Scheduling of head-dependent cascaded hydro systems: mixed-integer quadratic programming approach

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Abstract

This paper is on the problem of short-term hydro scheduling, particularly concerning head-dependent cascaded hydro systems. We propose a novel mixed-integer quadratic programming approach, considering not only head-dependency, but also discontinuous operating regions and discharge ramping constraints. Thus, an enhanced short-term hydro scheduling is provided due to the more realistic modeling presented in this paper. Numerical results from two case studies, based on Portuguese cascaded hydro systems, illustrate the proficiency of the proposed approach.

Keywords: Hydro scheduling; mixed-integer quadratic programming; head-dependency

1. Introduction

In this paper, the short-term hydro scheduling (STHS) problem of head-dependent cascaded hydro systems is considered. In hydro plants with a large storage capacity available, head variation has negligible influence on operating efficiency in the short-term [1]. In hydro plants with a small storage capacity available, also known as run-of-the-river hydro plants, operating efficiency is sensitive to the head: head change effect [2]. For instance, in the Portuguese system there are several cascaded hydro systems formed by several but small reservoirs. Hence, it is necessary to consider head-dependency on STHS. In a cascaded hydraulic configuration, where hydro plants can be connected in both series and parallel, the release of an upstream plant contributes to the inflow of the next downstream plants, implying spatial-temporal coupling among reservoirs. Head-dependency coupled with the cascaded hydraulic configuration augments the problem complexity and dimension.

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Hydro plants particularly run-of-the-river hydro plants are considered to provide an environmentally friendly energy option, while fossil-fuelled plants are considered to provide an environmentally aggressive energy option, but nevertheless still in nowadays a necessary option [3]. However, the rising demand for electricity, likely increases in fossil-fuel prices, and the need for clean emission-free generation sources, are trends in favor of increasing generation from renewable sources.

The Portuguese fossil fuels energy dependence is among the highest in the European Union. Portugal does not have endogenous thermal resources, which has a negative influence on Portuguese economy. Moreover, the Portuguese greenhouse emissions are already out of Kyoto target and must be reduced in the near future. Hence, promoting efficiency improvements in the exploitation of the Portuguese hydro resources reduces the reliance on fossil fuels and decreases greenhouse emissions.

In a deregulated profit-based environment, such as the Norwegian case [4] or concerning Portugal and Spain given the Iberian Electricity Market, a hydroelectric utility is usually an entity owning generation resources and participating in the electricity market with the ultimate goal of maximizing profits, without concern of the system, unless there is an incentive for it [5].

The optimal management of the water available in the reservoirs for power generation, without affecting future operation use, represents a major advantage for the hydroelectric utilities to face competition [6]. STHS models provide decision support for the operational task of bidding in the energy and system services markets [7].

In the STHS problem a time horizon of one to seven days is considered, usually divided in hourly intervals. Hence, the STHS problem is treated as a deterministic one. Where the problem includes stochastic quantities, such as inflows to reservoirs or electricity prices, the corresponding forecasts are used [8].

Dynamic programming (DP) is among the earliest methods applied to the STHS problem [9,10]. Although DP can handle the nonconvex, nonlinear characteristics present in the hydro model, direct application of DP methods for hydro systems with cascaded reservoirs is impractical due to the well-known DP curse of dimensionality, more difficult to avoid in short-term than in long-term optimization without losing the accuracy needed in the model [11].

Artificial intelligence techniques have also been applied to the STHS problem [12–15]. However, a significant computational effort is necessary to solve the problem for cascaded hydro systems. Also, due to the heuristics used in the search process only sub-optimal solutions can be reached.

A natural approach to STHS is to model the system as a network flow model, because of the underlying network structure subjacent in cascaded reservoirs [16]. This network flow model is often simplified to a linear or piecewise linear one [17]. Linear programming (LP) is a well-known optimization method and standard software can be found commercially. Mixed-integer linear programming (MILP) is becoming often used for STHS [18–21], where integer variables allow modeling of discrete hydro unit-commitment constraints.

However, LP typically considers that hydroelectric power generation is linearly dependent on water discharge, thus ignoring head-dependency to avoid nonlinearities. The discretization of the nonlinear dependence between power generation, water discharge and head, used in MILP to model head variations, augment the computational burden required to solve the STHS problem. Furthermore, methods based on successive linearization in an iterative scheme depend on the expertise of the operator to properly calibrate the parameters. For instance, the selection of the best under-relaxation factor in [21] is empiric and case-dependent, rendering some ambiguity to these methods.

A nonlinear model has advantages compared with a linear one. A nonlinear model expresses hydroelectric power generation characteristics more accurately and head-dependency on STHS can be taken into account [2,6,22]. Although there were considerable computational difficulties in the past to directly use nonlinear programming (NLP) methods to this sort of problem, with the drastic advancement in computing power and the development of more effective nonlinear solvers in recent years this disadvantage seems to be eliminated.

