

Electricity prices forecasting by a hybrid evolutionary-adaptive methodology

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Abstract

With the restructuring of the electricity sector in recent years, and the increased variability and uncertainty associated with electricity market prices, it has become necessary to develop forecasting tools with enhanced capabilities to support the decisions of market players in a competitive environment. Hence, this paper proposes a new hybrid evolutionary-adaptive methodology for electricity prices forecasting in the short-term, i.e., between 24 and 168 hours ahead, successfully combining mutual information, wavelet transform, evolutionary particle swarm optimization, and the adaptive neuro-fuzzy inference system. In order to determine the accuracy, competence and proficiency of the proposed methodology, results from real-world case studies using real data are presented, together with a thorough comparison considering the results obtained with previously reported forecasting tools. Not only is the accuracy an important factor, but also the computational burden is relevant in a comparative study. The results show that it is possible to reduce the uncertainty associated with electricity market prices prediction without using any exogenous data, just the historical values, thus requiring just a few seconds of computation time.

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1. Introduction

1.1. Aims and Difficulty

In a deregulated electricity market, the most important signal for all market players corresponds to the price. The evolution from a vertically integrated structure to a deregulated framework increased the complexity of electricity market prices behaviour [1, 2]. Several characteristics of electricity market prices series make their forecast harder than demand series, such as non-stationary behaviour, high volatility and frequency, seasonality and the calendar effect [3]. An accurate tool for short-term electricity market prices forecasting is needed to assist producers in designing their offering strategies to the electricity market to achieve maximum profits [4, 5], on the one hand, and to assist consumers in protecting themselves against elevated prices and for planning purposes, on the other [6, 7]. Forecasting electricity market prices has grown to be one of the main research areas in power engineering [8-10], but the corresponding tools or techniques have not yet reached maturity [11]. Predicting electricity market prices is indeed a crucial task for all market players [12] in their decision making, especially with the advent of smart grids [13], or even with the mitigation commitment of green house gases emissions around the world [14].

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35 **1.2. Literature Review**

36 In recent years, several forecasting methodologies have been described in the specialized literature. These can
37 be divided in two groups: hard and soft computing methodologies [15]. In hard computing, some known
38 methodologies can be found, such as auto regressive integrated moving average (ARIMA) [16], wavelet
39 transform (WT) with ARIMA [17], and transfer function models [18]. These methodologies usually need a large
40 number of physical data, requiring also the exact modelling of the system, and result in high computational
41 burden. In soft computing, the methodologies use an auto learning process from historical sets to identify future
42 patterns. Thus, a new soft computing methodology was developed in this paper. Several methodologies can be
43 found in the specialized literature, such as neural network (NN) [19], NN model based on similar days (SDNN)
44 [20], fuzzy NN (FNN) [21], weighted nearest neighbours (WNN) [22], a modification of WNN called pattern
45 sequence-based forecasting (PSF) [23], adaptive wavelet NN (AWNN) [6], hybrid intelligent systems (HIS)
46 [24], NN with WT (NNWT) [25], WT with ARIMA and radial basis function NN (RBFN) [15], Elman network
47 or simple recurrent network (SRN) [26], cascaded neuro-evolutionary algorithms (CNEA) [27], cascaded NN
48 (CNN) [28], hybrid neuro-evolutionary system (HNES) [29], and other hybrids [30].

49 Existing features selection algorithms include correlation analysis [31], mutual information [27] and modified
50 relief [32], among other combinations and two stage techniques [33]. More recently, there are some interesting
51 methodologies published in this field of knowledge, such as WT combined with chaotic least squares support
52 vector machine (CLSSVM) and exponential generalized autoregressive conditional heteroskedastic (EGARCH)
53 model (WT+CLSSVM+EGARCH) [34], singular spectrum analysis (SSA) method [35], a combination
54 technique based on wavelet transform fuzzy, firefly algorithm and fuzzy ARTMAP (WT+FF+FA) [36], a
55 recursive dynamic factor analysis combined with Kalman filter (RDFA+KF) [37], and a derived methodology by
56 integrating the kernel principal component analysis, combined with the local informative vector machine,
57 derived from a local regression method (KPCA+IVM) [38].

58 **1.3. Motivation and Contribution**

59 In this paper, a new hybrid evolutionary–adaptive (HEA) methodology is proposed for short-term electricity
60 market price forecasting, based on mutual information (MI), WT, evolutionary particle swarm optimization
61 (EPSO), and the adaptive neuro-fuzzy inference system (ANFIS). The HEA methodology is tested on real case
62 studies using electricity market prices from the Spanish and Pennsylvania-New Jersey-Maryland (PJM)
63 electricity markets. To prove its superior forecasting accuracy and reduced computational burden, a
64 comprehensive comparison with others methodologies previously published in the specialized literature was

65 undertaken. The comparison (illustrated in tabular form) will take into account AWNN [6], wavelet-PSO-ANFIS
66 (WPA) methodology [12], RBFN [15], ARIMA [16], wavelet-ARIMA [17], NN [19], SDNN [20], FNN [21],
67 WNN [22], PSF [23], HIS [24], NNWT [25], SRN [26], CNEA [27], CNN [28], HNES [29], other hybrids [30,
68 33, 36] and mixed models [39].

69 The new proposed HEA methodology, which combines relevant techniques (MI, WT, EPSO and ANFIS) for
70 the first time ever in electricity market prices forecasting, allows a timely and improved prediction with low
71 computational effort, avoiding the use of exogenous data such as load, oil prices, among others, using instead
72 just the historical values of electricity market prices available from public domain, which is an important new
73 contribution to the field.

74 **1.4. Layout of the paper**

75 The remainder of this paper is partitioned as follows: Section 2 describes the proposed methodology in detail,
76 i.e., all techniques that compose the proposed methodology and structure, Section 3 provides the forecasting
77 accuracy validation, which supports the comparative results obtained, Section 4 presents the case studies with
78 real-world historical data in different time horizons from Spanish and PJM markets, and finally, in Section 5 the
79 conclusions are drawn.

80 **2. Proposed Methodology**

81 The HEA methodology results from the innovative combination of MI, WT, EPSO and ANFIS. MI eliminates
82 the randomness in the selection of electricity market prices sets as inputs, increasing the robustness of the
83 methodology and helping to decrease the final forecasting error. Furthermore, MI is a nonlinear feature selection
84 algorithm, more adequate for electricity market prices (non-stationary, time varying and irregular sets), instead
85 of using a simple linear correlation analysis for input selection that can compromise the selection of the best
86 candidates [27]. The WT is employed to decompose the electricity price sets into new constitutive sets with
87 better behavior without losing the real behavior of input data signal. Then, the forthcoming values of those
88 constitutive sets are predicted with the ANFIS. The EPSO brings on augmented ANFIS performance by tuning
89 their membership functions to attain a lesser error. The evolutionary concepts can make a real difference in terms
90 of convergence properties, i.e., the EPSO is self-adaptive, more robust and less sensitive to parameter
91 initialization, comparatively to PSO. The evolutionary characteristics of EPSO and the adaptive characteristics of
92 ANFIS complement each other perfectly. Finally, the inverse WT is used to reconstruct the signal, obtaining then
93 the final forecasting results.

94 **2.1. Mutual Information**

95 The MI is based on the concept of entropy. This concept shows that random processes may have a complexity of
 96 such order that the signal cannot be compressed or reduced. The entropy is derived from statistical physics, which
 97 was used as a measure of the disorder state of a system. The entropy $H(X)$ is mathematically described as [27]:

$$H(X) = - \int P(X) \log_2(P(X)) dX \quad (1)$$

98 where X is a random continuous variable with distribution probability $P(X)$.

