# Short-term electricity prices forecasting in a competitive market by a hybrid intelligent approach

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#### Abstract

In this paper, a hybrid intelligent approach is proposed for short-term electricity prices forecasting in a competitive market. The proposed approach is based on the wavelet transform and a hybrid of neural networks and fuzzy logic. Results from a case study based on the electricity market of mainland Spain are presented. A thorough comparison is carried out, taking into account the results of previous publications. Conclusions are duly drawn. © 2010 Elsevier Ltd. All rights reserved.

Keywords: Electricity market; fuzzy logic; neural networks; price forecasting; wavelet transform

#### 1. Introduction

Deregulation processes during the last two decades across many developed economies have motivated the need for more accurate forecasting tools of electricity markets [1]. Short-term electricity prices forecasting is required by producers and consumers to derive their bidding strategies to the electricity market. Deregulation brings electricity prices uncertainty, placing higher requirements on forecasting [2]. Therefore, price forecasting tools are essential for all market participants for their survival under deregulated environment [3]. In most competitive electricity markets the series of prices presents the following features: high frequency, non-constant mean and variance, daily and weekly seasonality, calendar effect on weekend and public holidays, high volatility and high percentage of unusual prices [4].

Price forecast is a key issue in competitive electricity markets [5,6], and several techniques have been tried out in this task. In general, hard and soft computing techniques could be used to predict electricity prices.

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The hard computing techniques include auto regressive integrated moving average (ARIMA) [7], wavelet-ARIMA [8], and mixed model [9] approaches. Usually, an exact model of the system is required, and the solution is found using algorithms that consider the physical phenomena that govern the process. Although these approaches can be very accurate, they require a lot of information, and the computational cost is very high.

The soft computing techniques include neural networks (NN) [10–13], fuzzy neural networks (FNN) [14], weighted nearest neighbors (WNN) [15], adaptive wavelet neural network (AWNN) [16], and hybrid intelligent system (HIS) [17] approaches.

A combination of neural networks with wavelet transform (NNWT) has also been recently proposed [18]. Usually, an input-output mapping is learned from historical examples, thus there is no need to model the system. Hence, these approaches can be much more efficient computationally and as accurate as the first ones, if the correct inputs are considered [19].

In this paper, a hybrid intelligent approach is proposed for short-term electricity prices forecasting. The proposed approach is based on the wavelet transform and a hybrid of neural networks and fuzzy logic.

The proposed approach is examined on the electricity market of mainland Spain, commonly used as the test case in several price forecasting papers [7–9, 13–18]. It has been concluded that the Spanish market has a hard nonlinear behaviour and time variant functional relationship [8,14]. So, this market is a real world case study with sufficient complexity.

The proposed approach is compared with ARIMA, mixed-model, NN, wavelet-ARIMA, WNN, FNN, HIS, AWNN and NNWT approaches, to demonstrate its effectiveness regarding forecasting accuracy and computation time.

This paper is organized as follows. Section 2 presents the proposed approach to forecast electricity prices. Section 3 provides the different criterions used to evaluate the forecasting accuracy. Section 4 provides the numerical results from a real-world case study. Finally, concluding remarks are given in Section 5.

#### 2. Proposed approach

The proposed approach to forecast electricity prices is based on the wavelet transform (WT) and a hybrid of NN and fuzzy logic known as adaptive-network-based fuzzy inference system (ANFIS). The WT is used to decompose the usually ill-behaved price series into a set of better-behaved constitutive series. Then, the future values of these constitutive series are forecasted using ANFIS. In turn, the ANFIS forecasts allow, through the inverse WT, reconstructing the future behaviour of the price series and therefore to forecast prices.

#### 2.1 Wavelet transform

The WT convert a price series in a set of constitutive series. These constitutive series present a better behaviour than the original price series, and therefore, they can be predicted more accurately. The reason for the better behaviour of the constitutive series is the filtering effect of the WT [8].

A brief summary of WT is presented hereafter. For the sake of simplicity, one-dimensional wavelets are considered to illustrate the related concepts.

A wavelet is a waveform of effectively limited duration that has an average value of zero. Comparing wavelets with sine waves (which are the basis of Fourier analysis), sinusoids do not have limited duration (they extend from minus to plus infinity). Moreover, where sinusoids are smooth and predictable, wavelets tend to be irregular and asymmetric. Wavelet analysis is the breaking up of a signal into shifted and scaled versions of the mother wavelet. Signals with sharp changes might be better analyzed with an irregular wavelet than with a smooth sinusoid. Wavelet analysis does not use a time-frequency region (like the short-time Fourier transform), but rather a time-scale region.

