Scheduling of a hydro producer considering head-dependency, price scenarios and risk-aversion

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Abstract

In this paper, a mixed-integer quadratic programming approach is proposed for the short-term hydro scheduling problem, considering head-dependency, discontinuous operating regions and discharge ramping constraints. As new contributions to earlier studies, market uncertainty is introduced in the model via price scenarios, and risk aversion is also incorporated by limiting the volatility of the expected profit through the conditional value-at-risk. Our approach has been applied successfully to solve a case study based on one of the main Portuguese cascaded hydro systems, requiring a negligible computational time.

Keywords: Hydro scheduling; mixed-integer quadratic programming; head-dependency; price scenarios; risk-aversion

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1. Introduction

In this paper, the short-term hydro scheduling (STHS) problem of head-dependent cascaded hydro systems is considered. In hydro plants with a small storage capacity available, operating efficiency is sensitive to the head: head change effect [1]. For instance, in the Portuguese system there are several cascaded hydro systems formed by several but small reservoirs. Hence, it is necessary to consider head-dependency on STHS. One of the main difficulties lies precisely in the correct modeling of the head variation and its effects [2].

In a deregulated profit-based environment, a hydroelectric utility is usually an entity owning generation resources and participating in the electricity market with the ultimate goal of maximizing profits, without concern of the system, unless there is an incentive for it [3]. However, in many electricity markets, due to power market exercised by the participants, two kind of generating companies can be defined, namely price-makers [4] and price-takers [5]; in other words, either the company is able to influence market prices to its own profit, or not. This paper is focused on price-taker hydro plants.

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In the Iberian day-ahead electricity market, supply or generation bids comprise several blocks with increasing values of power and price. Based on supply bids and demand offers, the market operator defines the market-clearing prices and power quantities. By means of this procedure, the generating companies' profits will depend on their units scheduling and bidding strategies. Still, only the short-term hydro scheduling optimization is taken into account, where the goal is the efficient operation planning of hydro plants for the next day, while bidding strategy development is outside the scope of this paper. However, an example of a linear bid curve of a generating company can be found in [6].

The optimal management of the water available in the reservoirs for power generation, without affecting future operation use, delivers a self-schedule [7] and represents a major advantage for the hydroelectric utilities to face competition.

Dynamic programming (DP) is among the earliest methods applied to the STHS problem. Although DP can handle the nonconvex, nonlinear characteristics present in the hydro model, direct application of DP methods for hydro systems with cascaded reservoirs is impractical due to the well-known DP curse of dimensionality [8]. DP can only be effective for cases with a lower dimension [9].

Artificial intelligence techniques have also been applied to the STHS problem, namely multi-objective differential evolution [10], adaptive chaotic differential evolution [11], hybrid differential evolution [12], and multi-objective cultural algorithm [13]. However, a significant computational effort is necessary to solve the problem for cascaded hydro systems. Also, due to the heuristics used in the search process only sub-optimal solutions can be reached.

A natural approach to STHS is to model the system as a network flow model, because of the underlying network structure subjacent in cascaded reservoirs [14]. This network flow model is often simplified to a linear or piecewise linear one. Linear programming (LP) is a well-known optimization method and standard software can be found commercially. Mixed-integer linear programming (MILP) is becoming often used for STHS, where integer variables allow modeling of discrete hydro unit-commitment constraints [15]. In [16] the aim is to determine a feasible and realistic operation of a set of coupled hydro units using a MILP approach. The MILP approach implemented in [17] takes into account the head effects on power production through a linearization technique, while in [18] a multi-stage mixed-integer linear stochastic programming was developed. However, LP considers that hydroelectric power generation is linearly dependent on water discharge, ignoring head-dependency to avoid nonlinearities.

Besides, the discretization of the nonlinear dependence between power generation, water discharge and head, used in MILP to model head variations, augment the computational burden required to solve the STHS problem. The model proposed in [19] incorporates the operating zones, upper and lower bounds on hydro power, and the hydraulic coupling, among others. Still, this model relies on a decomposition approach, where DP is used in a second step to determine the operating zones, and after an iterative linearization of the constraints is employed. As a consequence, convergence and numerical problems may be encountered. Further, linearization methods based on successive iterations depend on operator know-how in parameters tuning, posing some ambiguities. It should be noted that, according to [20], the under relaxed iterative procedure used in [16] does not always reach convergence and that, in some cases, it converges to operation schedules with lower revenues than those obtained in previous iterations.