99 In the case where variable X is a random discrete variable, for example, (X_1, \dots, X_n) , with distribution
 100 probabilities $P(X_n)$, the entropy $H(X)$ is given by:

$$H(X) = - \sum_{i=1}^n P(X_i) \log_2(P(X_i)) \quad (2)$$

101 The following examples should be considered:

- 102 • “A given event is equal to 0”, when this event does not occur;
- 103 • “A given event is equal to 1”, when this event occurs;
- 104 • Consider the events: $X_1 = 0 \wedge X_2 = 1$;

105 The individual entropy is equal to 0, that is, $H(X_n) = 0$, if:

$$(P(X_1) = 0 \wedge P(X_2) = 1) \vee (P(X_1) = 1 \wedge P(X_2) = 0) \quad (3)$$

106 and the individual entropy is equal to 1, that is, $H(X_n) = 1$, if:

$$P(X_1) = 0.5 \wedge P(X_2) = 0.5 \quad (4)$$

107 By extending the definition of entropy for the case of joint distributions of random variables, where the value
 108 of a random continuous variable X is known, if the entropy of a random continuous variable Y is assumed to be
 109 known, (1) takes a new approach [28]:

$$H(X, Y) = - \iint P(X_n, Y_m) \log_2(P(X_n, Y_m)) \quad (5)$$

110 In the case where variables X and Y are random discrete variables, the joint entropy $H(X, Y)$ is given by:

$$H(X, Y) = - \sum_{i=1}^n \sum_{j=1}^m P(X_i, Y_j) \log_2(P(X_i, Y_j)) \quad (6)$$

111 However, it is not possible to compute (6) directly, so a new concept is necessary, which measures the level
 112 of uncertainty of the random discrete variable Y after having observed the value of random discrete variable X .
 113 The conditional entropy is defined as:

$$H(Y/X) = - \sum_{i=1}^n \sum_{j=1}^m P(X_i, Y_j) \log_2 (P(Y_j/X_i)) \quad (7)$$

114 The conditional entropy $H(Y/X)$ quantifies the remaining uncertainty of Y when X is known. The joint and
115 conditional entropies are related by:

$$H(X, Y) = H(X) + H(Y/X) = H(Y) + H(X/Y) \quad (8)$$

116 The MI measures the level of information between a set of information data. The discrete expression is
117 defined as:

$$MI(X, Y) = \sum_{i=1}^n \sum_{j=1}^m P(X_i, Y_j) \log_2 \left(\frac{P(X_i, Y_j)}{P(X_i)P(Y_j)} \right) \quad (9)$$

118 The MI results can be described by the following points:

- 119 • If $MI(X, Y) \approx 1$, then the sets are correlated.
- 120 • If $MI(X, Y) \approx 0$, then sets are not related.
- 121 • If $MI(X, Y) = 0$, then the sets are completely independent.

122 MI has a strong connection with the individual entropy described in (2), with the conditional entropy
123 described in (7), as well as with (8), so the MI in (9) can be expressed as:

$$MI(X, Y) = H(X) - H(X / Y) \quad (10)$$

$$MI(X, Y) = MI(Y, X) \quad (11)$$

124 To ensure the convergence of the HEA methodology, the bounds of MI are very important to guarantee the
125 best performance of the ANFIS. The MI helps to determine the best sets of candidates that will be inputs for
126 training the ANFIS architecture [40].

127 2.2. Wavelet Transform

128 The WT is commonly used to understand the non-stationary or time varying sets [41], being sensitive to the
129 irregularities of input sets [42].

130 WT tools are capable of illustrating different aspects in the sets that are beyond the capacities of other tools
131 without losing the signal [43], reducing the noise of the sets (smoothing effect) without appreciable degradation.

132 The discrete wavelet transform (DWT) is defined:

$$W(m, n) = 2^{-(m/2)} \sum_{t=0}^{T-1} f(t) \varphi \left(\frac{t - n2^m}{2^m} \right) \quad (12)$$

133 In (12) the variable T represents the signal length $f(t)$, the parameters of scaling and translation are given by

134 $a = 2^m$ and $b = n2^m$, respectively, and the time step is given by t . A DWT algorithm is used based on four filters
 135 divided into two groups: the decomposition in low-pass and high-pass filters and the reconstruction in low-pass and
 136 high-pass filters. The approximations and details of the original sets can be obtained via Mallat algorithm.
 137 Furthermore, Fig. 1 shows a three-level decomposition model of WT. The approximations are able to retain the
 138 general information of the original sets, that is, the low-frequency representation and description of the high-
 139 frequency component. The details are able to explain the difference between successive approximations. The
 140 fourth order Daubechies is chosen as the mother wavelet function due to a better trade-off between smoothness and
 141 length, as explained in [12].

142 **2.3. Evolutionary Particle Swarm Optimization**

143 EPSO is a meta-heuristic method where rules and optimization concepts are contained in the evolutionary
 144 strategies and self-adaptive properties [44]. It is usual to call by generation, the data with alternative solutions,
 145 and by individuals the particles data. Each particle is described by object parameters (the value of the variables
 146 describing the solution) and strategic parameters (the mutation coefficients of each variable, angle of correlation
 147 of mutation variables, or similar) [45].

148 In EPSO it should be noted the following ideas [46]:

- 149 • Each particle (data) is replicated as many times as necessary.
- 150 • The weight parameter of the particles is transformed.
- 151 • The object parameters of each particle are transformed into a new generated particle by strategic parameters.
- 152 • The new mutated particles generate new particles.
- 153 • For a group constituted by old particles and new particles, the best fit should lead to the generation of a new
 154 population of particles.

155 Hence, the formulation of EPSO is composed of an object parameters X and strategic parameters w that
 156 correspond to the weights. The movement rule of EPSO is defined as [47]:

$$157 X_i^{new} = X_i + V_i^{new} \tag{13}$$

$$158 V_i^{new} = w_{i0}^* V_i + w_{i1}^* (b_i - X_i) + w_{i2}^* (b_g^* - X_i) \tag{14}$$

159 Equations (13) and (14) are similar to their classical algorithm, that is, the movement rule keeps the inertia,
 158 memory and cooperation terms, which can be seen in Fig. 2.

159 The significant difference in EPSO is related to the weights w_{ik}^* , which undergo mutation, given as:

$$159 w_{ik}^* = w_{ik} + \tau N(0,1) \tag{15}$$

160 where $N(0, I)$ is a randomly Gaussian variable with mean 0 and variance I . The global best b_g^* is changed
 161 according to:

$$b_g^* = b_g + \tau' N(0,1) \quad (16)$$

162 In Equations (13–16), the parameters $\{X_i, V_i, b_i, k, \tau, \tau'\}$ represent the position X_i , velocity V_i , best point b_i
 163 found at generation k , the learning parameters τ and the mutated parameters τ' . EPSO usually presents much
 164 better convergence characteristics than PSO due to the fact that only the stronger particles survive in the
 165 evolutionary process [47].

166 2.4. Adaptive Neuro-Fuzzy Inference System

167 The NN and fuzzy systems are complementary tools that can be combined to create an adaptive architecture
 168 with fuzzy inference. The NN has the capability of self-learning which is essential for the fuzzy system to auto-
 169 adjust accordingly with the proposed problem. Due to the self-learning process, the membership functions are
 170 adjusted in an adaptive representation [48]. The general ANFIS architecture is represented in Fig. 3, showing a
 171 five-layered feed forward network with the fuzzification, rules, normalization, defuzzification, and single
 172 summation layers, respectively.