Wavelet analysis is capable of revealing aspects of data that other signal analysis techniques miss, such as trends, breakdown points, discontinuities in higher derivatives and self-similarity. Furthermore, wavelet analysis can often compress or de-noise a signal without appreciable degradation [20]. These capabilities of WT can be useful in short-term electricity prices forecasting.

WTs can be divided in two categories: continuous wavelet transform (CWT) and discrete wavelet transform (DWT). The CWT W(a,b) of signal f(x) with respect to a mother wavelet  $\phi(x)$  is given by [20]:

$$W(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(x) \phi\left(\frac{x-b}{a}\right) dx \tag{1}$$

where the scale parameter a controls the spread of the wavelet and translation parameter b determines its central position. The W(a,b) coefficient represents how well the original signal f(x) and the scaled/translated mother wavelet match. Thus, the set of all wavelet coefficients W(a,b), associated to a particular signal, is the wavelet representation of the signal with respect to the mother wavelet. Since the CWT is achieved by continuously scaling and translating the mother wavelet, substantial redundant information is generated. Therefore, instead of doing that, the mother wavelet can be scaled and translated using certain scales and positions usually based on powers of two. This scheme is more efficient and just as accurate as the CWT [21]. It is known as the DWT and defined as:

$$W(m,n) = 2^{-(m/2)} \sum_{t=0}^{T-1} f(t) \phi\left(\frac{t-n \cdot 2^m}{2^m}\right)$$
(2)

where T is the length of the signal f(t). The scaling and translation parameters are functions of the integer variables m and n ( $a = 2^m$ ,  $b = n \cdot 2^m$ ); t is the discrete time index.

A fast DWT algorithm based on the four filters (decomposition low-pass, decomposition high-pass, reconstruction low-pass, and reconstruction high-pass filters), developed by Mallat [22], is considered in this paper. Multiresolution via Mallat's algorithm is a procedure to obtain "approximations" and "details" from a given signal. An approximation is a low-frequency representation of the original signal, whereas a detail is the difference between two successive approximations. An approximation holds the general trend of the original signal, whereas a detail depicts high-frequency components of it [21]. By successive decomposition of the approximations (Fig. 1), a multilevel decomposition process can be achieved where the original signal is broken down into lower resolution components.

A wavelet function of type Daubechies of order 4 (abbreviated as Db4) is used as the mother wavelet  $\phi(t)$ . This wavelet offers an appropriate trade-off between wave-length and smoothness, resulting in an appropriate behaviour for short-term electricity prices forecasting.

Similar wavelets have been considered by previous researchers [8,20,21]. Also, three decomposition levels are considered, as shown in Fig. 1, since it describes the price series in a more thorough and meaningful way than the others [23].

"See Fig. 1 at the end of the manuscript".

#### 2.2 Neuro-fuzzy approach

NN are simple, but powerful and flexible tools for forecasting, provided that there are enough data for training, an adequate selection of the input-output samples, an appropriated number of hidden units and enough computational resources available. Also, NN have the well-known advantages of being able to approximate any nonlinear function and being able to solve problems where the input-output relationship is neither well defined nor easily computable, because NN are data-driven. Multi-layered feedforward NN are specially suited for forecasting, implementing nonlinearities using sigmoid functions for the hidden layer and linear functions for the output layer [13].

Just like NN, a fuzzy logic system is a nonlinear mapping of an input vector into a scalar output, but it can handle numerical values and linguistic knowledge. In general, a fuzzy logic system contains four components: fuzzifier, rules, inference engine, and defuzzifier. The fuzzifier converts a crisp input variable into a fuzzy representation, where membership functions give the degree of belonging of the variable to a given attribute. Fuzzy rules are of the type "if–then", and can be derived from numerical data or from expert linguistic. Mamdani and Sugeno inference engines are two of the main types of inference mechanisms. The Mamdani engine combines fuzzy rules into a mapping from fuzzy input sets to fuzzy output sets, while the Takagi–Sugeno type relates fuzzy inputs and crisp outputs. The defuzzifier converts a fuzzy set into a crisp number using the centroid of area, bisector of area, mean of maxima, or maximum criteria.

NN have the advantage over the fuzzy logic models that knowledge is automatically acquired during the learning process. However, this knowledge cannot be extracted from the trained network behaving as a black box. Fuzzy systems, on the other hand, can be understood through their rules, but these rules are difficult to define when the system has too many variables and their relations are complex [19].

A combination of NN and fuzzy systems has the advantages of each of them. In a neuro-fuzzy system, neural networks extract automatically fuzzy rules from numerical data and, through the learning process, the membership functions are adaptively adjusted.