In earlier works [2,6,22], the use of the nonlinear model in some case studies leads to a result that exceeds by at least 3 percent what is obtained by a linear model, requiring a negligible extra computation time. However, the nonlinear model cannot avoid water discharges at forbidden areas, and may give schedules unacceptable from an operation point of view. Moreover, it is important to notice that a minor change in the electricity price may give a significant change in the water discharge, and consequently in the power generation of plants. Therefore, ramp rate of water discharge is included in the constraints to keep a lesser and steady head variation, which is particularly important for reservoirs with a task of navigation.

Hence, in this paper we propose a novel mixed-integer quadratic programming (MIQP) approach to solve the STHS problem, where integer variables are used to model the on-off behavior of the hydro plants. The proposed approach considers head-dependency, discontinuous operating regions, and discharge ramping constraints, in order to obtain more realistic and feasible results.

This paper is organized as follows. In Section 2, the mathematical formulation of the STHS problem is provided. Section 3 presents the proposed MIQP approach to solve the STHS problem. In Section 4, the proposed MIQP approach is applied on two case studies, based on Portuguese cascaded hydro systems, to demonstrate its effectiveness. Finally, concluding remarks are given in Section 5.

2. Problem formulation

The notation used throughout the paper is stated as follows.

I, i	Set and index of reservoirs.
K, k	Set and index of hours in the time horizon.
π_k	Forecasted electricity price in hour *k*.
p_{ik}	Power generation of plant *i* in hour *k*.
Ψ_i	Future value of the water stored in reservoir *i*.
v_{ik}	Water storage of reservoir *i* at end of hour *k*.
a_{ik}	Inflow to reservoir *i* in hour *k*.
M_i	Set of upstream reservoirs of plant *i*.
q_{ik}	Water discharge of plant *i* in hour *k*.
s_{ik}	Water spillage by reservoir *i* in hour *k*.
η_{ik}	Power efficiency of plant *i* in hour *k*.
h_{ik}	Head of plant *i* in hour *k*.
l_{ik}	Water level in reservoir *i* in hour *k*.
$v_i^{\min}, v_i^{\max}$	Water storage limits of reservoir *i*.

$q_i^{\min}, q_i^{\max}$ Water discharge limits of plant *i*.

u_{ik} Commitment decision of plant *i* in hour *k*.

R_i Discharge ramping limit of plant *i*.

$\boldsymbol{H}$ Hessian matrix.

$\boldsymbol{f}$ Vector of coefficients for the linear term.

$\boldsymbol{x}$ Vector of decision variables.

$\boldsymbol{A}$ Constraint matrix.

$\boldsymbol{b}^{\min}, \boldsymbol{b}^{\max}$ Lower and upper bound vectors on constraints.

$\boldsymbol{x}^{\min}, \boldsymbol{x}^{\max}$ Lower and upper bound vectors on variables.

$\eta_i^{\min}, \eta_i^{\max}$ Power efficiency limits of plant *i*.

$h_i^{\min}, h_i^{\max}$ Head limits of plant *i*.

$l_i^{\min}, l_i^{\max}$ Water level limits of reservoir *i*.

The STHS problem can be stated as to find out the periodic water discharges, q_{ik}, the water storages, v_{ik}, and the water spillages, s_{ik}, for each reservoir, $i = 1, \ldots, I,$ at all hours of the time horizon, $k = 1, \ldots, K,$ that optimize an objective function subject to constraints. The water storages at the end of the time horizon, v_{iK}, must be decided according with future operations. Additionally, the commitment decision, u_{ik}, is ascertained.

In the STHS problem under consideration, the objective function is a measure of the profit attained by the conversion of potential energy into electric energy, without affecting future operations. Thus, the objective function to be maximized can be expressed as:

$$F = \sum_{i=1}^{I} \sum_{k=1}^{K} \pi_k \, p_{ik} + \sum_{i=1}^{I} \Psi_i \left(v_{iK} \right) \qquad (1)$$

The objective function in (1) is composed of two terms. The first term represents the profit with the hydro system during the short-term time horizon, where π_k is the forecasted electricity price in hour *k* and p_{ik} is the power generation of plant *i* in hour *k*.

The second term expresses the value of the water stored in the reservoirs for future operations. This second term is only needed if no final water storage requirement is specified. An appropriate representation when this term is explicitly taken into account can be seen for instance in [23]. The storage targets for the short-term time horizon can be established by medium-term planning studies.

