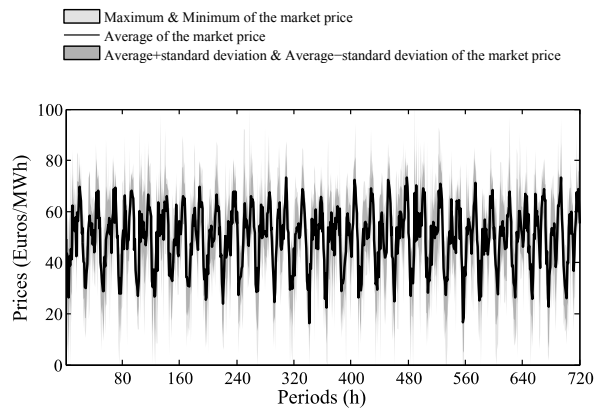


Figure 5. Maximum, average+standard deviation, average, average-standard deviation, and minimum of the market prices in €/MWh for 720 h.

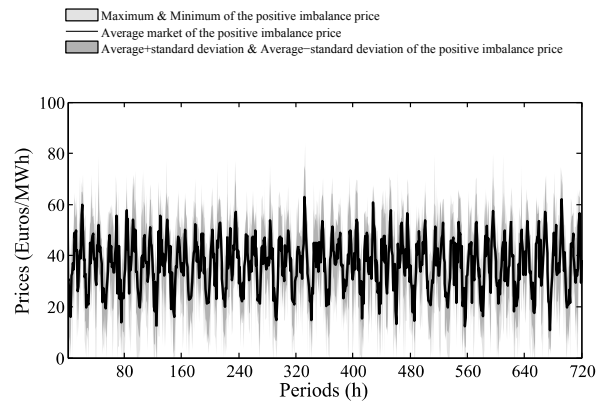


Figure 6. Maximum, average+standard deviation, average, average-standard deviation, and minimum of the positive imbalance prices in €/MWh for 720 h.

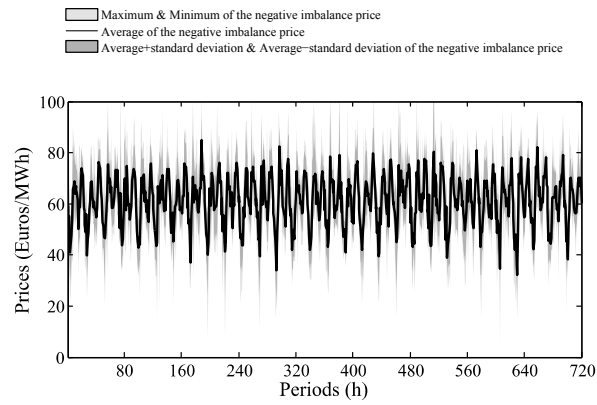


Figure 7. Maximum, average+standard deviation, average, average-standard deviation, and minimum of the negative imbalance prices in €/MWh for 720 h.

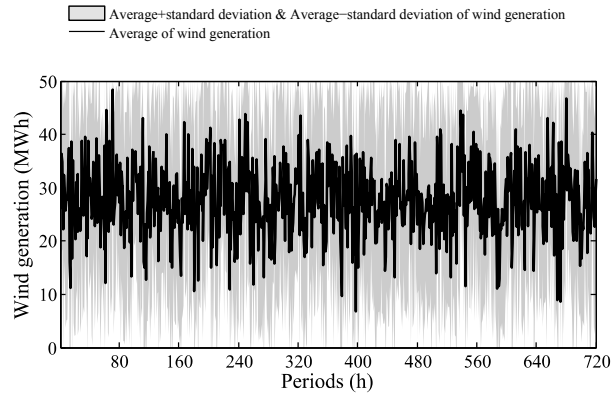


Figure 8. Average+standard deviation, average, and average-standard deviation of wind generation in MWh for 720 h.

1 fw_{pw} has to be parameterized. Due to this, the average prices per scenario of each time frame are shown
 2 in Fig. 9. Each of these average prices per scenario are multiplied by the fpf factor, where the fpf factor
 3 can be 0, 50, 100, or 400.

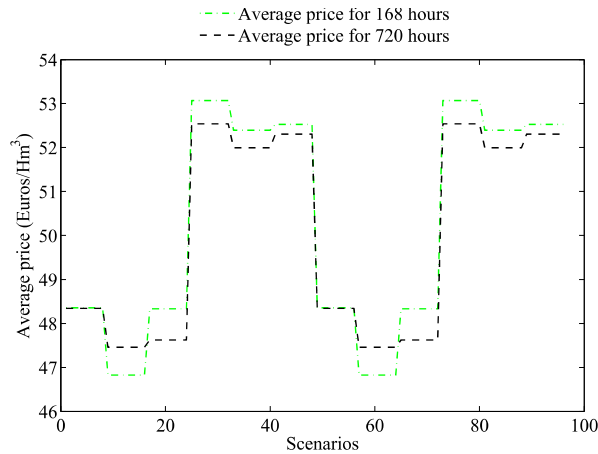


Figure 9. Average price per scenario in €/Hm³ for several time frames such as: 168 h and 720 h.

4. Results

5 This section shows the main results obtained from the simulations.

6 4.1. Effects of the FWV on the offer

7 The behavior of the two kinds of strategies are presented in 3D figures, where the sales and purchases of
 8 energy are shown. The axes are the risk aversion, the fpf factor and the quantity offered. The risk aversion

parameter is $\beta \in (0, 1)$, ranging from 0, 0.25, 0.5, 0.75 and 0.9. The fpf factor range is 0, 50, 100, and 400. The following figures are presented depending on the time frame: 1 week (168 h) and 1 month (720 h).

Fig. 10 and 11 show the single strategy offer for the two time frames and Fig. 12 and 13 present the separate strategy offer for the two time frames.

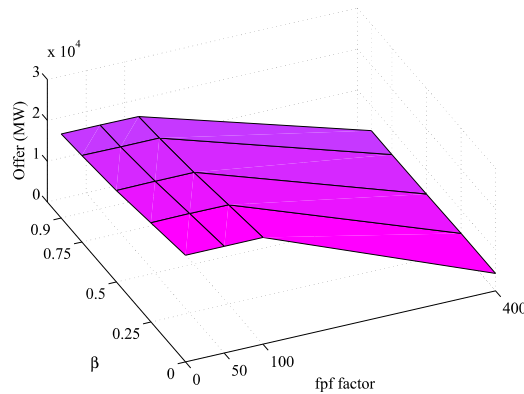


Figure 10. Single strategy offer for 720 h.