ANFIS is a class of adaptive multi-layer feedforward networks, applied to nonlinear forecasting where past samples are used to forecast the sample ahead. ANFIS incorporates the self-learning ability of NN with the linguistic expression function of fuzzy inference [24].

The ANFIS architecture is shown in Fig. 2. The ANFIS network is composed of five layers. Each layer contains several nodes described by the node function. The node function is described next. Let  $O_i^j$  denote the output of the *i*th node in layer *j*.

"See Fig. 2 at the end of the manuscript".

In layer 1, every node *i* is an adaptive node with node function:

$$O_i^1 = \mu A_i(x), \quad i = 1, 2$$
 (3)

or

$$O_i^1 = \mu B_{i-2}(y), \quad i = 3, 4$$
 (4)

where x (or y) is the input to the *i*th node and  $A_i$  (or  $B_{i-2}$ ) is a linguistic label associated with this node. Thus,  $O_i^1$  is the membership grade of a fuzzy set A (=  $A_1, A_2, B_1$ , or  $B_2$ ) and it specifies the degree to which the given input x (or y) satisfies the quantifier A. The membership functions for A and B are usually described by generalized bell functions, e.g.:

$$\mu A_{i}(x) = \frac{1}{1 + \left|\frac{x - r_{i}}{p_{i}}\right|^{2q_{i}}}$$
(5)

where  $\{p_i, q_i, r_i\}$  is the parameter set. As the values of these parameters change, the bell-shaped function varies accordingly, thus exhibiting various forms of membership functions on linguistic label  $A_i$ . In fact, any continuous and piecewise differentiable functions, such as triangular-shaped membership functions, are also qualified candidates for node functions in this layer [25]. Parameters in this layer are referred to as premise parameters.

In layer 2, each node  $\prod$  multiplies incoming signals and sends the product out:

$$O_i^2 = w_i = \mu A_i(x) \ \mu B_i(y), \quad i = 1, 2$$
(6)

Hence, each node output represents the firing strength of a rule.

In layer 3, each node N computes the ratio of the *i*th rules's firing strength to the sum of all rules' firing strengths:

$$O_i^3 = \overline{w}_i = \frac{w_i}{w_1 + w_2}, \quad i = 1, 2$$
 (7)

The outputs of this layer are called normalized firing strengths.

In layer 4, each node computes the contribution of the *i*th rule to the overall output:

$$O_i^4 = \overline{w}_i \ z_i = \overline{w}_i \left( a_i \ x + b_i \ y + c_i \right), \quad i = 1, 2$$
(8)

where  $\overline{w}_i$  is the output of layer 3 and  $\{a_i, b_i, c_i\}$  is the parameter set. Parameters of this layer are referred to as consequent parameters.

In layer 5, the single node  $\Sigma$  computes the final output as the summation of all incoming signals:

$$O_i^{\,\rm S} = \sum_i \overline{w}_i \ z_i = \frac{\sum_i w_i \ z_i}{\sum_i w_i} \tag{9}$$

Thus, an adaptive network is functionally equivalent to a Sugeno-type fuzzy inference system.

The ANFIS considered in this study uses a hybrid learning algorithm to identify parameters of Sugeno-type fuzzy inference systems. Thus, ANFIS uses a combination of the least-squares method (to determine consequent parameters) and the backpropagation gradient descent method (to learn the premise parameters). A step in the learning process has two passes: In the first pass training data is brought to the inputs, the premise parameters are assumed to be fixed and the optimal consequent parameters are estimated by an iterative least mean squares procedure. In the second pass the patterns are propagated again, but this time the consequent parameters are assumed to be fixed and backpropagation is used to modify the premise parameters [25]. The membership functions considered in this study are triangular-shaped. The selection of numbers of membership functions is a trade-off between refining and sparseness [26].

#### 3. Forecasting accuracy evaluation

To evaluate the accuracy in forecasting electricity prices, different criterions are used. This accuracy is computed in function of the actual prices that occurred.

The mean absolute percentage error (MAPE) criterion, the sum squared error (SSE) criterion, and the standard deviation of error (SDE) criterion, are defined as follows.

The MAPE criterion is defined as follows:

$$MAPE = \frac{100}{N} \sum_{h=1}^{N} \frac{\left| \hat{p}_{h} - p_{h} \right|}{\overline{p}}$$
(10)

$$\overline{p} = \frac{1}{N} \sum_{h=1}^{N} p_h \tag{11}$$

where  $\hat{p}_h$  and  $p_h$  are respectively the forecasted and actual electricity prices at hour h,  $\overline{p}$  is the average price of the forecasting period and N is the number of forecasted hours.