The optimal value of the objective function is determined subject to constraints of two kinds: equality constraints and inequality constraints or simple bounds on the variables. The constraints are indicated as follows:

$$v_{ik} = v_{i,k-1} + a_{ik} + \sum_{m \in M_i} (q_{mk} + s_{mk}) - q_{ik} - s_{ik} \tag{2}$$

$$p_{ik} = q_{ik}\, \eta_{ik}(h_{ik}) \tag{3}$$

$$h_{ik} = l_{f(i)k}(v_{f(i)k}) - l_{t(i)k}(v_{t(i)k}) \tag{4}$$

$$v_i^{\min} \le v_{ik} \le v_i^{\max} \tag{5}$$

$$u_{ik}\, q_i^{\min} \le q_{ik} \le u_{ik}\, q_i^{\max} \tag{6}$$

$$q_{ik} - R_i \le q_{i,k+1} \le q_{ik} + R_i \tag{7}$$

$$s_{ik} \ge 0 \tag{8}$$

Equation (2) corresponds to the water balance equation for each reservoir, assuming that the time required for water to travel from a reservoir to a reservoir directly downstream is less than the one hour period, independently of water discharge, due to the small distance between consecutive reservoirs. In (2) v_{ik} is the water storage of reservoir *i* at end of hour *k*, a_{ik} is the inflow to reservoir *i* in hour *k*, q_{ik} is the water discharge of plant *i* in hour *k*, s_{ik} is the water spillage by reservoir *i* in hour *k*, and M_i is the set of upstream reservoirs of plant *i*. Time-delay is a difficult issue, depending on the distance between the reservoirs and on the water discharge, deserving particular attention and research. Time-delay can be accounted for by considering a different model structure for different flow levels in an iterative procedure, which is outside the scope of this paper. In (3) hydroelectric power generation, p_{ik}, is considered a function of water discharge and efficiency, η_{ik}. We consider efficiency given by the output-input ratio, depending on the head, h_{ik}.

In (4) the head is considered a function of the water levels in the upstream reservoir, denoted by $f(i)$ in subscript, and downstream reservoir, denoted by $t(i)$ in subscript, depending on the water storages in the respectively reservoirs. Typically for a powerhouse with a reaction turbine, where the tail water elevation is not constant, the head is modeled as in (4), and for a powerhouse with an impulse turbine, where the tail water elevation remains constant, the head depends only on the upstream reservoir water level as in [21]. Hence, tailrace effects can be considered by including a correction in the data regarding reservoir water levels. In (5) water storage has lower and upper bounds. Here for each reservoir i, $v_i^{\min}$ is the minimum storage, and $v_i^{\max}$ is the maximum storage. In (6) water discharge has lower and upper bounds. Here for each reservoir i, $q_i^{\min}$ is the minimum discharge, and $q_i^{\max}$ is the maximum discharge. The maximum discharge may be considered a function of the head, as in [6,22]. As a new contribution to earlier studies, we consider the commitment decision of each hydro plant. Hence, the binary variable, u_{ik}, is equal to 1 if plant i is on-line in hour k, otherwise is equal to 0. Also, we consider discharge ramping constraints, in (7), which may be imposed due to requirements of navigation, environment, and recreation [24]. In (8) a null lower bound is considered for water spillage. Normally, water spillage by the reservoirs occurs when without it the water storage exceeds its upper bound, so spilling is necessary to avoid damage. The initial water storages, v_{i0}, and the inflows to reservoirs are known input data.

3 Solution methodology

The STHS problem can be formulated as a mixed-integer quadratic problem, given by:

$$Max\ \boldsymbol{F}(\boldsymbol{x}) = \boldsymbol{f}^{\mathrm{T}}\, \boldsymbol{x} + 1/2\, \boldsymbol{x}^{\mathrm{T}}\, \boldsymbol{H}\, \boldsymbol{x} \tag{9}$$

$$\text{subject to} \qquad \boldsymbol{b}^{\min} \le \boldsymbol{A}\,\boldsymbol{x} \le \boldsymbol{b}^{\max} \tag{10}$$

$$-\infty \le \boldsymbol{x}^{\min} \le \boldsymbol{x} \le \boldsymbol{x}^{\max} \le \infty \tag{11}$$

$$\boldsymbol{x}_j \text{ integer}, \quad j \in J \tag{12}$$

In (9) the function $\boldsymbol{F}(\cdot)$ is a quadratic objective function of decision variables, where $\boldsymbol{f}$ is the vector of coefficients for the linear term and $\boldsymbol{H}$ is the Hessian matrix. In (10) $\boldsymbol{A}$ is the constraint matrix, $\boldsymbol{b}^{\min}$ and $\boldsymbol{b}^{\max}$ are the lower and upper bound vectors on constraints. Equality constraints are defined by setting the lower bound equal to the upper bound, i.e. $\boldsymbol{b}^{\min} = \boldsymbol{b}^{\max}$. In (11) $\boldsymbol{x}^{\min}$ and $\boldsymbol{x}^{\max}$ are the lower and upper bound vectors on variables. The variables $\boldsymbol{x} \in J$ are restricted to be integers. The lower and upper bounds for water discharge imply new inequality constraints that will be rewritten into (10).