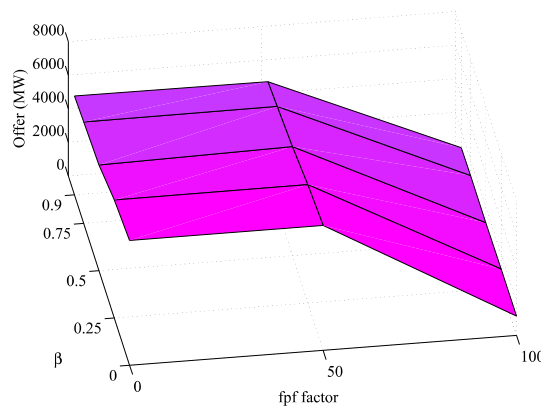


Figure 11. Single strategy offer for 168 h.

Fig. 10 and 11 present two behaviors, one of them describes the evolution of the offer with the β value and the other is the offer evolution with the fpf factor. Normally, the offer is higher for a higher risk ($\beta = 0$). Also, the behavior of the offer with respect to the fpf factor is stable (similar offers for different fpf factors) until fpf factor reaches a threshold value; then, the offer is reduced because it is more profitable to store water in the reservoir than to generate energy by the hydro unit. Hence, the offer decreases for an fpf factor of 400 in the 720 h case and for an fpf factor of 100 in the 168 h case.

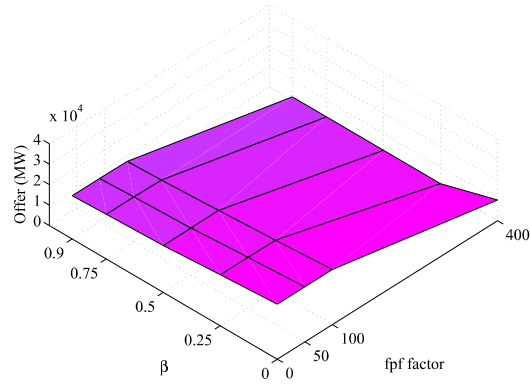


Figure 12. Separate strategy offer for 720 h.

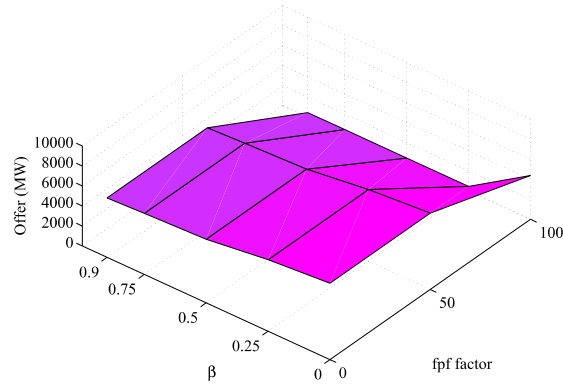


Figure 13. Separate strategy offer for 168 h.

1 Fig. 12 and 13 show a similar behavior, except for values of risk below $\beta = 0.25$, where there is a small
 2 increase in the offer after the threshold of the *fpf* factor is reached.

3 Fig. 14 presents the purchase bid by the pumping unit in the separate strategy for 720 h. The purchase
 4 bid of a separate strategy is zero for an *fpf* factor = 400 in the 720 h case, where the separate purchase bid
 5 is zero for 168 h and the purchase of the single strategy is zero for the two time frames.

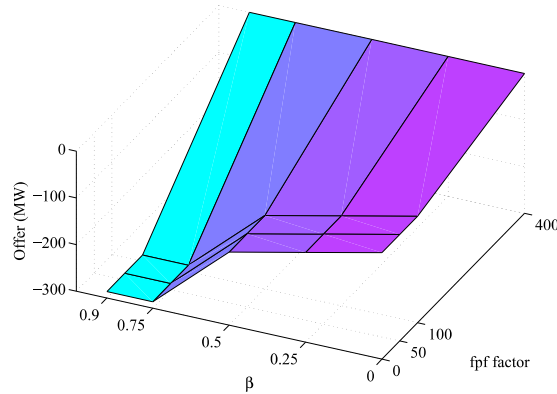


Figure 14. Purchase for the separate strategy for 720 h.

1 The purchase bid is equal to zero in the separate strategy for 168 h because, for 168 h, it is more profitable
 2 to sell energy, since the water reservoir can be equal in the first and last periods without buying energy
 3 to pump. But in the 720 h case, the unit can purchase energy to pump, improving the profit and making
 4 the final reserve equal to or greater than the initial reserve. The purchase bid with a single strategy is zero
 5 because, with this strategy, the unit can pump without buying energy. The pumped energy can come from
 6 a purchase bid and from the excess of wind energy with respect to a possible offer, reducing the imbalances.
 7 Due to this, the purchase bid is zero for the two time frames in the single strategy.

8 *4.2. Effects of the FWV on imbalances*

9 The main effects of the *FWV* in the positive and negative imbalances are shown in the following figures.
 10 Fig. 15 and 16 show the positive imbalances of the single strategy for the two time frames.

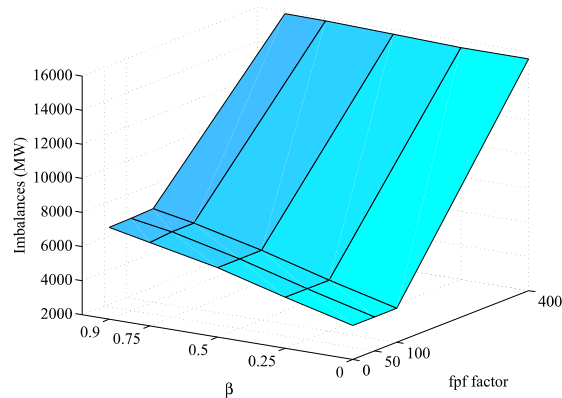


Figure 15. Positive imbalances for the single strategy for 720 h.

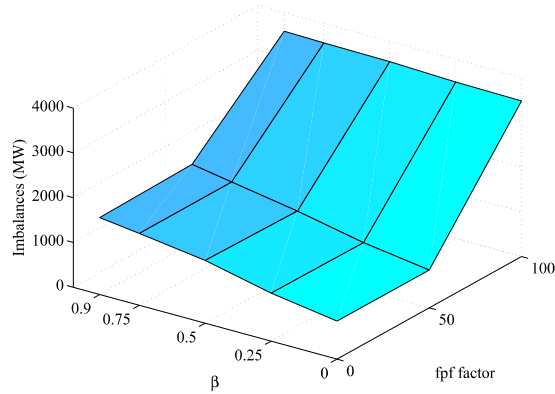


Figure 16. Positive imbalances for the single strategy for 168 h.

1

Fig. 17 and 18 show the positive imbalances of the separate strategy for the two time frames.