173 In each layer, Ln_i is the output of the i_{th} node in layer n . In Layer 1 all nodes i are adaptive nodes with node
 174 function $L1_i$ given by:

$$L1_i = \mu A_i(x), \quad i = 1, 2 \quad (17)$$

175 or

$$L1_i = \mu B_{i-2}(y), \quad i = 3, 4 \quad (18)$$

176 where x or y is the input of the i_{th} node and A_i or B_{i-2} are the linguistic labels associated with these nodes.

177 The membership functions in A or B are typically described by a bell function where $\{p_i, q_i, r_i\}$ are the set
 178 parameters defined as:

$$\mu A_i(x) = \frac{1}{1 + \left| \frac{x - r_i}{p_i} \right|^{2q_i}} \quad (19)$$

179 A triangular membership function is considered in this paper as a node function due to being a continuous and
 180 piecewise differentiable function.

181 In Layer 2 all output nodes represent the firing strength of the rule w_i , where each node is represented by Π ,
 182 that is, the output signals are multiplied by the previous inputs signals:

$$L2_i = w_i = \mu A_i(x) \mu B_i(y), \quad i = 1, 2 \quad (20)$$

183 In Layer 3 every node N calculates the ratio of firing rules strength i_{th} with the sum of all firing strength rules:

$$L3_i = \bar{w}_i = \frac{w_i}{w_1 + w_2}, \quad i = 1, 2 \quad (21)$$

184 In Layer 4 all nodes compute the contribution of the rule i_{th} to the global output, where $\{a_i, b_i, c_i\}$ is the
185 parameter set and \bar{w}_i is the layer output:

$$L4_i = \bar{w}_i z_i = \bar{w}_i (a_i x + b_i y + c_i), \quad i = 1, 2 \quad (22)$$

186 Layer 5 corresponds to the output node of the ANFIS tool where the summation Σ is made:

$$L5_i = \sum_i \bar{w}_i z_i = \frac{\sum_i w_i z_i}{\sum_i w_i} \quad (23)$$

187 The ANFIS used in this paper employs the least-squares and back-propagation gradient descent method.
188 Furthermore, the EPSO assists the ANFIS in the tuning of their membership function parameters.

189 2.5. Hybrid Evolutionary-Adaptive Methodology

190 The HEA methodology will now be described in successive steps. Fig. 4 shows the structure of the HEA
191 methodology in the form of a detailed flowchart. The information provided in the successive steps (given below)
192 and Fig. 4 complement each other and provide all the necessary information.

- 193 • Step 1: Initialize the HEA methodology with an historical data matrix of electricity prices considering the
194 previous six weeks/days. Each of the above six columns of the matrix represents the electricity price profile
195 for one week or one day.
- 196 • Step 2: The matrix with the six previous weeks/days data will be normalized in $\{0,1\}$ intervals, to find the set
197 of historical electricity prices in the same scale, which will be later used by the MI in future candidate
198 selections. This step is important to avoid the loss of relevant information.
- 199 • Step 3: Constitute data groups for the MI. The number of these groups is defined by combinatorial
200 optimization in order to avoid compromising the computational burden. The formation of these groups must be
201 performed in a balanced way, thus avoiding compromising the ANFIS performance.
- 202 • Step 4: Compute the entropy and conditional entropy of each group by using (2) and (8) previously described,
203 where $P(X_n)$ is given by a binomial distribution function.
- 204 • Step 5: Compute the MI, given by (10) previously described, of each group.
- 205 • Step 6: Compute the best group subset data. The best group found will be recombined in electricity data-sets.
206 The selected sets are inputs for the WT.
- 207 • Step 7: Train the ANFIS with the previous constitutive sets. The optimization of the membership function

208 parameters is achieved by EPSO. Table 1 shows the parameters considered for MI, ANFIS and EPSO. These
 209 parameters result from the expertise acquired in the simulations, taking also into account previous
 210 publications. The approach developed in this paper uses A3 [17], along with D3 and D1, as inputs for the
 211 ANFIS. The inference rules of ANFIS are put into automatic mode to achieve the best performance. This is
 212 done due to the nature of the data, which requires a large number of inference rules to obtain the best results.

- 213 • Step 8: Until the best results or convergence are not reached, jump to Step 7. When the best results are found
 214 or convergence is reached, the inverse WT is applied and the output of the methodology is reached, that is, the
 215 electricity prices are forecasted for the next week/day.
- 216 • Step 9: Compute the price forecasting errors with different criteria to validate the methodology, comparing the
 217 results obtained with other results already available in the scientific literature.

218 3. Forecasting Accuracy Validation

219 To compare the proposed methodology with other methodologies used for electricity market price
 220 forecasting, previously published in the specialized literature, the mean absolute percentage error (MAPE)
 221 criterion is commonly used. This criterion is given as:

$$222 \quad MAPE = \frac{100}{N} \sum_{h=1}^N \frac{|\hat{p}_h - p_h|}{\bar{p}} \quad (24)$$

$$223 \quad \bar{p} = \frac{1}{N} \sum_{h=1}^N p_h \quad (25)$$

224 where \hat{p}_h is the electricity market price forecast at hour h , p_h is the actual electricity market price at hour h , \bar{p} is
 225 the average electricity market price for the forecasting horizon, and N is the number of hours. The uncertainty of
 the proposed methodology is also evaluated using the error variance estimation. The smaller the value for this
 criterion, the more exact the methodology is [17].

226 In accordance with the MAPE criterion expressed in (24), the error variance criterion is given by:

$$227 \quad \sigma_{e,t}^2 = \frac{1}{N} \sum_{h=1}^N \left(\frac{|\hat{p}_h - p_h|}{\bar{p}} - e_t \right)^2 \quad (26)$$

$$228 \quad e_t = \frac{1}{N} \sum_{h=1}^N \frac{|\hat{p}_h - p_h|}{\bar{p}} \quad (27)$$

Both MAPE and error variance criteria are employed in this paper, where N corresponds to 24 or 168 hours.
 Note that the average price is used in Eq.(24) to elude the instability caused when the prices are near to zero [16].

229 4. Case Studies

230 4.1. Spanish Market

231 The HEA methodology is utilized first to predict electricity market prices for the next 24/168 hours (next
232 day/week) for mainland Spain. The historical data of electricity prices are available in [49]. As mentioned in
233 [17], this market is difficult to predict due to the changes in prices that occur as a result of the strategies of the
234 dominant player.

235 The electricity price sets used for the Spanish market date back to the year 2002, to allow a clear and fair
236 comparison with the results already obtained using other published methodologies, that is, the same four test
237 weeks of the year 2002 were selected, each corresponding to a different season (winter, spring, summer, and
238 fall). Moreover, for a clear and fair comparison with the results already obtained using other published
239 methodologies, only historical data sets of electricity market prices were used, that is, no exogenous sets, such as
240 load, oil prices, or others are taken into account. Otherwise a correct comparative study would not be possible.
241 Moreover, demand data does not significantly improve predictions [50], The HEA methodology predicts the next
242 168 hours electricity market prices taking into account the previous 1008 hours (i.e. six weeks or 42 days for
243 each season), which in turn will be the input sets. Very large training sets are not used to avoid over-training
244 during the learning process. The output of the methodology corresponds directly to a set with 168 values, equal
245 to the forecasting horizon. For day-ahead (24 hours) forecasts, the previous six days are considered. The results
246 with the HEA methodology are initially provided in Figs. 5 to 8 for the four test weeks of 2002. Table 2 shows
247 the MAPE criterion comparative results between HEA methodology and eighteen other methodologies.