In (3) the efficiency depends on the head. We consider it given by:

$$\eta_{ik} = \alpha_i \, h_{ik} + \eta_i^0 \tag{13}$$

where the parameters α_i and η_i^0 are given by:

$$\alpha_i = (\eta_i^{\max} - \eta_i^{\min}) \, / \, (h_i^{\max} - h_i^{\min}) \tag{14}$$

$$\eta_i^0 = \eta_i^{\max} - \alpha_i \, h_i^{\max} \tag{15}$$

In (4) the water level depends on the water storage. We consider it given by:

$$l_{ik} = \beta_i \, v_{ik} + l_i^0 \tag{16}$$

where the parameters β_i and l_i^0 are given by:

$$\beta_i = (l_i^{\max} - l_i^{\min}) \, / \, (v_i^{\max} - v_i^{\min}) \tag{17}$$

$$l_i^0 = l_i^{\max} - \beta_i \, v_i^{\max} \tag{18}$$

Substituting (13) into (3) we have:

$$p_{ik} = q_{ik} \, (\alpha_i \, h_{ik} + \eta_i^0) \tag{19}$$

Therefore, substituting (4) and (16) into (19), hydroelectric power generation becomes a nonlinear function of water discharge and water storage, given by:

$$p_{ik} = \alpha_i \, \beta_{f(i)} \, q_{ik} \, v_{f(i)k} - \alpha_i \, \beta_{t(i)} \, q_{ik} \, v_{t(i)k} + \delta_i \, q_{ik} \tag{20}$$

where the parameter δ_i is given by:

$$\delta_i = \alpha_i \, (l_{f(i)}^0 - l_{t(i)}^0) + \eta_i^0 \tag{21}$$

The parameters given by the product of $\alpha's$ by $\beta's$ are of crucial importance for the behavior of head-dependent reservoirs in a cascaded hydro system, setting optimal reservoirs storage trajectories in accordance to their relative position in the cascade. It should be noted that these parameters are not related to the solution procedure. Instead, they are determined only by physical data defining the hydro system [2].

Equation (20) can be converted in the format of (9), with the parameter δ_i multiplied by the forecasted electricity price π_k appearing in the vector $\boldsymbol{f}$, and the parameters $\alpha_i \beta_{f(i)}$ and $\alpha_i \beta_{t(i)}$ also multiplied by the forecasted electricity price π_k appearing in the matrix $\boldsymbol{H}$.

A major advantage of our novel MIQP approach is to consider the head change effect in a single function (20) of water discharge and water storage that can be used in a straightforward way, instead of deriving several curves for different heads.

As a new contribution to earlier studies, we model the on-off behavior of the hydro plants using integer variables. Thus, the unit commitment is considered in (6), allowing for multiple operating regions. Since we can achieve a solution faster for a MILP approach than for the proposed MIQP approach, the MILP approach is used to find a starting point for the MIQP approach. Afterwards, we check for an enhanced objective function value using our novel MIQP approach. In our case studies we always arrive at convergence to a superior solution.

4. Case studies

The proposed MIQP approach, considering head-dependency, discontinuous operating regions, and discharge ramping constraints, has been applied on two case studies based on Portuguese cascaded hydro systems: a) hydro system with three cascaded reservoirs; b) hydro system with seven cascaded reservoirs. A comparison with a MILP approach is carried out, making clear the advantages of the proposed MIQP approach.

Our novel MIQP approach has been developed and implemented in MATLAB and solved using the optimization solver package Xpress-MP. The numerical testing has been performed on a 600-MHz-based processor with 256 MB of RAM.

The competitive environment coming from the deregulation of the electricity markets brings electricity prices uncertainty, placing higher requirements on forecasting. A good forecasting tool reduces the risk of under/over estimating the profit of the utilities and provides better risk management. Several forecasting procedures are available for forecasting electricity prices [25–27], but for the STHS problem the prices are considered as deterministic input data.

4.1 Case 1

The hydro system has three cascaded reservoirs and is shown in Fig. 1.

"See Fig. 1 at the end of the manuscript".

Inflow is considered only on reservoir 1. The final water storage in the reservoirs is constrained to be equal to the initial water storage, chosen as 60% of maximum storage.

The time horizon considered is one week, divided into 168 hourly intervals. The electricity price profile considered over the time horizon is shown in Fig. 2 ($ is a symbolic economic quantity).

"See Fig. 2 at the end of the manuscript".

The optimal storage trajectories for the reservoirs are shown in Fig. 3. The dash-dot lines denote the results obtained by a MILP approach while the solid lines denote the results obtained by the proposed MIQP approach.

"See Fig. 3 at the end of the manuscript".