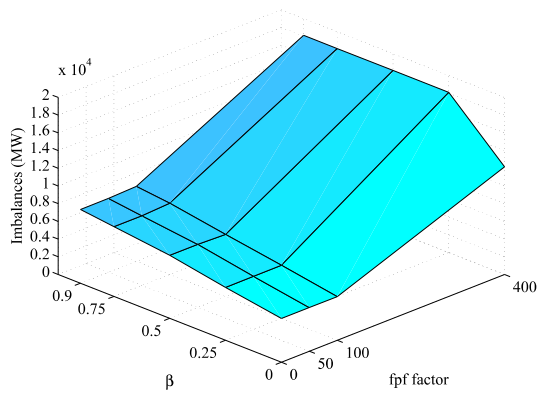


Figure 17. Positive imbalances for the separate strategy for 720 h.

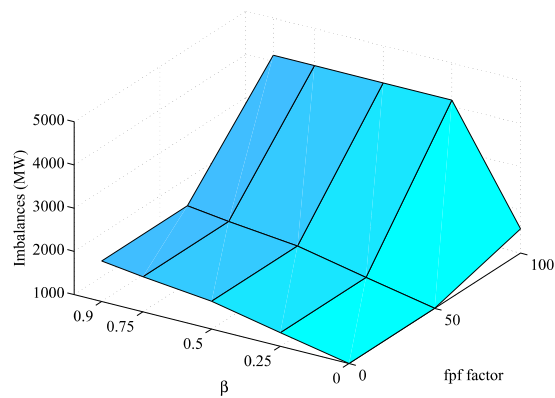


Figure 18. Positive imbalances for the separate strategy for 168 h.

1 Fig. 19 and 20 show the negative imbalances of the single strategy for the two time frames and Fig. 21
2 and 22 present the negative imbalances of the separate strategy for the two time frames.

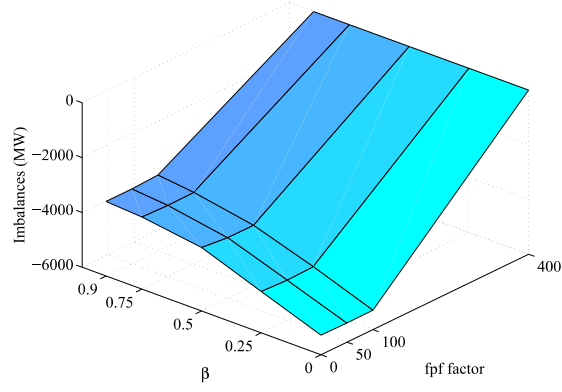


Figure 19. Negative imbalances for the single strategy for 720 h.

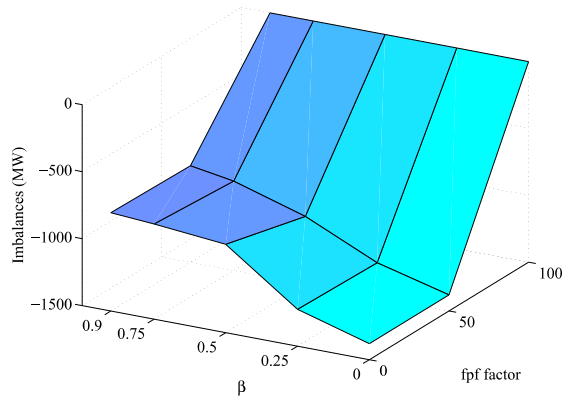


Figure 20. Negative imbalances for the single strategy for 168 h.

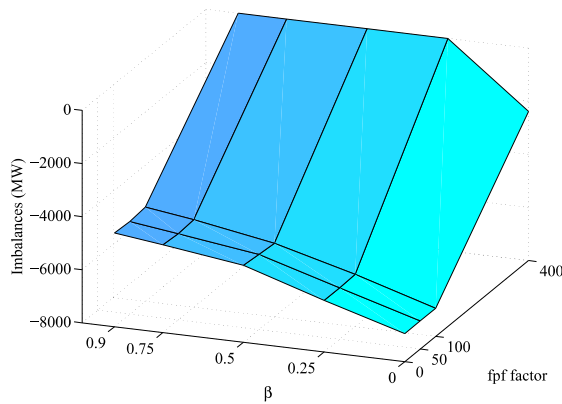


Figure 21. Negative imbalances for the separate strategy for 720 h.

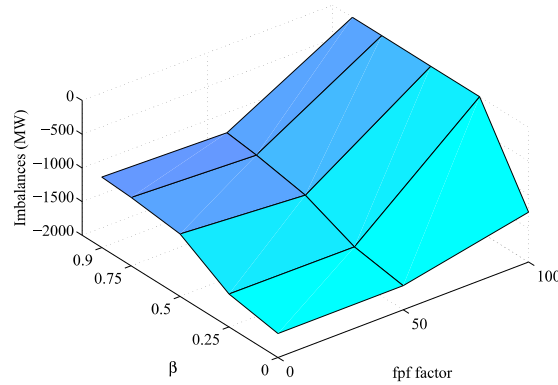


Figure 22. Negative imbalances for the separate strategy for 168 h.

1 The positive imbalance is higher for a high *fpf* factor and for high β values, however, the negative
 2 imbalance is higher when β is low and for a low *fpf* factor.

3 When there is a high offer, the possible negative imbalance will be higher and the possible positive
 4 imbalance will be lower, but, for a low offer the possible negative imbalance will be lower and the possible
 5 positive imbalance will be higher. This behavior comes from the difference between the total capacity of the
 6 units and the offer in case of a possible positive imbalance, and from the difference between the generation
 7 (equal to zero) and the offer in case of a possible negative imbalance, as shown in Fig. 23. In Fig. 23 the
 8 three possibilities of the possible imbalances are presented: Case 1 (high offer), Case 2 (intermediate offer)
 9 and Case 3 (low offer).

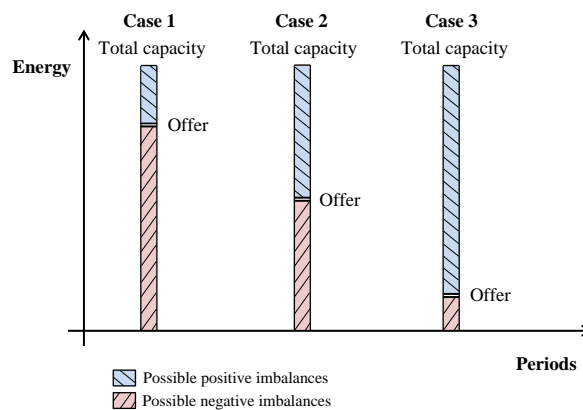


Figure 23. Cases of the possible imbalances depending on the offer.

1 4.3. Effects of the *FWV* on the discharge and pumping of reversible hydro power

2 The amount of energy generated or pumped is different depending on the time frame. Table 1 shows the
3 energy generated or pumped.

4 The energy pumped for 720 h is not proportional to the 168 h time frame, being much higher for 720 h
5 than for 168 h.

6 This is observed in Table 1 that presents the values of generation and pumping, showing that the
7 reversible hydro unit is more flexible for 720 h. Also, the single strategy is more flexible due to the pumping
8 of more water.