248 The enhancements between HEA and the other methodologies are 58.0%, 55.1%, 53.1%, 48.5%, 48.1%,
249 44.4%, 43.7%, 40.0%, 38.1%, 37.1%, 36.3%, 27.2%, 21.4%, 19.9%, 18.7%, 18.5%, 17.6% and
250 15.6%, respectively. The MAPE criterion using HEA has an average value of only 4.18%, the lowest one of all,
251 which is significant. Even if each week is analysed per se, the results are always better. Hence, although the
252 proposed methodology is not specifically designed for price spike forecasting, which is the main goal of other
253 papers [51-52], it behaves quite well in their presence with excellent overall results.

254 Table 3 shows the error variance criterion comparative results between the HEA methodology and fourteen
255 other methodologies. The enhancements between HEA and the other methodologies are 83.7%, 78.6%, 76.6%,
256 72.2%, 68.8%, 59.5%, 58.3%, 58.3%, 57.1%, 54.5%, 44.4%, 28.6%, 28.6% and 28.6% respectively. Results for
257 the mixed-model, FNN, PSF and SRN are not available in their papers. The average value is only 0.0015, again
258 the lowest one of all, indicating reduced uncertainty in the forecasts, which is another important feature.

259 More recent data (year 2006) for the Spanish market has also been considered. Moreover, the best and worst
260 forecasts generated by PSF and HEA methodologies for year 2006 data have been compared. The best forecast
261 for PSF occurred on June 23rd, 2006, in which the MAPE was 3.10%, while using the HEA methodology the
262 MAPE decreases to 2.31%. The worst forecast for PSF occurred on May 8th, 2006, in which the MAPE was
263 9.39%, while using the HEA methodology (as illustrated in Fig. 10) the MAPE decreases to 4.37%. Hence, the
264 forecasting trends for the year 2006 are in agreement with those previously observed for the year 2002:
265 enhancements range from 25.5% to 53.5%, which is significant.

266 Furthermore, the HEA methodology requires a low computational burden: the average computation time for
267 a 168-hours forecast is less than 40 sec. using MATLAB platform on a standard PC with a 1.8 GHz-based-
268 processor and 1.5 GB RAM. Not only is the training time less, but also the accuracy is higher and the uncertainty
269 is lower with the HEA methodology. This is the major added value the paper provides. More recently, the
270 WT+CLSSVM+EGARCH methodology in [34] presented a lower MAPE but required a computation time of
271 about 10 min. Hence, the proposed HEA methodology presents, indeed, the best trade-off between computation
272 time and average MAPE, which is crucial for real-life and real-time applications.

273 Fig. 9 shows the daily error between the HEA methodology results and the results previously reported for the
274 NN, NNWT, and WPA methodologies, for the four seasons of the year. It can be seen that, for most days, the
275 HEA methodology presents better forecasting results, that is, lower errors, comparatively to the other three
276 methodologies.

277 4.2. PJM Market

278 The HEA methodology is also utilized to predict electricity market prices for the next 24/168 hours (next
279 day/week) for the PJM market. The historical data of electricity prices are available in [53]. Like in Spanish
280 Market no exogenous data such as load, oil prices, and others sets are taken into account. The results with the
281 HEA methodology for the PJM market are provided in Figs. 11 to 17 for five days and two weeks of the year
282 2006. The same test days/weeks of previous published papers have been considered to allow a clear and fair
283 comparison with the results already obtained using other published methodologies. Otherwise a correct
284 comparative study would not be possible. Table 4 and Table 5 show the MAPE and error variance results,
285 respectively, for the HEA methodology and five other methodologies.

286 The MAPE enhancements between HEA and the other methodologies are 59.1%, 40.2%, 28.2%, 25.9% and
287 25.7%, respectively. The error variance enhancements between HEA and the other methodologies are 75.5%,
288 64.7%, 45.5%, 42.9% and 25.0%, respectively. The HEA methodology clearly outperforms, again, all other

289 methodologies in every day/week analysed. Moreover, the electricity price forecast results for 168 hours are
290 provided in about 40 seconds, while 24 hours forecasts require even less computation time. Hence, this second
291 case study further and unequivocally demonstrates and validates the proficiency of the proposed methodology.

292 **5. Conclusion**

293 A new hybrid evolutionary–adaptive methodology, called HEA, was proposed in this paper for short-term
294 electricity market price forecasting. The HEA methodology results from the valuable and innovative joint
295 characteristics of WT (bringing a filtering effect), EPSO (bringing evolutionary optimization) and ANFIS
296 (bringing an adaptive architecture), considering also MI in the selection of the best input data. For a fair and
297 clear comparison, identical test days/weeks used by other methods were considered, but without exogenous
298 variables. The application of the proposed HEA methodology was revealed to be accurate and effective, helping
299 to reduce the uncertainty associated with market prices. The results for the Spanish and PJM markets
300 demonstrated the superiority of the HEA methodology, regarding both average MAPE and error variance
301 criteria. Even if each day/week is analysed per se the results are always better. The low computational burden
302 is also demonstrated, providing 168 hours electricity price forecast results in less than 40 seconds. Hence, it can
303 be concluded that the proposed methodology is proficient taking into account previously reported results in the
304 specialized literature, with the best trade-off between computation time and average MAPE.

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311 **References**

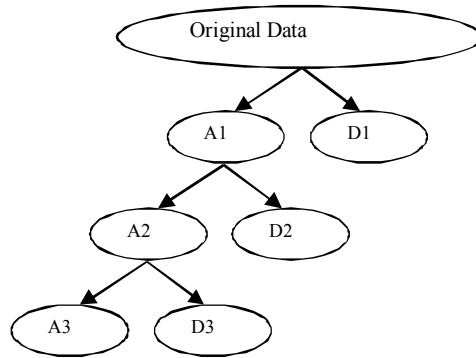
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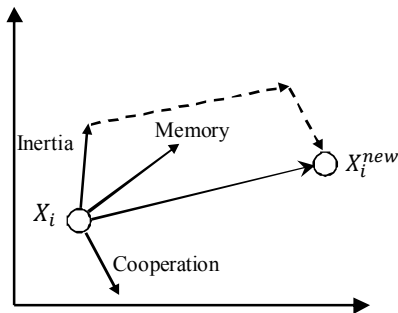
420 **Figure captions**



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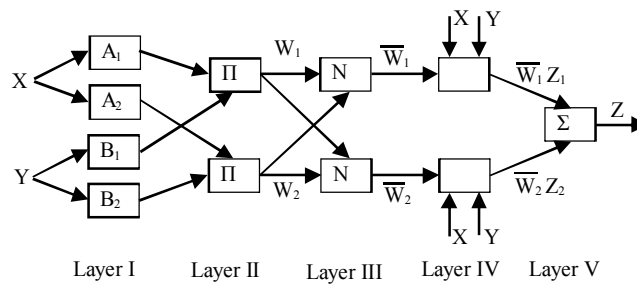
Fig. 1. Three-level decomposition model of WT.



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Fig. 2. Movement rule of a particle.