The SSE criterion is given by:

$$SSE = \sum_{h=1}^{N} (\hat{p}_h - p_h)^2$$
(12)

The SDE criterion is given by:

$$SDE = \sqrt{\frac{1}{N} \sum_{h=1}^{N} (e_h - \bar{e})^2}$$
 (13)

$$e_h = \hat{p}_h - p_h \tag{14}$$

$$\overline{e} = \frac{1}{N} \sum_{h=1}^{N} e_h \tag{15}$$

where  $e_h$  is the forecast error at hour h and  $\bar{e}$  is the average error of the forecasting period.

#### 4. Numerical results

The proposed hybrid wavelet-neuro-fuzzy (WNF) approach is applied to forecast next-week prices in the electricity market of mainland Spain. Price forecasting is computed using historical data of year 2002 for the Spanish market. For the sake of simplicity and clear comparison, no exogenous variables are considered. Also, for the sake of a fair comparison, the same test weeks as in [7–9, 13–18] are selected, which correspond to the four seasons of year 2002. To build the forecasting model, the hourly historical price data of the 42 days previous to the day of the week whose prices are to be forecasted have been considered.

Numerical results with the proposed hybrid WNF approach are shown in Figs. 3 to 6 respectively for the winter, spring, summer and fall weeks. Each figure shows the actual prices, solid line, together with the forecasted prices, dashed line.

"See Fig. 3 at the end of the manuscript". "See Fig. 4 at the end of the manuscript". "See Fig. 5 at the end of the manuscript". "See Fig. 6 at the end of the manuscript". Table 1 presents the values for the criterions to evaluate the accuracy of the proposed hybrid WNF approach in forecasting electricity prices. The first column indicates the week, the second column presents the MAPE, the third column presents the square root of the SSE, and the fourth column presents the SDE. A good accuracy was ascertained: the MAPE for the Spanish market has an average value of 6.53%.

#### "See Table 1 at the end of the manuscript".

Table 2 shows a comparison between the proposed hybrid WNF approach and nine other approaches (ARIMA, mixed-model, NN, wavelet-ARIMA, WNN, FNN, HIS, AWNN and NNWT), in what regards the MAPE criterion.

"See Table 2 at the end of the manuscript".

The proposed approach presents better forecasting accuracy over the other approaches. Moreover, the average computation time is less than 5 seconds using MATLAB on a PC with 1 GB of RAM and a 2.0-GHz-based processor. Nevertheless, an average value of 5.32% for the MAPE has been recently reported using a cascaded neuro-evolutionary algorithm (CNEA) [27] approach. However, this algorithm has a major drawback: the computation time of about 40 minutes.

Hence, the proposed approach presents the best trade-off between forecasting accuracy and computation time, which can be of the utmost importance for real-life applications.

#### 5. Conclusions

A hybrid intelligent approach is proposed for electricity prices forecasting on the Spanish market. The application of the proposed approach to price forecasting is both novel and effective. The MAPE has an average value of 6.53%, while the average computation time is less than 5 seconds. Hence, the proposed approach presents the best trade-off between forecasting accuracy and computation time, taking into account the results of previous publications.

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**Figure captions** 



Fig. 1. Multilevel decomposition process.



Fig. 2. ANFIS architecture.



Fig. 3. Winter week: actual prices, solid line, together with the forecasted prices, dashed line, in euro per megawatt



Fig. 4 Spring week: actual prices, solid line, together with the forecasted prices, dashed line, in euro per megawatt



Fig. 5 Summer week: actual prices, solid line, together with the forecasted prices, dashed line, in euro per megawatt



Fig. 6 Fall week: actual prices, solid line, together with the forecasted prices, dashed line, in euro per megawatt

# Tables

# Table 1

Statistical analysis of the weekly forecasting error

Week	MAPE	$\sqrt{\text{SSE}}$	SDE
Winter	3.38	25.20	1.26
Spring	4.01	31.44	1.64
Summer	9.47	72.92	4.12
Fall	9.27	55.90	3.04

# Table 2

Comparative MAPE results

	Winter	Spring	Summer	Fall	Average
ARIMA [7]	6.32	6.36	13.39	13.78	9.96
Mixed-model [9]	6.15	4.46	14.90	11.68	9.30
NN [13]	5.23	5.36	11.40	13.65	8.91
Wavelet-ARIMA [8]	4.78	5.69	10.70	11.27	8.11
WNN [15]	5.15	4.34	10.89	11.83	8.05
FNN [14]	4.62	5.30	9.84	10.32	7.52
HIS [17]	6.06	7.07	7.47	7.30	6.97
AWNN [16]	3.43	4.67	9.64	9.29	6.75
NNWT [18]	3.61	4.22	9.50	9.28	6.65
WNF	3.38	4.01	9.47	9.27	6.53