9 4.4. Effects of the *FWV* on the expected profits, *CVaR* and standard deviation

10 The objective function aims to maximize the total expected profits and the *CVaR*. Table 2 shows the
11 profits coming from selling energy in the electricity market as well as the *FWV*.

12 When the *fpf* factor reaches a value which is high enough, the profit, *PF*, decreases to reduce the
13 consumption of water because it is more profitable to have the water stored. *FWV* is less volatile because
14 it depends on the reservoir level. Table 2 shows that the profit of the single strategy is always higher than
15 the one of the separate strategy except for *fpf* factors where *FWV* is more profitable without risk aversion,
16 i.e. $\beta=0$; nevertheless, *FWV*s for the two strategies are very similar because they depend on the reservoir
17 levels.

		fpf factor	Risk aversion (β)					
			0	0.25	0.5	0.75	0.9	
Time frame 720 hours	SINGLE OFFER	0	Generation (MWh)	3960	4174	4149	4161	4153
		0	Pumping (MWh)	88	95	97	103	100
		50	Generation (MWh)	4172	4563	4521	4514	4489
		50	Pumping (MWh)	88	93	96	99	97
		100	Generation (MWh)	4158	4571	4522	4504	4496
		100	Pumping (MWh)	88	94	95	103	100
	SEPARATE OFFER	400	Generation (MWh)	0	0	0	0	0
		400	Pumping (MWh)	61	61	61	61	61
		0	Generation (MWh)	4832	4835	4839	4847	4848
		0	Pumping (MWh)	29	38	47	70	70
		50	Generation (MWh)	4843	4846	4850	4858	4858
		50	Pumping (MWh)	29	37	47	70	70
Time frame 168 hours	SINGLE OFFER	100	Generation (MWh)	4843	4847	4850	4857	4858
		100	Pumping (MWh)	28	37	46	70	70
		400	Generation (MWh)	0	0	0	0	0
		400	Pumping (MWh)	0	0	0	0	0
		0	Generation (MWh)	1948	1913	1933	1912	1926
		0	Pumping (MWh)	8	7	7	7	6
	SEPARATE OFFER	50	Generation (MWh)	2068	1934	2067	1929	1919
		50	Pumping (MWh)	8	5	3	6	4
		100	Generation (MWh)	0	0	0	0	0
		100	Pumping (MWh)	0	0	0	0	0
		0	Generation (MWh)	2210	2199	2176	2181	2180
		0	Pumping (MWh)	0	0	0	0	0
SEPARATE OFFER	50	Generation (MWh)	2210	2202	2181	2190	2185	
	50	Pumping (MWh)	0	0	0	0	0	
	100	Generation (MWh)	0	0	0	0	0	
	100	Pumping (MWh)	0	0	0	0	0	

Table 1. Values of the hydro generation/pumping for each strategy and each factor considered in the simulations.

		fpf factor	Risk aversion (β)				
			0	0.25	0.5	0.75	0.9
Time frame 720 hours	SINGLE OFFER	0 Profit (€)	807945	803994	794925	786899	781681
		0 FWV (€)	0	0	0	0	0
		50 Profit (€)	807965	804155	793943	785257	781218
		50 FWV (€)	472924	472924	472922	472937	472934
	100 Profit (€)	807938	803970	794461	785317	779942	
	100 FWV (€)	945872	945866	945860	945873	945873	
	400 Profit (€)	440662	440572	439868	438948	438439	
	400 FWV (€)	3783522	3783522	3783522	3783522	3783522	
	SEPARATE OFFER	0 Profit (€)	793286	789414	779997	771193	765829
		0 FWV (€)	0	0	0	0	0
		50 Profit (€)	793274	789293	779475	770253	764967
		50 FWV (€)	472940	472940	472940	472940	472940
		100 Profit (€)	793229	789308	779988	768925	763752
		100 FWV (€)	945880	945880	945880	945880	945880
		400 Profit (€)	481573	417267	417267	417267	417267
		400 FWV (€)	3783522	3783522	3783522	3783522	3783522
Time frame 168 hours	SINGLE OFFER	0 Profit (€)	230132	229053	225747	223192	222145
		0 FWV (€)	0	0	0	0	0
		50 Profit (€)	230098	229089	226648	224054	222775
		50 FWV (€)	419184	419184	419184	419184	419182
	100 Profit (€)	104692	104664	104557	104507	104148	
	100 FWV (€)	838624	838624	838624	838624	838624	
	SEPARATE OFFER	0 Profit (€)	227096	225771	221926	219789	218547
		0 FWV (€)	0	0	0	0	0
		50 Profit (€)	227096	226003	223182	220117	218972
		50 FWV (€)	419158	419158	419158	419158	419158
		100 Profit (€)	125429	97543	97543	97543	97543
		100 FWV (€)	838624	838624	838624	838624	838624

Table 2. Values of the profits from selling the energy and the *FWV* for each strategy and each factor considered in the simulations.

1 The behavior of the efficient frontier, expected profits vs. standard deviation, is consistent, i.e., "more
2 profit, more risk", as shown in Table 3. Also, for high values of β , *CVaR* is higher and the total expected
3 profit is reduced. Normally, the total expected profit of the single strategy is higher than the one of the
4 separate strategy, but, when the *fpf* factor is equal to 400 or 100 (depending on the time frames), this
5 changes due to the importance of the reservoir levels. Nevertheless, the standard deviation continues to be
6 lower in the single strategy compared to the separate strategy.