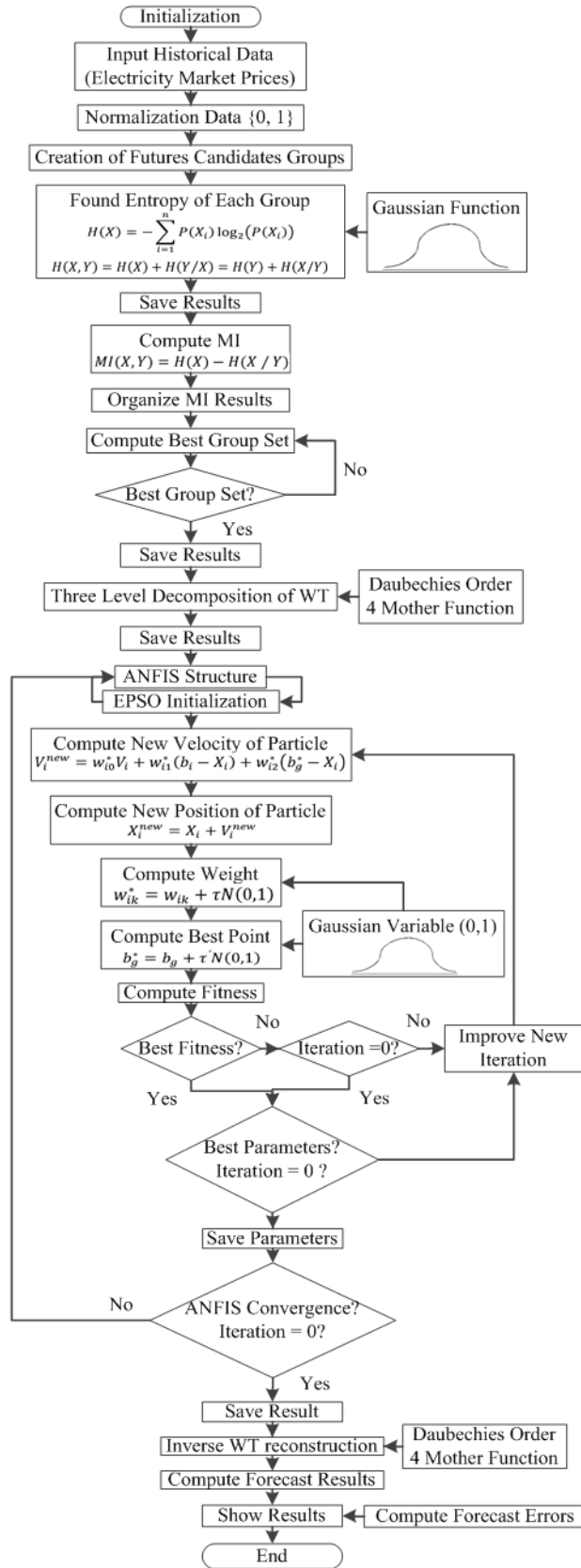


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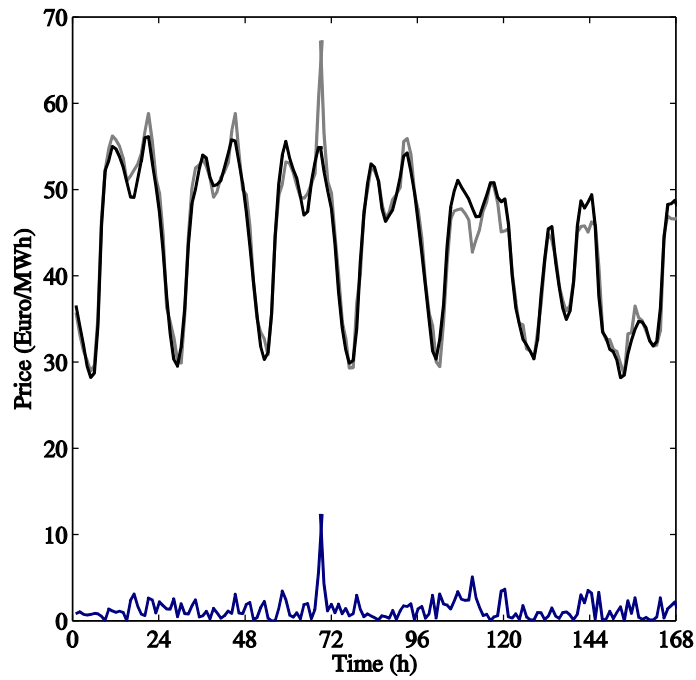
Fig. 3. ANFIS architecture.



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Fig. 4. Detailed flowchart of the proposed methodology.

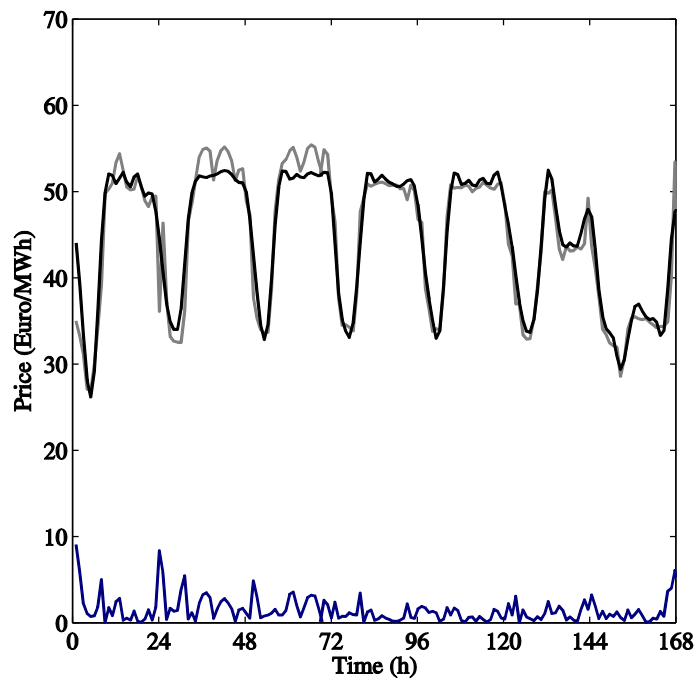


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Fig. 5. Winter week 2002 results for the Spanish market: The gray and black lines represent the actual and forecasted prices, respectively, while the dark-blue line at the bottom represents the errors in absolute value.

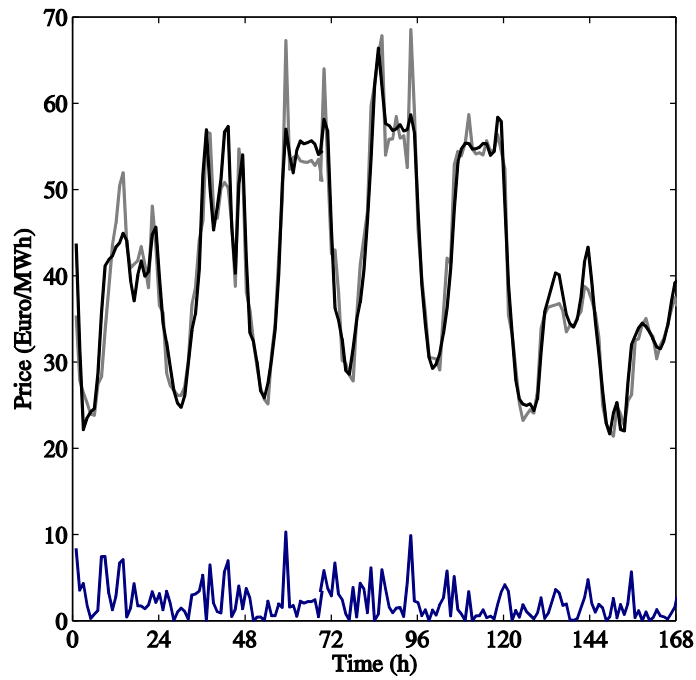


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Fig. 6. Spring week 2002 results for the Spanish market: The gray and black lines represent the actual and forecasted prices, respectively, while the dark-blue line at the bottom represents the errors in absolute value.