The comparison between MILP and MIQP approaches reveals the influence of considering head-dependency on the behavior of the reservoirs. The upstream reservoir should operate at a suitable high storage level in order to benefit the operating efficiency of its associated plants, due to the head change effect. The storage level in the last downstream reservoir is lower with the proposed MIQP approach than with the MILP approach, thereby improving the head for the immediately upstream reservoir. Hence, a higher efficiency of the last downstream plant is not important for the overall profit in this realistic hydro system.

The optimal discharge profiles for the reservoirs are shown in Fig. 4. Again, the dash-dot lines denote the results obtained by a MILP approach while the solid lines denote the results obtained by the proposed MIQP approach.

"See Fig. 4 at the end of the manuscript".

The comparison between MILP and MIQP approaches reveals that the water discharge changes more quickly from the lower value to the upper value with the MILP approach than with the proposed MIQP approach, thus ignoring the head-dependency. The water discharge and consequently the hydroelectric power generation tend to follow the shape of the electricity price profile shown in Fig. 2. As a new contribution to earlier studies [2,6,22], the water discharges at forbidden intervals are avoided, namely between 0 and $q_i^{\min}$. Also, ramp rate of water discharge is included in the constraints. Thus, an enhanced STHS is provided due to the more realistic modeling presented in this paper.

4.2 Case 2

The hydro system has seven cascaded reservoirs and is shown in Fig. 5.

"See Fig. 5 at the end of the manuscript".

The hydro plants numbered in Fig. 5 as 1, 2, 4, 5 and 7 are run-of-the-river hydro plants. The hydro plants numbered as 3 and 6 are storage hydro plants. Hence, for the storage hydro plants head-dependency may be neglected, due to the small head variation during the short-term time horizon.

Inflow is considered only on reservoirs 1 to 6. The final water storage in the reservoirs is constrained to be equal to the initial water storage, chosen as 80% of maximum storage. Also, the minimum storage is constrained to be equal to 30% of maximum storage.

The time horizon considered is one day, divided into 24 hourly intervals. The electricity price profile considered over the time horizon is shown in Fig. 6.

"See Fig. 6 at the end of the manuscript".

The optimal storage trajectories for the reservoirs are shown in Fig. 7. The dash-dot lines denote the results obtained by a MILP approach while the solid lines denote the results obtained by the proposed MIQP approach.

"See Fig. 7 at the end of the manuscript".

The optimal discharge profiles for the reservoirs are shown in Fig. 8. Again, the dash-dot lines denote the results obtained by a MILP approach while the solid lines denote the results obtained by the proposed MIQP approach.

"See Fig. 8 at the end of the manuscript".

Table 1 summarizes an overall comparison between the numerical results obtained by both optimization methods.

"See Table 1 at the end of the manuscript".

Although the average water discharge is as expected the same for both optimization methods, the average storage is superior with the proposed MIQP approach due to the consideration of head-dependency. Thus, regardless of the price scenario considered, with the proposed MIQP approach we have a higher total profit for the hydroelectric utility, about 4.4%. Moreover, the extra computation time required is negligible, converging rapidly to the optimal solution.

The benefits of considering head-dependency are shown by providing a MILP approach that does not consider the impact of variable head. In order to model head variations in MILP, the discretization of the nonlinear dependence between power generation, water discharge and head is required. However, such discretization augments the computational burden required to solve the STHS problem. For instance, the optimal solution reported in [19] required 22 minutes of CPU time, on a 400-MHz-based processor with 500 MB of RAM. A major advantage of our novel MIQP approach is to consider the head change effect in a single function of water discharge and water storage that can be used in a straightforward way, instead of deriving several curves for different heads. Hence, the proposed MIQP approach provides better results for cascaded hydro systems, where head-dependency plays a major role on the behavior of the reservoirs.

5. Conclusions

A novel MIQP approach is proposed in this paper to solve the STHS problem. Our approach allows an efficient consideration of the nonlinear dependence between power generation, water discharge and head. As a new contribution to earlier studies, integer variables are used to model the on-off behavior of the hydro plants. Also, discharge ramping constraints are included to keep a lesser and steady head variation. Numerical testing results show that the proposed approach is computationally adequate for hydro systems with run-of-the-river hydro plants, considering head-dependency, discontinuous operating regions, and discharge ramping constraints, in order to obtain more realistic and feasible results. The additional computation time required is negligible, converging rapidly to the optimal solution. Hence, the proposed approach is both accurate and computationally acceptable, providing an enhanced STHS.