7 Table 3 presents the expected profit (EP), *CVaR* and the standard deviation (STD).

		fpf factor	Risk aversion (β)				
			0	0.25	0.5	0.75	0.9
Time frame 720 hours	SINGLE OFFER	0 EP (€)	807945	803994	794925	786899	781681
		0 CVaR (€)	558338	604975	618731	624372	625653
		0 STD (€)	180868	163892	149136	140425	136090
		50 EP (€)	1280889	1277079	1266865	1258194	1254152
		50 CVaR (€)	1003394	1049832	1064687	1071657	1072438
		50 STD (€)	180932	164154	149400	139866	136634
		100 EP (€)	1753810	1749836	1740320	1731190	1725815
		100 CVaR (€)	1448305	1494308	1508921	1515485	1516542
		100 STD (€)	180938	164041	149738	140167	136116
	400 EP (€)	4224184	4224094	4223390	4222470	4221961	
	400 CVaR (€)	3729148	3817451	3818446	3818942	3819044	
	400 STD (€)	128383	128163	127750	127383	127125	
	SEPARATE OFFER	0 EP (€)	793286	789414	779997	771193	765829
		0 CVaR (€)	542948	589280	605506	611364	612513
		0 STD (€)	183493	165378	148096	139168	134861
		50 EP (€)	1266214	1262233	1252415	1243193	1237908
		50 CVaR (€)	987906	1035331	1052796	1058741	1059819
		50 STD (€)	183470	165391	148517	139254	134833
		100 EP (€)	1739109	1735188	1725869	1714805	1709633
		100 CVaR (€)	1432720	1479353	1496252	1503473	1504466
		100 STD (€)	183472	165210	149462	138743	134518
		400 EP (€)	4265095	4200789	4200789	4200789	4200789
		400 CVaR (€)	3736799	3790597	3790597	3790597	3790597
		400 STD (€)	149794	128693	128693	128693	128693
SINGLE OFFER		0 EP (€)	230132	229053	225747	223192	222145
		0 CVaR (€)	151457	169346	175201	176876	177107
		0 STD (€)	49484	45272	39712	37413	36717
	50 EP (€)	649282	648272	645832	643238	641957	
	50 CVaR (€)	539257	563424	568095	569869	570177	
	50 STD (€)	49496	45737	41007	38173	37194	
	100 EP (€)	943316	943288	943180	943131	942771	
	100 CVaR (€)	813567	840526	840698	840736	840833	
	100 STD (€)	34939	34903	34747	34744	34586	
SEPARATE OFFER	0 EP (€)	227096	225771	221926	219789	218547	
	0 CVaR (€)	146604	165284	172097	173495	173784	
	0 STD (€)	50256	45753	39462	37480	36665	
	50 EP (€)	646254	645161	642340	639275	638130	
	50 CVaR (€)	536882	559562	564874	566909	567121	
	50 STD (€)	50256	46457	41267	37926	37021	
	100 EP (€)	964052	936167	936167	936167	936167	
	100 CVaR (€)	821610	833056	833056	833056	833056	
	100 STD (€)	46239	34770	34770	34770	34770	

Table 3. Values of the expected profit, CVaR and standard deviation for each strategy and each factor considered in the simulations.

1 From Fig. 9, the average future water price, $F\bar{W}P = \sum_w (fwp_w)/ts$, is equal to €50.04/Hm³ for 720
2 h and €50.25/Hm³ for 168 h. Hence, $F\bar{W}P$ provides enough information to obtain the water price. In
3 Table 3 the fpf factors are obtained when the standard deviation of the expected profit remains constant
4 for several β values of the separate offer, producing two limits: an fpf factor of 400 in 720 h and an fpf
5 factor of 100 in 168 h.

6 Thus, the fpf factor multiplied by $F\bar{W}P$ for 720 h is $400 \times 50.04 = \text{€}20,016/\text{Hm}^3$ and $100 \times 50.25 =$
7 $\text{€}5,025/\text{Hm}^3$ for 168 h. In this case, the monthly value is four times the weekly value, approximately.

8 In general, the offer is reduced for a specific value of the fpf factor depending on the time frame. This
9 behavior is similar for the two strategies with the exception of the separate strategy that shows a small

change for $\beta = 0$ and with the last fpf factor evaluated for the two time frames. This effect can also be observed in Table 3 for $\beta = 0$ with an fpf factor = 400 for a time frame of 720 h and with an fpf factor=100 for a time frame of 168 h. Thus, in Figs. 10, 11, 17, 18 and Table 3 it is observed that, for $\beta = 0$ and the highest fpf factor of the separate strategy the profits that come from sales of energy are higher than in the single strategy case because the separate strategy is more risky (the measure of it being STD) and the units prefer to offer more energy and increase the negative imbalance, also reducing the positive imbalance. Thus, the energy profit is only higher for the separate strategy with the highest risk and fpf factor. This behavior is a result of considering risk in the models.

Another interesting feature is the effect of the time frame. Ratios are obtained by dividing the values of the same variable for different time frames. The reference ratio is $720/168=4.28$. Ratios higher or lower than 4.28 imply a different evolution per hour between the 720 and 168 h cases. Ratios higher than the reference ratio mean an increment in the value per hour of the variables (the total expected profit, $CVaR$ and standard deviation). Instead, ratios lower than the reference ratio mean a reduction in the value of the variables. Table 4 presents the ratios of the total expected profits, $CVaR$ and standard deviation of the total expected profits.

		Ratios	Risk aversion (β)				
			0	0.25	0.5	0.75	0.9
SINGLE OFFER	0	EP	3.5	3.5	3.5	3.5	3.5
		CVaR	3.7	3.6	3.5	3.5	3.5
		STD	3.7	3.6	3.8	3.8	3.7
	50	EP	2.0	2.0	2.0	2.0	2.0
		CVaR	1.9	1.9	1.9	1.9	1.9
		STD	3.7	3.6	3.6	3.7	3.7
	100	EP	1.9	1.9	1.8	1.8	1.8
		CVaR	1.8	1.8	1.8	1.8	1.8
		STD	5.2	4.7	4.3	4.0	3.9
SEPARATE OFFER	0	EP	3.5	3.5	3.5	3.5	3.5
		CVaR	3.7	3.6	3.5	3.5	3.5
		STD	3.7	3.6	3.8	3.7	3.7
	50	EP	2.0	2.0	1.9	1.9	1.9
		CVaR	1.8	1.9	1.9	1.9	1.9
		STD	3.7	3.6	3.6	3.7	3.6
	100	EP	1.8	1.9	1.8	1.8	1.8
		CVaR	1.7	1.8	1.8	1.8	1.8
		STD	4.0	4.8	4.3	4.0	3.9

Table 4. Ratios of the total expected profits, $CVaR$ and standard deviation.

The standard deviation ratio is higher when the fpf factor is equal to 100, but the total expected profit ratio and the $CVaR$ ratio are lower for high fpf factors. Thus, the behavior of the total expected profit ratio and the $CVaR$ ratio vs. the standard deviation is the opposite. The standard deviation ratio of the two strategies is similar to the reference ratio for an fpf factor of 100, but, sometimes, it is slightly higher

1 than the reference ratio. The other ratios are lower, where the minimum ratio is 1.7, therefore, these values
 2 are not increased proportionally (4.28 times) with respect to the 168 h case, they can even be reduced.

3 5. Conclusions

4 This paper presented two strategies to offer wind generation and hydro generation/pumping in day-
 5 ahead electricity markets. Moreover, the *FWV* was included affecting the behavior of the strategies. A new
 6 formulation including the *FWV* was presented and results were obtained for two time frames.

7 The *FWV* and the time horizon can considerably affect wind and reversible hydro power. The *FWV*
 8 depends on fwp_w , if fwp_w is high enough, the *FWV* is more important in the objective function. Moreover,
 9 due to the lower volatility of the water reservoir level, the standard deviation of the objective function is
 10 reduced. In this way, to reduce the *PF* of the objective function, the generator offers a lower amount
 11 of energy increasing the possibility of obtaining both a higher positive imbalance and a lower negative
 12 imbalance. Regarding hydro pumping and generation, when the *fpf* factor is increased, the pumping is
 13 constant or reduced. Regarding the evolution of pumping with risk aversion, lower β values reduce the
 14 pumping of water to the upper reservoir to generate and sell more energy.