A nonlinear model has advantages compared with a linear one. A nonlinear model expresses hydroelectric power generation characteristics more accurately and head-dependency on STHS can be taken into account. The use of the nonlinear model in some case studies leads to a result that exceeds by at least 3 percent what is obtained by a linear model, requiring a negligible extra computation time [21]. However, the nonlinear model cannot avoid water discharges at forbidden areas, and may give schedules unacceptable from an operation point of view. Moreover, it is important to notice that a minor change in the electricity price may give a significant change in the water discharge, and consequently in the power generation of plants.

The previous concerns lead to the mixed-integer quadratic programming (MIQP) approach proposed in [1] to solve the STHS problem, where integer variables are used to model the on-off behavior of the hydro plants. In [1] the STHS problem was treated as a deterministic one, thus ignoring uncertainties. However, this may not be a realistic assumption, given the highly volatile electricity prices even for the short-term time horizon. Hence, in this paper the STHS problem is considered as a stochastic one.

Any producer participating in an electric energy pool should self-schedule its units to maximize its expected profit assuming a given level of risk. This optimal self-schedule is then used by the producer to derive an appropriate bidding strategy to the pool.

As new contributions to earlier studies, market uncertainty is introduced in the model via price scenarios, and risk-aversion is also incorporated by limiting the volatility of the expected profit through the conditional value-at-risk (CVaR). Furthermore, no previous approach using CVaR as risk measure has modeled effectively the head-dependency of the hydro plants.

This paper is organized as follows. In Section 2, the mathematical formulation of the STHS problem is provided. Section 3 presents the MIQP approach to solve the STHS problem. In Section 4, the proposed MIQP approach is applied on a case study, based on one of the main Portuguese cascaded hydro systems, to demonstrate its effectiveness. Finally, concluding remarks are given in Section 5.

2. Problem formulation

The notation used throughout the paper is stated as follows.

Sets

I, i	Set and index of reservoirs.
K, k	Set and index of hours in the time horizon.
N, n	Set and index of scenarios.

Constants and data

δ	Per unit confidence level.
ρ_n	Probability of occurrence of scenario n .
α	Weighting positive factor to achieve appropriate tradeoff profit versus risk.
λ_{kn}	Electricity price for scenario n in hour k.
a_{ik}	Inflow to reservoir i in hour k.
M_i	Set of upstream reservoirs of plant i.
R_i	Discharge ramping limit of plant i.
$v_i^{\min}, v_i^{\max}$	Water storage limits of reservoir i.
$q_i^{\min}, q_i^{\max}$	Water discharge limits of plant i.
$\eta_i^{\min}, \eta_i^{\max}$	Power efficiency limits of plant i.
$h_i^{\min}, h_i^{\max}$	Head limits of plant i.
$l_i^{\min}, l_i^{\max}$	Water level limits of reservoir i.

Variables

B_n	Benefit in scenario n .
ζ	Value-at-risk.

η_n	Auxiliary variable used to compute the conditional value-at-risk.
p_{ik}	Power generation of plant i in hour k.
v_{ik}	Water storage of reservoir i at end of hour k.
q_{ik}	Water discharge of plant i in hour k.
s_{ik}	Water spillage by reservoir i in hour k.
η_{ik}	Power efficiency of plant i in hour k.
h_{ik}	Head of plant i in hour k.
l_{ik}	Water level in reservoir i in hour k.
u_{ik}	Commitment decision of plant i in hour k.

Vectors and matrixes

$\boldsymbol{H}$	Hessian matrix.
$\boldsymbol{f}$	Vector of coefficients for the linear term.
$\boldsymbol{x}$	Vector of decision variables.
$\boldsymbol{A}$	Constraint matrix.
$\boldsymbol{b}^{\min}, \boldsymbol{b}^{\max}$	Lower and upper bound vectors on constraints.
$\boldsymbol{x}^{\min}, \boldsymbol{x}^{\max}$	Lower and upper bound vectors on variables.

2.1 Conditional Value at Risk

A hydro power producer is subject to many uncertainties such as available water and the price of electricity. The electricity price is highly volatile and presents large variations within a short time horizon. Hence, the electricity price is a determinant risk factor for future revenue. Typically, the main goal for a hydro power producer is only to maximize the expected profit over time, ignoring the rest of parameters characterizing the distribution of the profit: the so-called risk-neutral producer. However, considering the uncertainty, these market agents seek to hedge against the risk associated with price volatility: the so-called risk-averse producer. As most power producers are, in fact, risk-averse, they may prefer a lower income with high certainty rather than a higher income with low probability. Therefore, a risk measure is required for power producers in order to avoid solutions that carry with them the possibility of low profits, allowing the power producer to face the trade-off between expected profit and risk aversion.