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Figure captions

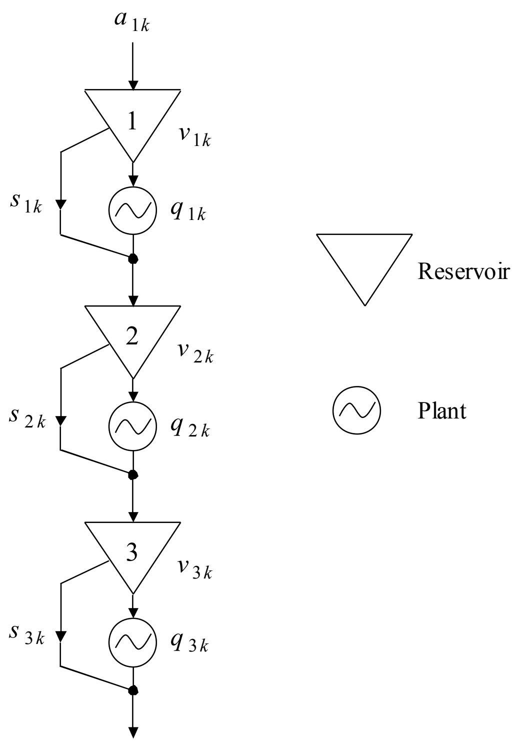

Fig. 1. Hydro system with three cascaded reservoir.

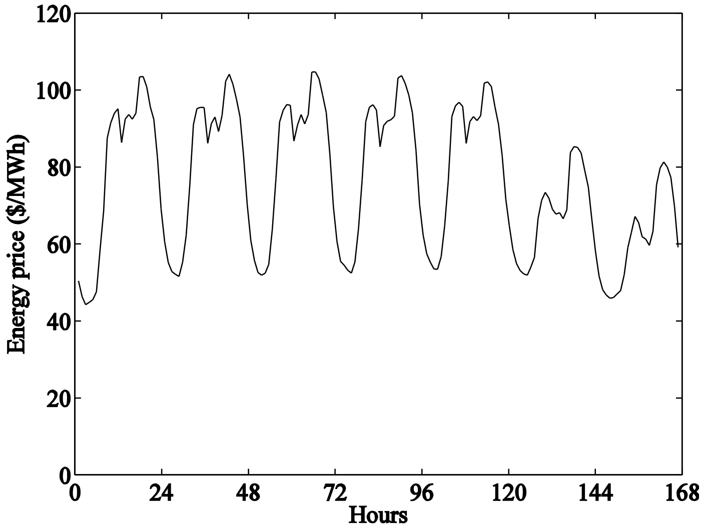

Fig. 2. Electricity price profile considering a one week time horizon.

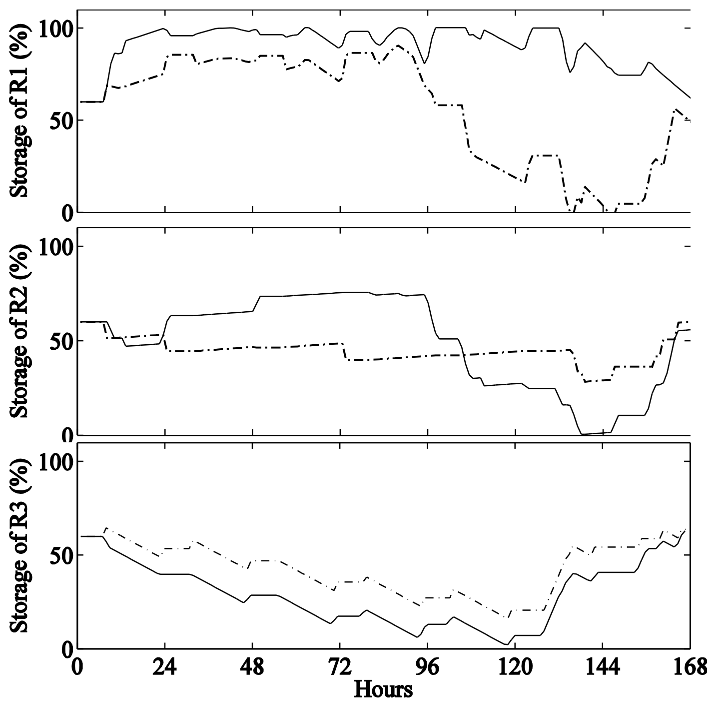

Fig. 3. Optimal storage trajectories for the reservoirs — case 1.