15 In addition, the prices calculated for water are: i) €20,016/Hm³ for 720 h and ii) €5,025/Hm³ for 168 h,
 16 where the price per hour for each time frame is €20,016/Hm³/720 h = €27.8/Hm³· h and €5,025/Hm³/168
 17 h = €29.9/Hm³· h, respectively. Therefore, the price of water per hour to reduce the offer of the generators
 18 is higher in the 168-h time horizon than in the 720-h one.

19 The results obtained by the models illustrated the following generic conclusions.

- 20 • A single strategy is more profitable than a separate strategy in all time frames. But, when the *FWV*
 21 is more relevant than the profit from selling energy and there is no risk aversion ($\beta=0$), the total
 22 expected profit is higher for the separate strategy.
- 23 • The profits from selling energy are more volatile than the profits from the *FWV* due to the lower
 24 volatility of the reservoir levels.
- 25 • In the single strategy case, wind imbalances and reversible hydro imbalances are lower.
- 26 • The ratios of variables for different time frames are not proportional to the reference ratio (4.28).
 27 Nevertheless, the standard deviation ratio has an opposite behaviour compared to the other ratios of
 28 the variables: total expected profits and *CVaR*.

- 1 • A longer time frame makes the reversible hydro unit more flexible because enlarging the time frame
2 relaxes the reservoir constraints.
- 3 • The ratios calculated have demonstrated that the time horizons affect the decisions of the problem,
4 where the temporal relation in the water reservoir is the relevant issue to compare between several
5 time frames and also the consequence of some differences in the decisions. Hence, the capacity to store
6 energy can take advantage of the volatility of the market prices for different time frames.

7 Acknowledgement

8 The research leading to these results has received funding from the EU Seventh Framework Programme
9 FP7/2007-2013 under grant agreement no. 309048. The work of João Catalão was also supported by
10 FEDER funds (European Union) through COMPETE and by Portuguese funds through FCT, under Projects
11 FCOMP-01-0124-FEDER-020282 (Ref. PTDC/EEA-EEL/118519/2010) and UID/CEC/50021/2013.

12 References

- 13 [1] J. P. S. Catalão, H. M. I. Pousinho, V. M. F. Mendes, Scheduling of head-dependent cascaded hydro systems: Mixed-integer
14 quadratic programming approach, *Energy Conversion and Management* 51 (3) (2010) 524 – 530.
- 15 [2] X. Yuan, Y. Wang, J. Xie, X. Qi, H. Nie, A. Su, Optimal self-scheduling of hydro producer in the electricity market,
16 *Energy Conversion and Management* 51 (12) (2010) 2523 – 2530.
- 17 [3] J. S. Anagnostopoulos, D. E. Papantonis, Pumping station design for a pumped-storage wind-hydro power plant, *Energy*
18 *Conversion and Management* 48 (11) (2007) 3009 – 3017.
- 19 [4] J. Kaldellis, K. Kavadias, E. Christinakis, Evaluation of the wind-hydro energy solution for remote islands, *Energy*
20 *Conversion and Management* 42 (9) (2001) 1105 – 1120.
- 21 [5] J. M. Morales, A. J. Conejo, H. Madsen, P. Pinson, M. Zugno, *Integrating Renewables in Electricity Markets: Operational*
22 *Problems*, Springer, 2014.
- 23 [6] N. Hatziaargyriou, A. Zervos, Wind power development in Europe, *Proceedings of the IEEE* 89 (12) (2001) 1765–1782.
- 24 [7] A. J. Conejo, J. M. Arroyo, J. Contreras, F. A. Villamor, Self-scheduling of a hydro producer in a pool-based electricity
25 market, *IEEE Transactions on Power Systems* 17 (4) (2002) 1265–1272.
- 26 [8] D. W. Bunn, *Modelling prices in competitive electricity markets*, J. Wiley, 2004.
- 27 [9] A. Ku, *Risk and flexibility in electricity: introduction to the fundamentals and techniques*, Risk Books, 2003.
- 28 [10] E. Castillo, R. Mínguez, A. Conejo, R. García-Bertrand, *Decomposition techniques in mathematical programming*,
29 Springer, 2006.
- 30 [11] J. M. Morales, A. J. Conejo, J. Pérez-Ruiz, Short-term trading for a wind power producer, *IEEE Transactions on Power*
31 *Systems* 25 (1) (2010) 554–564.
- 32 [12] H. M. I. Pousinho, V. M. F. Mendes, J. P. S. Catalão, A risk-averse optimization model for trading wind energy in a
33 market environment under uncertainty, *Energy* 36 (8) (2011) 4935–4942.