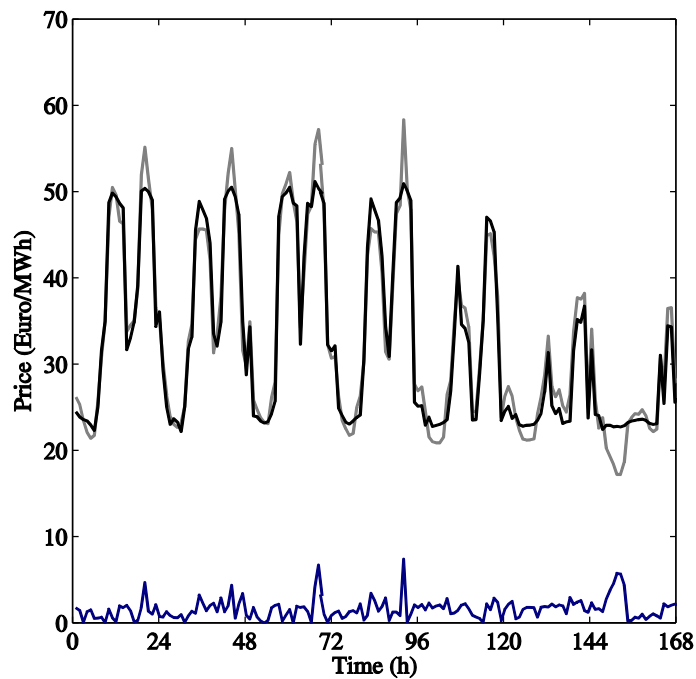


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Fig. 7. Summer week 2002 results for the Spanish market: The gray and black lines represent the actual and forecasted prices, respectively, while the dark-blue line at the bottom represents the errors in absolute value.

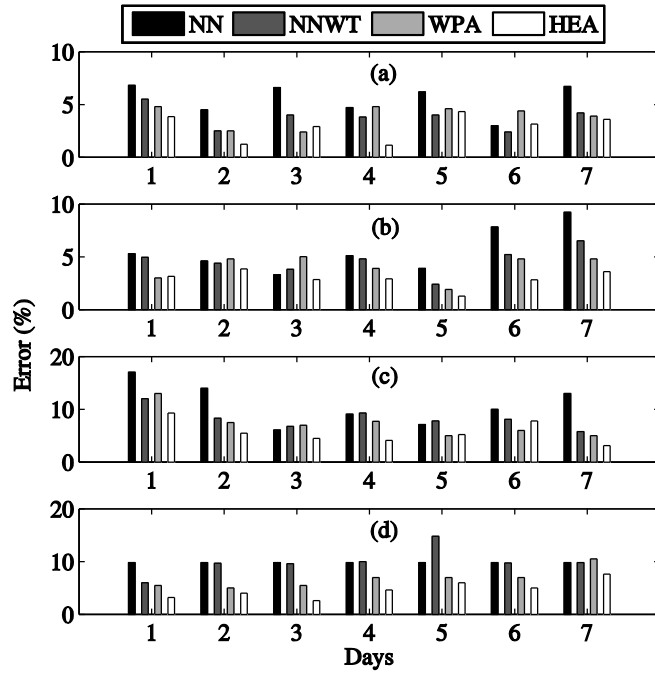


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Fig. 8. Fall week 2002 results for the Spanish market: The gray and black lines represent the actual and forecasted prices, respectively, while the dark-blue line at the bottom represents the errors in absolute value.



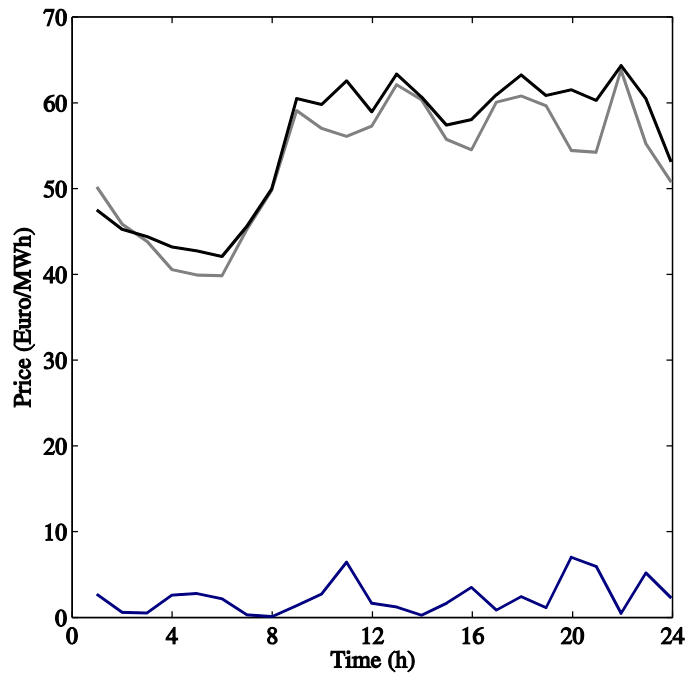
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Fig. 9. Daily error comparative results between NN, NNWT, WPA and HEA methodologies, regarding the

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four seasons of year 2002 for the Spanish market: (a) winter; (b) spring; (c) summer; (d) fall.

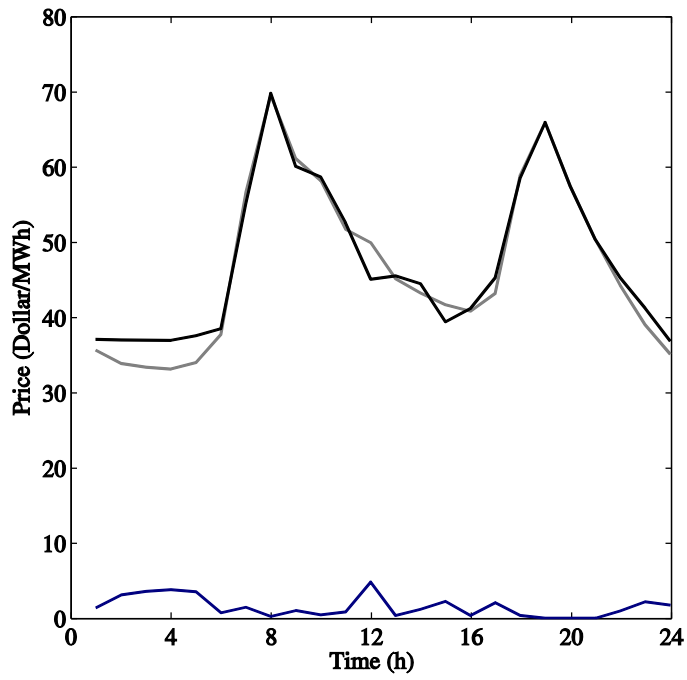


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Fig. 10. May 8, 2006, results for the Spanish market: The gray and black lines represent the actual and forecasted prices, respectively, while the dark-blue line at the bottom represents the errors in absolute value.

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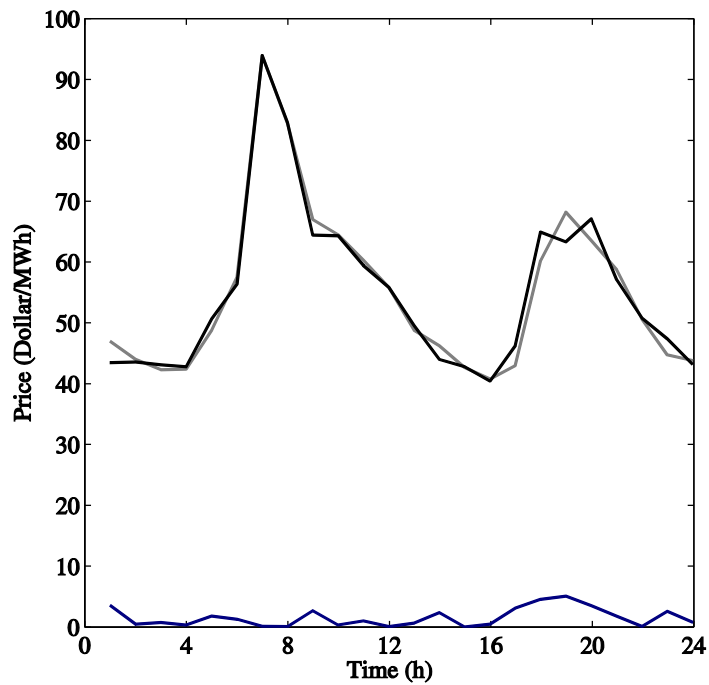


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Fig. 11. January 20, 2006, results for the PJM market: The gray and black lines represent the actual and forecasted prices, respectively, while the dark-blue line at the bottom represents the errors in absolute value.