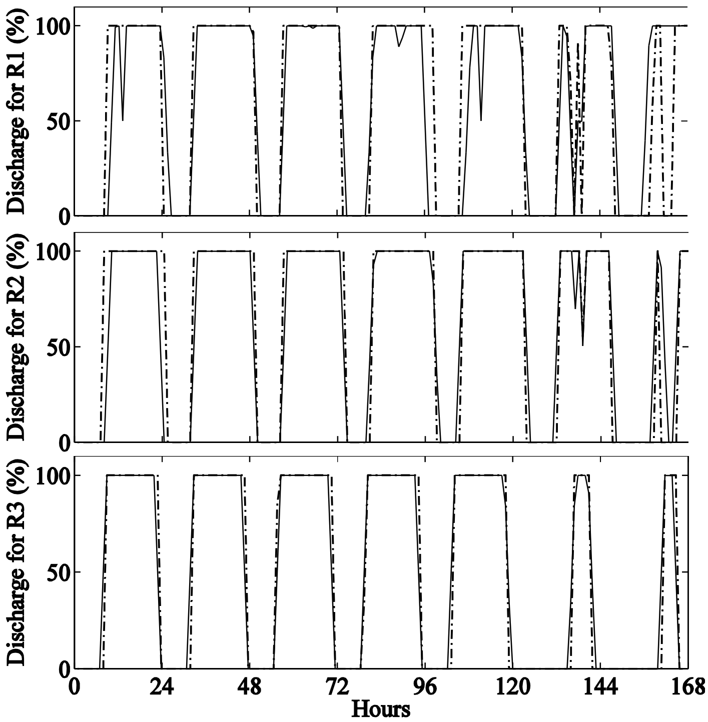

Fig. 4. Optimal discharge profiles for the reservoirs — case 1.

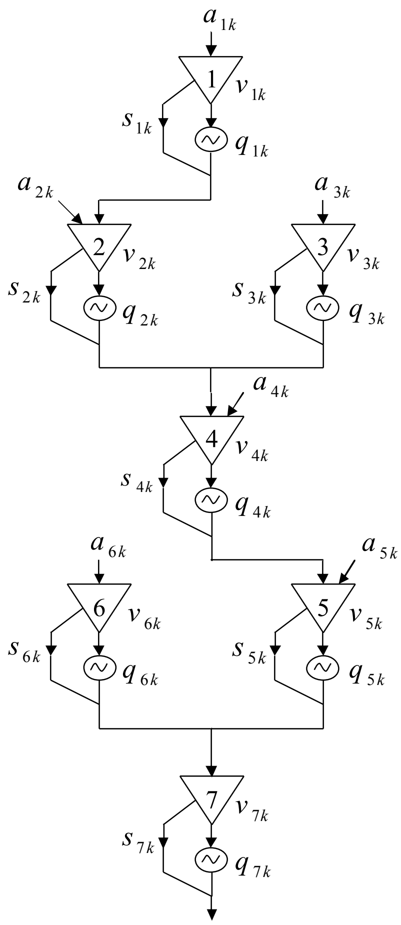

Fig. 5. Hydro system with seven cascaded reservoirs.

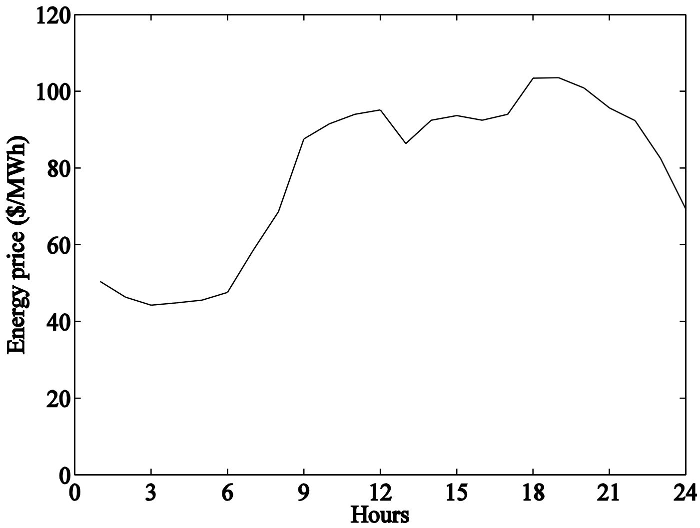

Fig. 6. Electricity price profile considering a one day time horizon.