- 1 [13] G. Steeger, L. Barroso, S. Rebennack, Optimal bidding strategies for hydro-electric producers: A literature survey, *IEEE*
2 *Transactions on Power Systems* 29 (4) (2014) 1758–1766.
- 3 [14] R. T. Rockafellar, S. Uryasev, Optimization of conditional value-at-risk, *Journal of Risk* 2 (2000) 21–42.
- 4 [15] S. Sarykalin, G. Serraino, S. Uryasev, Value-at-risk vs. conditional value-at-risk in risk management and optimization,
5 *Tutorials in Operations Research*. INFORMS, Hanover, MD (2008) 270–294.
- 6 [16] J. M. Angarita, J. G. Usaola, Combining hydro-generation and wind energy: Biddings and operation on electricity spot
7 markets, *Electric Power Systems Research* 77 (5-6) (2007) 393–400.
- 8 [17] J. Matevosyan, M. Olsson, L. Söder, Hydropower planning coordinated with wind power in areas with congestion problems
9 for trading on the spot and the regulating market, *Electric Power Systems Research* 79 (1) (2009) 39–48.
- 10 [18] H. Ding, Z. Hu, Y. Song, Stochastic optimization of the daily operation of wind farm and pumped-hydro-storage plant,
11 *Renewable Energy* 48 (0) (2012) 571 – 578.
- 12 [19] M. Marzband, F. Azarnejadian, M. Savaghebi, J. Guerrero, An optimal energy management system for islanded microgrids
13 based on multiperiod artificial bee colony combined with markov chain, *IEEE Systems Journal* PP (99) (2015) 1–11.
14 doi:10.1109/JSYST.2015.2422253.
- 15 [20] M. Marzband, A. Sumper, A. Ruiz-Álvarez, J. L. Domínguez-García, B. Tomoiagă, Experimental evaluation of a real time
16 energy management system for stand-alone microgrids in day-ahead markets, *Applied Energy* 106 (2013) 365–376.
- 17 [21] M. Marzband, A. Sumper, J. L. Domínguez-García, R. Gumara-Ferret, Experimental validation of a real time energy man-
18 agement system for microgrids in islanded mode using a local day-ahead electricity market and minlp, *Energy Conversion*
19 *and Management* 76 (2013) 314–322.
- 20 [22] M. Marzband, M. Ghadimi, A. Sumper, J. L. Domínguez-García, Experimental validation of a real-time energy man-
21 agement system using multi-period gravitational search algorithm for microgrids in islanded mode, *Applied Energy* 128
22 (2014) 164–174.
- 23 [23] A. Sánchez de la Nieta, J. Contreras, J. I. Muñoz, Optimal coordinated wind-hydro bidding strategies in day-ahead
24 markets, *IEEE Transactions on Power Systems* 28 (2) (2013) 798–809.
- 25 [24] J. García-González, R. R. de la Muela, L. M. Santos, A. M. González, Stochastic joint optimization of wind generation
26 and pumped-storage units in an electricity market, *IEEE Transactions on Power Systems* 23 (2) (2008) 460–468.
- 27 [25] E. D. Castronuovo, J. P. Lopes, On the optimization of the daily operation of a wind-hydro power plant, *IEEE Transactions*
28 *on Power Systems* 19 (3) (2004) 1599–1606.
- 29 [26] M. Denault, D. Dupuis, S. Couture-Cardinal, Complementarity of hydro and wind power: Improving the risk profile of
30 energy inflows, *Energy Policy* 37 (12) (2009) 5376–5384.
- 31 [27] J. Dhillon, A. Kumar, S. Singal, Optimization methods applied for wind-*psp* operation and scheduling under deregulated
32 market: A review, *Renewable and Sustainable Energy Reviews* 30 (2014) 682–700.
- 33 [28] L. V. Abreu, M. E. Khodayar, M. Shahidepour, L. Wu, Risk-constrained coordination of cascaded hydro units with
34 variable wind power generation, *IEEE Transactions on Sustainable Energy* 3 (3) (2012) 359–368.
- 35 [29] F. J. Díaz, J. Contreras, J. I. Muñoz, D. Pozo, Optimal scheduling of a price-taker cascaded reservoir system in a pool-based
36 electricity market, *IEEE Transactions on Power Systems* 26 (2) (2011) 604–615.
- 37 [30] C. Bueno, J. A. Carta, Wind powered pumped hydro storage systems, a means of increasing the penetration of renewable
38 energy in the Canary Islands, *Renewable and Sustainable Energy Reviews* 10 (4) (2006) 312–340.
- 39 [31] Wind-hydro energy integration project, "Wind-hydro integration: pumped storage to support wind", [Available

1 online]: [http://http://www.hydroworld.com/articles/print/volume-17/issue-3/Articles/wind-hydro-integration-pumped-](http://http://www.hydroworld.com/articles/print/volume-17/issue-3/Articles/wind-hydro-integration-pumped-storage-to-support-wind.html)
2 [storage-to-support-wind.html](http://http://www.hydroworld.com/articles/print/volume-17/issue-3/Articles/wind-hydro-integration-pumped-storage-to-support-wind.html).

3 [32] J. Dhillon, A. Kumar, S. Singal, Optimization methods applied for windpsp operation and scheduling under deregulated
4 market: A review, *Renewable and Sustainable Energy Reviews* 30 (0) (2014) 682 – 700.

5 [33] J. G. Manwell, J. F. MacGowan, A. L. Rogers, *Wind Energy Explained: Theory, Design and Application*, Second Edition,
6 John Wiley and Sons, 2009.

7 [34] Spanish Renewable Energy Plan for 2005–2010,

8 [Available online]:

9 [http://www.idae.es/uploads/documentos/documentos_PER.2005-2010.8.de.gosto-2005.Completo.\(modificacionpag_63\)-](http://www.idae.es/uploads/documentos/documentos_PER.2005-2010.8.de.gosto-2005.Completo.(modificacionpag_63).Copia.2.301254a0.pdf)
10 [Copia.2.301254a0.pdf](http://www.idae.es/uploads/documentos/documentos_PER.2005-2010.8.de.gosto-2005.Completo.(modificacionpag_63).Copia.2.301254a0.pdf).

11 [35] The Mathworks Inc., Matlab, [Available online]:

12 <http://www.mathworks.com>.

13 [36] A. Brooke, D. Kendrick, A. Meeraus, R. Raman, *GAMS/CPLEX: A Users Guide*, GAMS Development Corporation
14 (2003).

15 [37] Operador del Mercado Ibérico de Energía-Polo Español, S. A., [Available online]:

16 <http://www.omie.es/files/flash/ResultadosMercado.swf>.

17 [38] Red Eléctrica de España, e-sios, [Available online]:

18 <http://www.esios.ree.es/web-publica>.

Nomenclature

Indexes

i	Index referring to a hydro unit.
l	Index referring to each block resulting from the linearization of the production curve of a hydro turbine.
t	Index referring to a period [hour].
w	Index referring to a scenario.

Parameters

α	Per unit confidence level.
$b_{t,w}^+$	Upper limit of the wind farm power offer in period t and scenario w [MW].
$b_{t,w}^-$	Lower limit of the wind farm power offer in period t and scenario w [MW].
$B0_i$	Hydro unit i power capacity [MW].
β	Risk aversion of the producer, $\beta \in [0, 1]$.
c_i	Start-up cost of hydro unit i [€].
c_i^H	Generating cost of hydro unit i [€/MWh].
c_i^p	Pumping cost of hydro unit i [€/MWh].
cv	Conversion factor [$\text{Hm}^3 \cdot \text{s} / \text{m}^3 \cdot \text{h}$].
c^W	Wind farm generation cost [€/MWh].
eff	Hydro pumping efficiency.
fpf	Future water price factor.
$fwpw$	Future water price for each scenario w [€/Hm ³].
$F\bar{W}P$	Average future water price [€/Hm ³].
$if_{t,i,w}$	Incoming flow associated with hydro unit i , period t , and scenario w [Hm ³ /h].
$gw_{t,w}$	Power produced by the wind farm using a Weibull distribution in period t and scenario w [MW].
$\bar{\lambda}_w$	Average price of $\lambda_{t,w}$ per scenario w by Hm ³ [€/Hm ³].
$\lambda_{t,w}$	Day-ahead market price in period t and scenario w [€/MWh].
$\lambda_{t,w}^+$	Positive imbalance market price in period t and scenario w [€/MWh].
$\lambda_{t,w}^-$	Negative imbalance market price in period t and scenario w [€/MWh].
$Pmax$	Maximum installed power of the wind farm [MW].