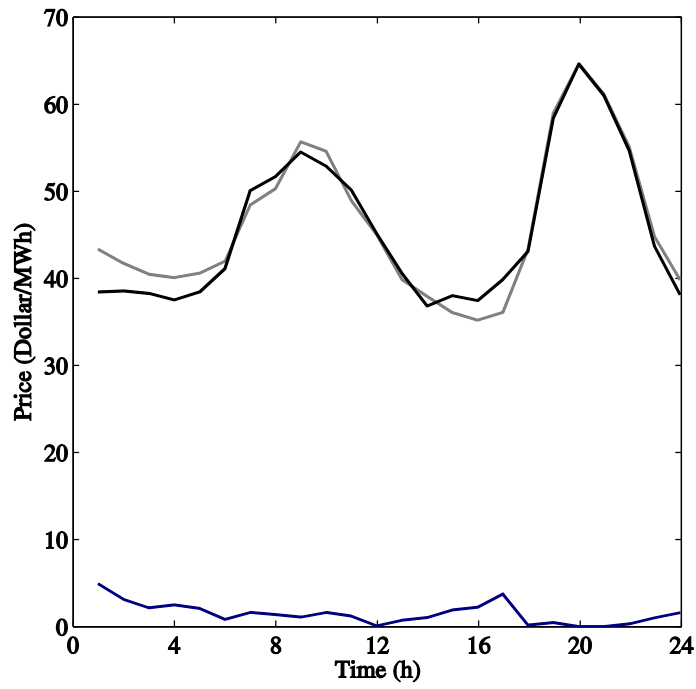


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Fig. 12. February 10, 2006, results for the PJM market: The gray and black lines represent the actual and forecasted prices, respectively, while the dark-blue line at the bottom represents the errors in absolute value.

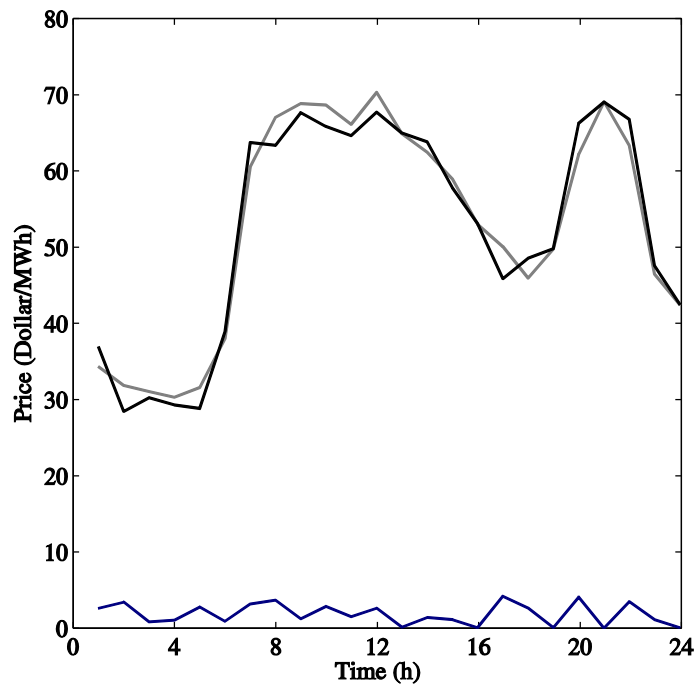


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Fig. 13. March5, 2006, results for the PJM market: The gray and black lines represent the actual and forecasted prices, respectively, while the dark-blue line at the bottom represents the errors in absolute value.

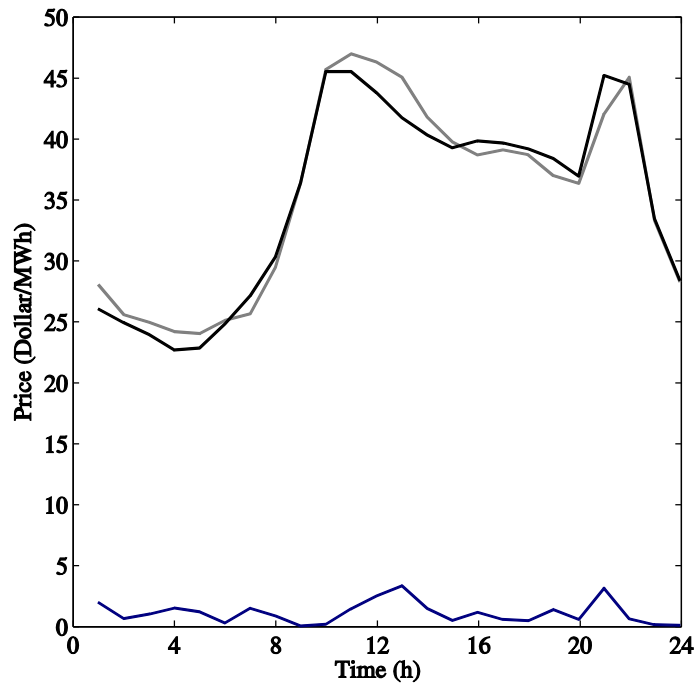


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Fig. 14. April7, 2006, results for the PJM market: The gray and black lines represent the actual and forecasted prices, respectively, while the dark-blue line at the bottom represents the errors in absolute value.

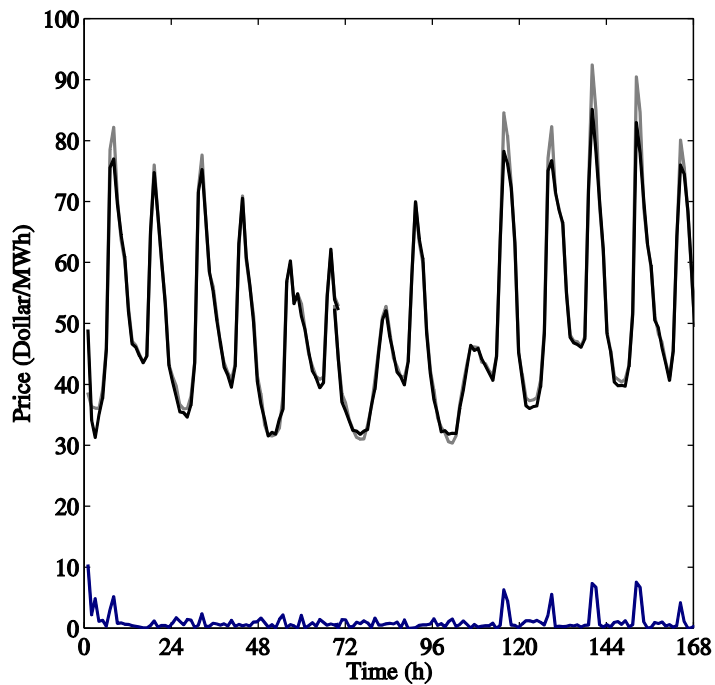


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Fig. 15. May13, 2006, results for the PJM market: The gray and black lines represent the actual and forecasted prices, respectively, while the dark-blue line at the bottom represents the errors in absolute value.