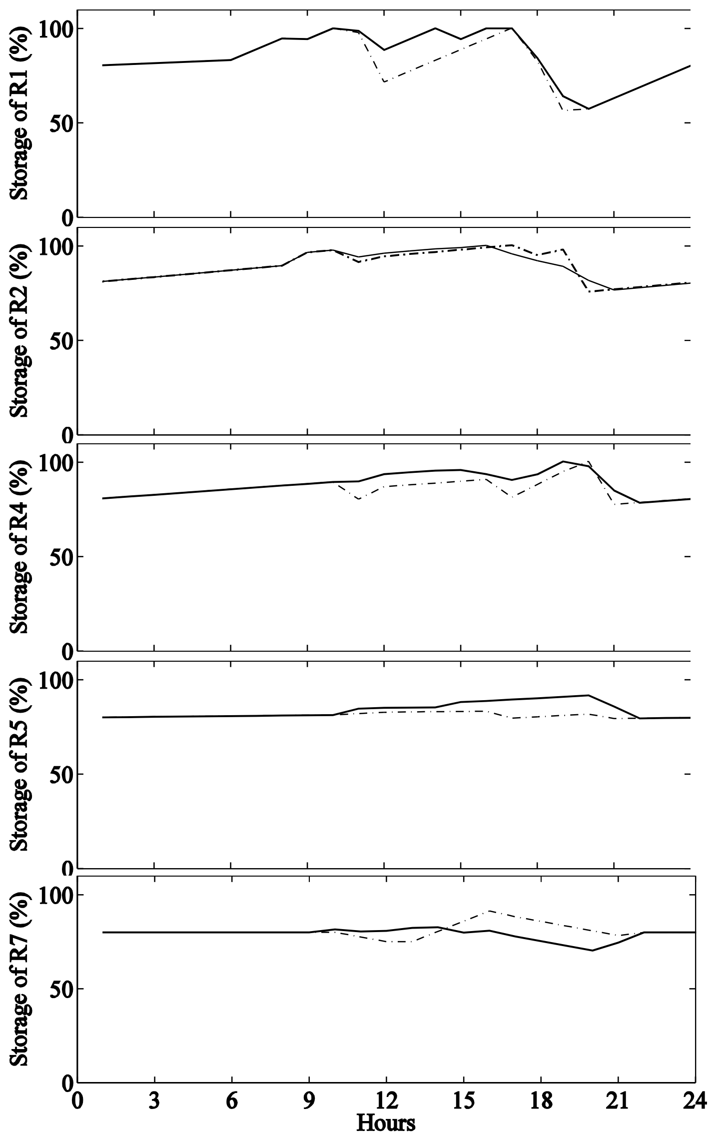

Fig. 7. Optimal storage trajectories for the reservoirs — case 2.

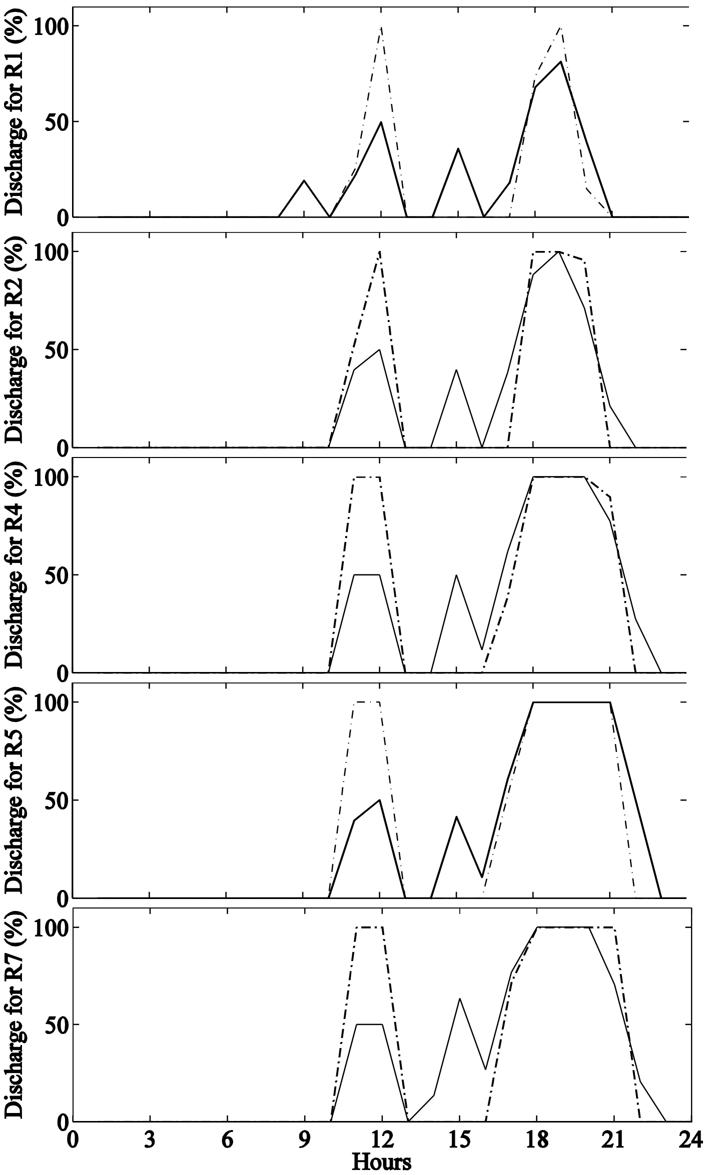

Fig. 8. Optimal discharge profiles for the reservoirs — case 2.

Tables

Table 1

Comparison of results

	Optimization method	Average discharge (%)	Average storage (%)	Total profit ($\$\times 10^3$)	CPU (s)
Case 1	MILP	41.58	33.69	5258.37	2.16
	MIQP	41.58	46.18	5477.53	6.22
Case 2	MILP	25.00	83.08	716.64	1.59
	MIQP	25.00	84.06	749.59	5.17