It is known from portfolio optimization theory that risk management can be more effectively done using tools as Value-at-risk (VaR) and CVaR [3]. Still, VaR has the additional difficulty, for stochastic problems, that it requires the use of binary variables for its modeling, whereas CVaR computation does not require the use of binary variables and it can be modeled by the simple use of linear constraints. Hence, we manage risk in our approach by imposing a lower limit and confidence level on CVaR. CVaR presents a new approach to address the integrated risk management problem of a hydro producer. This risk measure has also been applied to find the optimal strategies in electricity markets [22].

For a given time horizon K and confidence level δ, CVaR is the conditional expectation of the profit above VaR. The concept of VaR and CVaR is illustrated in Fig. 1.

"See Fig. 1 at the end of the manuscript".

Value-at-Risk, ζ, is a measure defined as the maximum profit value such that the probability of the profit being lower than or equal to this value is lower than or equal to $1-\delta$:

$$VaR = \max(x \mid \mathrm{p}\{B \leq x\} \leq 1-\delta) \tag{1}$$

CVaR is the expected profit not exceeding a measure called Value-at-Risk:

$$CVaR = E(B \mid B < \zeta) \tag{2}$$

The discrete formulation is the following:

$$CVaR = \frac{1}{1-\delta} \sum\nolimits_{n \in N \ \mid B_n \leq \zeta} \rho_n B_n \tag{3}$$

An equivalent representation of CVaR can be used in order to be computationally modeled by solvers. The CVaR assumes the expression:

$$CVaR = \frac{1}{1-\delta} \sum_{n=1}^{N} \rho_n \eta_n \tag{4}$$

where

$$\eta_n = \begin{cases} B_n & if \quad B_n \leq \zeta \\ 0 & if \quad B_n > \zeta \end{cases} \tag{5}$$

The technical literature refers that δ assumes values usually between 0.90 and 0.99 [23]. In this paper, δ is equal to 0.95. Hence, CVaR can be mathematically defined as:

$$\max \quad \zeta - \frac{1}{1-\delta} \sum_{n}^{N} \rho_n \eta_n \tag{6}$$

subject to

$$-B_n + \zeta - \eta_n \leq 0 \tag{7}$$

$$\eta_n \geq 0 \tag{8}$$

The objective function (6) is composed for two terms. The first term is a variable whose optimal value is equal to VaR, and the second term corresponds to the average value of η_n. This average is determined for the η_n different from 0, where the summation is divided by the sum of the probabilities of the lower profits covered by the accumulated probability equal to 1 - δ.

In constraint (7), η_n is a variable whose value is equal to zero if the scenario n has a profit greater than ζ. For the rest of scenarios, η_n is equal to the difference of ζ and the corresponding profit. In other words, η_n provides the excess of ζ over the profit in scenario *n* if this excess is positive.

2.2 Objective function

In the STHS problem under consideration, the objective function takes into account all the price scenarios at once, weighed by their occurrence probability. In order to consider risk, the objective function (6) is added to the original objective function of the considered STHS problem using the weighting factor α. Thus, the objective function to be maximized can be expressed as:

$$F = \sum_{n=1}^{N} \rho_n B_n + \alpha \left(\zeta - \frac{1}{1-\delta} \sum_{n=1}^{N} \rho_n \eta_n \right) \tag{9}$$

where ρ_n is the probability associated to scenario *n*, α is the weighting positive factor to achieve an appropriate trade-off between profit and risk, ζ is the Value-at-Risk at a confidence level of δ, and η is an auxiliary variable used to compute the CVaR. B_n is the benefit for each price scenario, given by:

$$B_n = \sum_{k=1}^{K} \lambda_{kn} \sum_{i=1}^{I} p_{ik} \tag{10}$$

where λ_{kn} is the electricity price for scenario *n* at the period *k*, and p_{ik} is the power generation of plant *i* during the period *k*.

The optimal value of variable ζ is computed to find the maximum profit corresponding to the value of the distribution equal to (1 − δ). Hence, the variable VaR does not directly depend on B_n, but instead on the worst-scenarios derived of the profits distribution determined for each B_n.