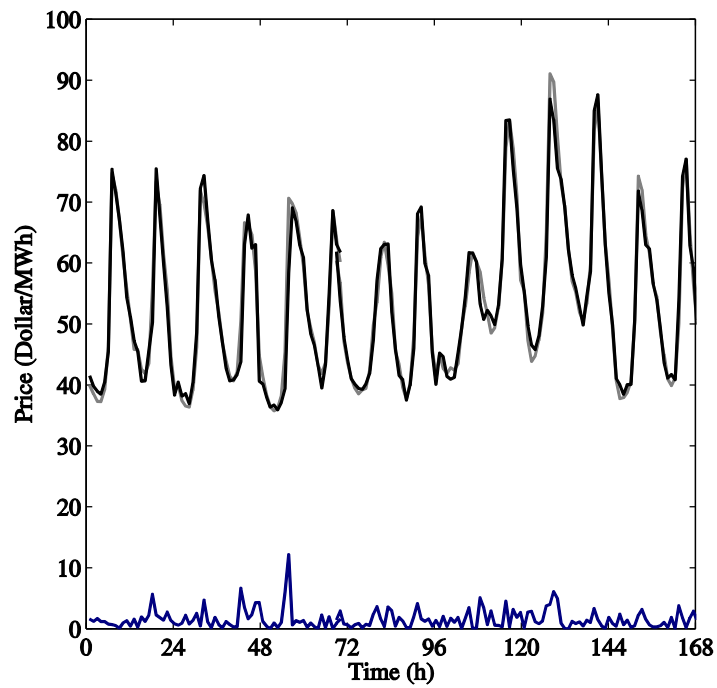


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Fig. 16. February 1-7, 2006, results for the PJM market: The gray and black lines represent the actual and forecasted prices, respectively, while the dark-blue line at the bottom represents the errors in absolute value.



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Fig. 17. February 22-28, 2006, results for the PJM market: The gray and black lines represent the actual and forecasted prices, respectively, while the dark-blue line at the bottom represents the errors in absolute value.

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469 **Tables**

470 **Table 1**

471 Parameters used in MI, ANFIS and EPSO techniques

	Parameters	Type or Size
MI	Best Lower Bound of Set	0.15
	Best Upper Bound of Set	0.65
ANFIS	Membership Functions	2-7
	Necessary Iterations	3-50
	Type of Membership Function	Triangular-format
EPSO	Fitness Acceleration	2
	Sharing Acceleration	2
	Initial Inertia Weight of Population	0.9
	Final Inertia Weight of Population	0.4
	Population Size	24-168
	Maximum Generation	48-326
	Number of New Particles	24-168
	Generation for Each New Particle	2
	Necessary Iterations	48-326
	Minimum Value of New Position	20
Maximum Value of New Position	70-120	

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474 **Table 2**

475 MAPE Criterion: Comparative results for Spanish market

	Winter	Spring	Summer	Fall	Average
ARIMA [16], 2003	6.32	6.36	13.39	13.78	9.96
Mixed-model [39], 2007	6.15	4.46	14.90	11.68	9.30
NN [19], 2007	5.23	5.36	11.40	13.65	8.91
Wavelet-ARIMA [17], 2005	4.78	5.69	10.70	11.27	8.11
WNN [22], 2007	5.15	4.34	10.89	11.83	8.05
FNN [21], 2006	4.62	5.30	9.84	10.32	7.52
PSF [23], 2011	5.98	4.51	9.11	10.07	7.42
HIS [24], 2009	6.06	7.07	7.47	7.30	6.97
AWNN [6], 2008	3.43	4.67	9.64	9.29	6.75
NNWT [25], 2010	3.61	4.22	9.50	9.28	6.65
SRN [26], 2013	4.11	4.37	9.09	8.66	6.56
RBFN [15], 2011	4.27	4.58	6.76	7.35	5.74
CNEA [27], 2009	4.88	4.65	5.79	5.96	5.32
CNN [28], 2009	4.21	4.76	6.01	5.88	5.22
HNES [29], 2010	4.28	4.39	6.53	5.37	5.14
MI+CNN [33], 2012	4.51	4.28	6.47	5.27	5.13
WPA [12], 2011	3.37	3.91	6.50	6.51	5.07
MI-MI+CNN [33], 2012	4.29	4.20	6.31	5.01	4.95
HEA, 2013	3.04	3.33	5.38	4.97	4.18

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478 **Table 3**

479 Weakly Error Variance Criterion: Comparative results for Spanish market

	Winter	Spring	Summer	Fall	Average
ARIMA [16], 2003	0.0034	0.0020	0.0158	0.0157	0.0092
NN [19], 2007	0.0017	0.0018	0.0109	0.0136	0.0070
Wavelet-ARIMA [17], 2005	0.0019	0.0025	0.0108	0.0103	0.0064
FNN [21], 2006	0.0018	0.0019	0.0092	0.0088	0.0054
AWNN [6], 2008	0.0012	0.0031	0.0074	0.0075	0.0048
NNWT [25], 2010	0.0009	0.0017	0.0074	0.0049	0.0037
HIS [24], 2009	0.0034	0.0049	0.0029	0.0031	0.0036
CNEA [27], 2009	0.0036	0.0027	0.0043	0.0039	0.0036
CNN [28], 2009	0.0014	0.0033	0.0045	0.0048	0.0035
RBFN [15], 2011	0.0015	0.0019	0.0047	0.0049	0.0033
WPA [12], 2011	0.0008	0.0013	0.0056	0.0033	0.0027
MI+CNN [33], 2012	0.0014	0.0014	0.0033	0.0022	0.0021
HNES [29], 2010	0.0013	0.0015	0.0033	0.0022	0.0021
MI-MI+CNN [33], 2012	0.0014	0.0014	0.0032	0.0023	0.0021
HEA, 2013	0.0008	0.0011	0.0026	0.0014	0.0015

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481 **Table 4**

482 MAPE Criterion: Comparative results for PJM market

	SDNN [20], 2007	WT+FF+FA [36], 2013	HNES [29], 2010	Hybrid [30], 2010	CNEA [27], 2009	HEA, 2013
Jan. 20	6.93	5.04	4.98	3.71	4.73	3.29
Feb. 10	7.96	5.43	4.10	2.85	4.50	2.80
Mar. 5	7.88	4.82	4.45	5.48	4.92	3.32
Apr. 7	9.02	6.24	4.67	4.17	4.22	3.55
May 13	6.91	4.11	4.05	4.06	3.96	3.43
Feb. 1-7	7.66	6.07	4.62	5.27	4.02	3.11
Feb. 22-28	8.88	6.12	4.66	5.01	4.13	3.08
Average	7.89	5.40	4.50	4.36	4.35	3.23

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485 **Table 5**

486 Error Variance Criterion: Comparative results for PJM market

	SDNN [20], 2007	CNEA [27], 2009	WT+FF+FA [36], 2013	Hybrid [30], 2010	HNES [29], 2010	HEA, 2013
Jan. 20	0.0034	0.0031	0.0016	0.0010	0.0020	0.0010
Feb. 10	0.0050	0.0036	0.0021	0.0015	0.0012	0.0009
Mar. 5	0.0061	0.0042	0.0032	0.0033	0.0015	0.0011
Apr. 7	0.0038	0.0022	0.0019	0.0013	0.0018	0.0011
May 13	0.0049	0.0027	0.0016	0.0015	0.0013	0.0012
Feb. 1-7	0.0066	0.0044	0.0023	0.0037	0.0016	0.0012
Feb. 22-28	0.0047	0.0035	0.0024	0.0025	0.0017	0.0017
Average	0.0049	0.0034	0.0022	0.0021	0.0016	0.0012

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