$porhoh_i$	Minimum power of hydro unit i for the upper curve [MW].
$porhol_i$	Minimum power of hydro unit i for the lower curve [MW].
$porhom_i$	Minimum power of hydro unit i for the intermediate curve [MW].
ppm_i	Pumping upper limit of hydro unit i [MW].
$r_{t=0,i,w}^{initial}$	Initial reservoir volume of hydro unit i and scenario w [Hm ³].
ρ_w	Probability of occurrence of scenario w .
$rhoh_{l,i}$	Slope of block l of hydro unit i for the upper curve [MW/(m ³ /s)].
$rhoh_{l,i}$	Slope of block l of hydro unit i for the lower curve [MW/(m ³ /s)].
$rhoh_{l,i}$	Slope of block l of hydro unit i for the intermediate curve [MW/(m ³ /s)].
$rhopp_i$	Conversion factor from total hydro unit i capacity in MWh to m ³ /s [MW/(m ³ /s)].
tp	Total number of periods.
ts	Total number of scenarios.
$umax_i$	Maximum water discharge of hydro unit i [m ³ /s].
$umin_i$	Minimum water discharge of hydro unit i [m ³ /s].
$umax_{l,i}$	Maximum water discharge of block l of hydro unit i [m ³ /s].
$vc1_i$	Lower level of the reservoir associated with hydro unit i used in the discretization of the hydro production curves [Hm ³].
$vc2_i$	Upper level of the reservoir associated with hydro unit i used in the discretization of the hydro production curves [Hm ³].
$Vmax_i$	Maximum volume of the reservoir of hydro unit i [Hm ³].
$Vmin_i$	Minimum volume of the reservoir of hydro unit i [Hm ³].

Continuous Variables

$b_{t,i}$	Joint power offer in the day-ahead market associated to the wind farm and hydro unit i in period t [MW].
$bh_{t,i}$	Power offer in the day-ahead market associated to hydro unit i in period t [MW].

bw_t	Power offer in the day-ahead market associated to the wind farm in period t [MW].	HPF	Sum of all profits of the hydro units [€].
$CVaR$	Conditional value at risk [€].	$np_{t,i,w}$	Power produced by hydro unit i in period t and scenario w to eliminate the negative imbalance [MW].
$\Delta_{t,i,w}$	Imbalance between the actual joint production and the joint power offer of hydro unit i in period t and scenario w [MW].	$p_{t,i,w}$	Power produced by hydro unit i in period t and scenario w [MW].
$\Delta_{t,i,w}^-$	Negative imbalance between the actual joint production and the joint power offer of hydro unit i in period t and scenario w [MW].	PF	Sum of all profits of the wind and hydro units [€].
$\Delta_{t,i,w}^+$	Positive imbalance between the actual joint production and the joint power offer of hydro unit i in period t and scenario w [MW].	$pn_{t,i,w}$	Net pumping of hydro unit i in period t and scenario w [MW].
$\Delta h_{t,i,w}$	Imbalance between actual hydro production and the power offer associated with the hydro unit i , in period t , and scenario w [MW].	$pp_{t,i,w}$	Total pumping of hydro unit i in period t and scenario w [MW].
$\Delta h_{t,i,w}^-$	Negative imbalance between the actual hydro production and the power offer associated with the hydro unit i in period t and scenario w [MW].	$ppb_{t,i,w}$	Power purchased by hydro unit i , in the day-ahead market that is pumped in period t and scenario w [MW].
$\Delta h_{t,i,w}^+$	Positive imbalance between the actual hydro production and the power offer associated with the hydro unit i in period t and scenario w [MW].	$ppw_{t,i,w}$	Power produced by the wind farm that is pumped to hydro unit i in period t and scenario w [MW].
$\Delta w_{t,w}$	Imbalance between actual wind production and the power offer associated to the wind farm in period t , and scenario w [MW].	$ppw_{t,i,w}^\pm$	Auxiliary variable associated with $ppw_{t,i,w}$ [MW].
$\Delta w_{t,w}^-$	Negative imbalance between the actual wind production and the power offer associated to the wind farm in period t and scenario w [MW].	$ppw_{t,i,w}^-$	Wind power that is pumped by hydro unit i when there is a joint offer to purchase power in period t and scenario w [MW].
$\Delta w_{t,w}^+$	Positive imbalance between the actual wind production and the power offer associated to the wind farm in period t and scenario w [MW].	$ppw_{t,i,w}^+$	Excess wind power that is pumped by hydro unit i , when there is a joint offer to sell power in period t and scenario w [MW].
η_w	Auxiliary variable in scenario w used to compute $CVaR$ [€].	$ppwm_{t,i,w}$	Excess of wind power that can be pumped by hydro unit i in period t and scenario w [MW].
FWV	Future water value of the volume of the reservoirs [€].	$pr_{t,i,w}$	Auxiliary variable associated with $pp_{t,i,w}$ [MW].
		$r_{t,i,w}$	Reservoir of hydro unit i in period t and scenario w [Hm ³].
		$r_{t=tp,i,w}$	Final reservoir volume in period tp of hydro unit i and scenario w [Hm ³].
		$s_{t,i,w}$	Spillage of hydro unit i in period t and scenario w [m ³ /s].
		$u_{t,i,w}$	Water discharge of hydro unit i in period t and scenario w [m ³ /s].

$ul_{l,t,i,w}$	Water discharge of block l of hydro unit i in period t and scenario w [m^3/s].	$d_{t,i,w}^2$	0/1 variable used in the discretization of the hydro production curves of hydro unit i in period t and scenario w .
VaR	Value at risk [€].		
$wf_{t,i,w}$	Water flow pumped by hydro unit i in period t and scenario w [m^3/s].	$j_{t,w}$	0/1 variable that is equal to 1 if the imbalance in period t , is negative, and 0 otherwise. Used in separate strategy.
WPF	Sum of all profits of the wind farm [€].		
<i>Binary Variables</i>			
$a_{t,i}$	0/1 variable that is equal to 0 if hydro unit i , pumps in period t , and 1 otherwise. Used in separate strategy.	$v_{t,i,w}$	0/1 variable that is equal to 1 if hydro unit i generates in period t and scenario w and 0 if the unit is pumping.
$a_{t,i,w}$	0/1 variable that is equal to 0 if hydro unit i , pumps in period t and scenario w , and 1 otherwise.	$w_{l,t,i,w}$	0/1 variable that is equal to 1 if the water discharged by hydro unit i has exceeded block l in period t and scenario w and 0 otherwise.
$bd_{t,i}$	0/1 variable that is equal to 1 if there is joint sale in period t , and 0 otherwise (purchase).	$y_{t,i,w}$	0/1 variable that is equal to 1 if hydro unit i is started-up in period t and scenario w , and 0 otherwise.
$d_{t,i,w}$	0/1 variable that is equal to 1 if the imbalance is negative, and 1 if the imbalance is positive.	$z_{t,i,w}$	0/1 variable that is equal to 1 if hydro unit i is shutdown in period t and scenario w , and 0 otherwise.
$d_{t,i,w}^1$	0/1 variable used in the discretization of the hydro production curves of hydro unit i in period t and scenario w .		