The confidence level is also a parameter that influences the results, since increasing the value of δ, we are placing additional weight on the average of the profit outcomes below ζ, and less weight on the actual value of ζ [24].

Hence, the higher the value of δ , the higher the risk aversion of the power producer due only to its concern about extremely poor outcomes, being all effort placed on maximizing profit in these worst-case scenarios.

On the other hand, the risk-neutral case is achieved for a δ tending to 0, since the corresponding value of the CVaR is equal to the expected value of the probability distribution of profit [25].

2.3 Hydro constraints

The optimal value of the objective function is determined subject to constraints of two kinds: equality constraints and inequality constraints or simple bounds on the variables. The constraints are as follows:

$$v_{ik} = v_{i,k-1} + a_{ik} + \sum_{m \in M_i} (q_{mk} + s_{mk}) - q_{ik} - s_{ik} \tag{11}$$

$$p_{ik} = q_{ik} \, \eta_{ik} (h_{ik}) \tag{12}$$

$$h_{ik} = l_{f(i)k} (v_{f(i)k}) - l_{t(i)k} (v_{t(i)k}) \tag{13}$$

$$v_i^{\min} \le v_{ik} \le v_i^{\max} \tag{14}$$

$$u_{ik} \, q_i^{\min} \le q_{ik} \le u_{ik} \, q_i^{\max} \tag{15}$$

$$q_{ik} - R_i \le q_{i,k+1} \le q_{ik} + R_i \tag{16}$$

$$s_{ik} \ge 0 \tag{17}$$

Equation (11) corresponds to the water balance equation for each reservoir, assuming that the time required for water to travel from a reservoir to a reservoir directly downstream is less than the one hour period, independently of water discharge, due to the small distance between consecutive reservoirs. In (11) v_{ik} is the water storage of reservoir *i* at end of hour *k*, a_{ik} is the inflow to reservoir *i* in hour *k*, q_{ik} is the water discharge of plant *i* in hour *k*, s_{ik} is the water spillage by reservoir *i* in hour *k*, and M_i is the set of upstream reservoirs of plant *i*.

In (12) hydroelectric power generation, p_{ik}, is considered a function of water discharge and efficiency, η_{ik}. We consider efficiency given by the output-input ratio, depending on the head, h_{ik}.

In (13) the head is considered a function of the water levels in the upstream reservoir, denoted by $f(i)$ in subscript, and downstream reservoir, denoted by $t(i)$ in subscript, depending on the water storages in the respective reservoirs.

In (14) water storage has lower and upper bounds. Here for each reservoir i, $v_i^{\min}$ is the minimum storage, and $v_i^{\max}$ is the maximum storage.

In (15) water discharge has lower and upper bounds. Here for each reservoir i, $q_i^{\min}$ is the minimum discharge, and $q_i^{\max}$ is the maximum discharge.

In (16) discharge ramping constraints are considered, which may be imposed due to requirements of navigation, environment, and recreation [26].

In (17) a null lower bound is considered for water spillage. Normally, water spillage by the reservoirs occurs when without it the water storage exceeds its upper bound, so spilling is necessary to avoid damage. The initial water storages, v_{i0}, and the inflows to reservoirs are known input data.

Finally, constraints (7) and (8) are used to compute the CVaR. The linearization of constraint (7) is taken into account in order to solve the problem, considering at the same time the quadratic objective function that guarantees the convergence of the procedure. Hence, the original problem is formulated in our paper as one with linear constraints and quadratic objective function, which can be solved by MIQP.

3. Solution methodology

The STHS problem can be formulated as a MIPQ problem, given by:

$$Max\ \boldsymbol{F}(\boldsymbol{x}) = \boldsymbol{f}^{\mathrm{T}}\ \boldsymbol{x} + 1/2\, \boldsymbol{x}^{\mathrm{T}}\ \boldsymbol{H}\ \boldsymbol{x} \tag{18}$$

$$\text{subject to} \qquad \boldsymbol{b}^{\min} \le \boldsymbol{A}\,\boldsymbol{x} \le \boldsymbol{b}^{\max} \tag{19}$$

$$-\infty \le \boldsymbol{x}^{\min} \le \boldsymbol{x} \le \boldsymbol{x}^{\max} \le \infty \tag{20}$$

$$\boldsymbol{x}_j \text{ integer}, \quad j \in J \tag{21}$$

In (18) the function $\boldsymbol{F}(\cdot)$ is a quadratic objective function of decision variables, where $\boldsymbol{f}$ is the vector of coefficients for the linear term and $\boldsymbol{H}$ is the Hessian matrix.

In (19) $\boldsymbol{A}$ is the constraint matrix, $\boldsymbol{b}^{\min}$ and $\boldsymbol{b}^{\max}$ are the lower and upper bound vectors on constraints. Equality constraints are defined by setting the lower bound equal to the upper bound, i.e. $\boldsymbol{b}^{\min} = \boldsymbol{b}^{\max}$. In (20) $\boldsymbol{x}^{\min}$ and $\boldsymbol{x}^{\max}$ are the lower and upper bound vectors on variables. The variables $\boldsymbol{x} \in J$ are restricted to be integers. The lower and upper bounds for water discharge imply new inequality constraints that will be rewritten into (19).

In (12) the efficiency depends on the head. We consider it given by:

$$\eta_{ik} = \alpha_i \, h_{ik} + \eta_i^0 \tag{22}$$

where the parameters α_i and η_i^0 are given by:

$$\alpha_i = (\eta_i^{\max} - \eta_i^{\min}) \,/\, (h_i^{\max} - h_i^{\min}) \tag{23}$$

$$\eta_i^0 = \eta_i^{\max} - \alpha_i \, h_i^{\max} \tag{24}$$

In (13) the water level depends on the water storage. We consider it given by:

$$l_{ik} = \beta_i \, v_{ik} + l_i^0 \tag{25}$$

where the parameters β_i and l_i^0 are given by:

$$\beta_i = (l_i^{\max} - l_i^{\min}) \,/\, (v_i^{\max} - v_i^{\min}) \tag{26}$$

$$l_i^0 = l_i^{\max} - \beta_i \, v_i^{\max} \tag{27}$$

Substituting (22) into (12) we have:

$$p_{ik} = q_{ik} \, (\alpha_i \, h_{ik} + \eta_i^0) \tag{28}$$

Therefore, substituting (13) and (25) into (28), hydroelectric power generation becomes a nonlinear function of water discharge and water storage, given by:

$$p_{ik} = \alpha_i \, \beta_{f(i)} \, q_{ik} \, v_{f(i)k} - \alpha_i \, \beta_{t(i)} \, q_{ik} \, v_{t(i)k} + \delta_i \, q_{ik} \tag{29}$$

where the parameter δ_i is given by:

$$\delta_i = \alpha_i \, (l_{f(i)}^0 - l_{t(i)}^0) + \eta_i^0 \tag{30}$$

A major advantage of our novel MIQP approach is to consider the head change effect in a single function (29) of water discharge and water storage that can be used in a straightforward way, instead of deriving several curves for different heads.

Our Hessian matrix is not semi definite, but rather indefinite. If a solution is found, there is no guarantee that it is the global optimal. Instead, a local optimal is usually obtained. Therefore, we consider a starting point given by the MILP approach, and afterwards we check for an enhanced objective function value using the proposed MIQP approach. In our case study we always arrive at convergence to a better solution. The starting point provides an improvement in the efficiency of the MIQP approach, since it can be faster to get the optimal solution in terms of the computation time. Still, it should be noted that no convergence problem occurs when solving the problem directly with the MIQP model.

As new contributions to earlier studies, market uncertainty is introduced in the model via price scenarios, and risk-aversion is also incorporated through CVaR in (9). Therefore, the trade-off of maximum profit versus minimum risk is now properly addressed.

4. Case study

The proposed MIQP approach, which considers not only the head change effect, discontinuous operating regions and discharge ramping constraints, but also price scenarios and risk-aversion, has been applied on a case study based on one of the main Portuguese cascaded hydro systems. This approach has been developed and implemented in MATLAB and solved using the optimization solver package Xpress-MP. The numerical testing has been performed on a 600-MHz-based processor with 256 MB of RAM.

In restructured markets, price forecasting is extremely important for all market players. Several forecasting procedures are available for predicting electricity prices, mainly based on time series models such as autoregressive integrated moving average modeling [27], or on artificial intelligence techniques such as neural network (NN) [28], enhanced probability NN [29], neuro-fuzzy approach [30], hybrid forecast method [31] and improved hybrid model [32]. In [1], prices were considered as deterministic input data. Instead, in this paper, several prices scenarios are now considered using the NN approach.

The prices scenarios over the time horizon are shown in Fig. 2 (where $ is a symbolic economic quantity). The number of prices scenarios generated in the optimization problem is $N = 20$. This number has been selected arbitrarily, and the probability of each generated scenario will be $1/N$.

"See Fig. 2 at the end of the manuscript".

The hydro system has seven cascaded reservoirs and is shown in Fig. 3. In [21], the hydro plants data is shown.

"See Fig. 3 at the end of the manuscript".

The hydro plants numbered in Fig. 3 as 1, 2, 4, 5 and 7 are hydro plants with a small storage capacity available. The hydro plants numbered as 3 and 6 are storage hydro plants. Hence, for the storage hydro plants head-dependency may be neglected, due to the small head variation during the short-term time horizon. Inflow is considered only on reservoirs 1 to 6.

The final water storage in the reservoirs is constrained to be equal to the initial water storage, chosen as 80% of maximum storage. The storage targets for the short-term time horizon, which are established by medium-term planning studies, may be represented either by a penalty on water storage or by a previously determined ‘future cost function’ [33].

The size of the MIQP problem, expressed as the number of continuous variables, binary variables and constraints, is provided in Table 1.

"See Table 1 at the end of the manuscript".

The expected profit versus profit standard deviation is presented in Fig. 4, considering four values for α. This figure provides the maximum achievable expected profit for each level of risk or, alternatively, the minimum achievable level of risk for each expected profit.

"See Fig. 4 at the end of the manuscript".

An analysis of Fig. 4, known as efficient frontier or Markowitz frontier, reveals that for a risk-neutral producer ($\alpha = 0$), the expected profit is \$323,848 with a standard deviation of \$37,105. On the other hand, a risk-averse producer ($\alpha = 1$) expects to achieve a profit of \$314,764 with a lower standard deviation of \$29,885.

Table 2 establishes a numerical comparison of the increase in profit for several levels of risk. The maximum profit represents an increase of 2.89% corresponding to level of risk $\alpha = 0$. Hence, different hydro power producers may choose different behaviors towards risk.

"See Table 2 at the end of the manuscript".

A thorough comparison of the optimal scheduling for two levels of risk is presented thereafter, highlighting the contributions modeled in this paper.

The optimal storage trajectories and the discharge profiles for the reservoirs are shown in Fig. 5. The solid line denotes the results obtained using a level of risk $\alpha = 0$, while the dashed line denotes the results obtained using a level of risk $\alpha = 1$.

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Fig. 5 shows the different storage trajectories for both levels of risk. We verify that some different behavior is possible to be observed for some reservoirs. The results for the optimal discharge profiles are consistent with the results for the optimal storage trajectories. The risk-neutral producer aims at discharging mostly during peak-hours, obtaining maximum profit.

Table 3 shows the scheduling results for the two levels of risk in the second hydro plant of this case study.

"See Table 3 at the end of the manuscript".

The results obtained in Table 3 show that different risk levels provide a different scheduling. In time periods in which the hydro plant is online, the production is usually slightly higher for $\alpha = 0$ than for $\alpha = 1$.

Fig. 6 presents the histograms of the obtained profits for $\alpha = 0$ and $\alpha = 1$.

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Analyzing Fig. 6, it can be verified that the risk level correspondent to $\alpha = 0$ implies a higher expected profit than for $\alpha = 1$. However, $\alpha = 0$ is riskier than $\alpha = 1$, because financial loss can occur under some scenarios. Hence, our model allows the decision maker to obtain solutions according to the desired risk exposure level.

The effort made to properly model the head-dependency on the STHS problem can be quantified through the objective function value, demonstrated in Table 4 for MIQP (considering head-dependency) and MILP (ignoring head-dependency).

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The average errors associated with the linearization curves, efficiency vs. head and water level vs. water storage, are presented in Table 5. It can be seen that the average errors are relatively small, concerning the cascaded hydro system studied.

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Due to the more realistic modeling presented in this paper, considering not only the head change effect, discontinuous operating regions and discharge ramping constraints, but also but also price scenarios and risk-aversion, a better STHS is provided, assuring simultaneously a negligible computation time.

5. Conclusions

A MIQP approach is proposed in this paper to solve the STHS problem, considering head-dependency, the on-off behavior of the hydro plants, and discharge ramping constraints. As new contributions to earlier studies, price scenarios and risk-aversion are also taken into account. Numerical testing results show that the proposed approach is computationally adequate to provide solutions to the decision maker according to the desired risk exposure level. The computation time required is negligible, converging rapidly to the optimal solution. Hence, the proposed approach provides an enhanced STHS.

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Figure captions

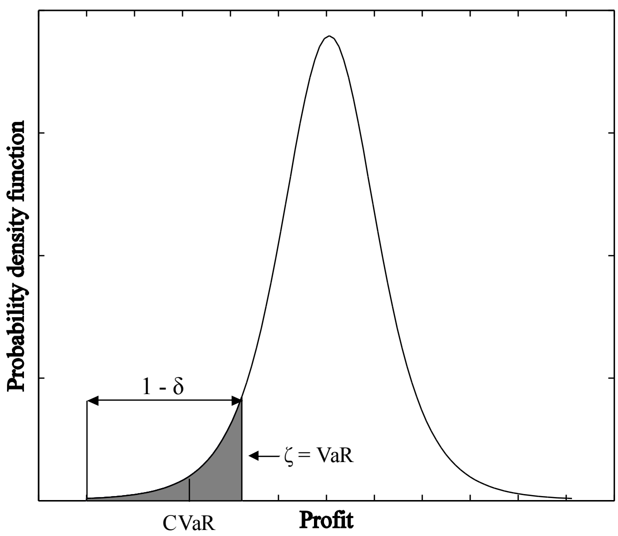

Fig. 1. VaR and CVaR illustration.

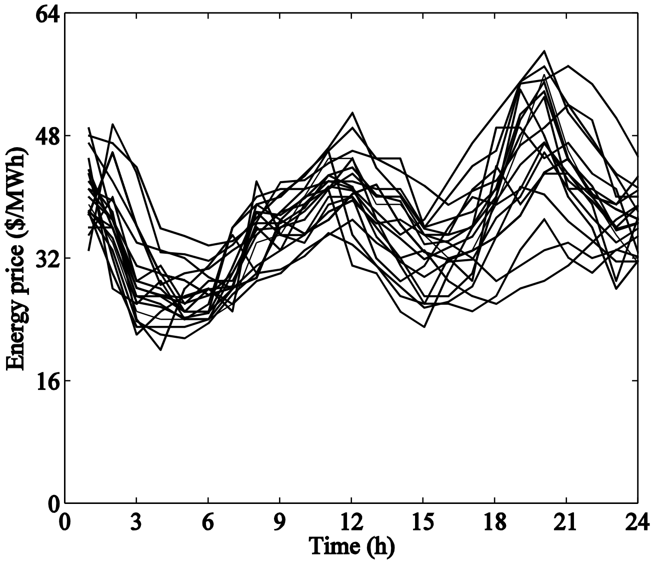

Fig. 2. Price scenarios over the 24-hours time horizon.

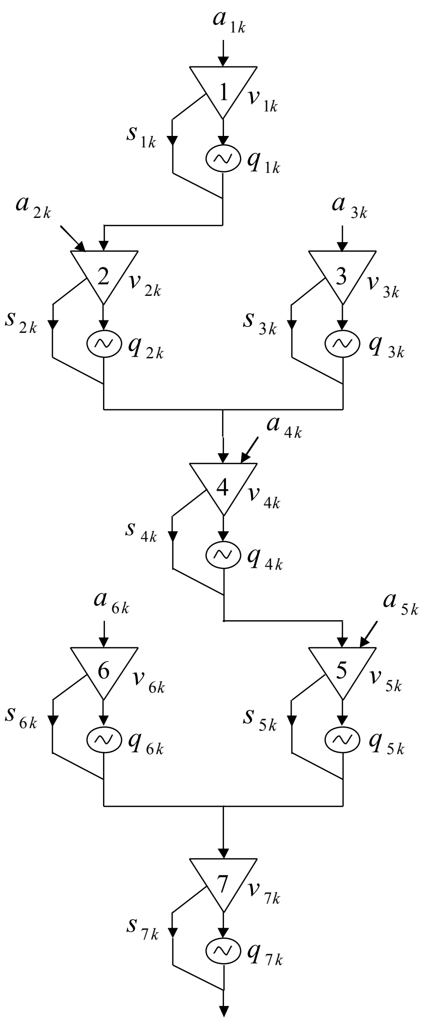

Fig. 3. Hydro system with seven cascaded reservoirs.

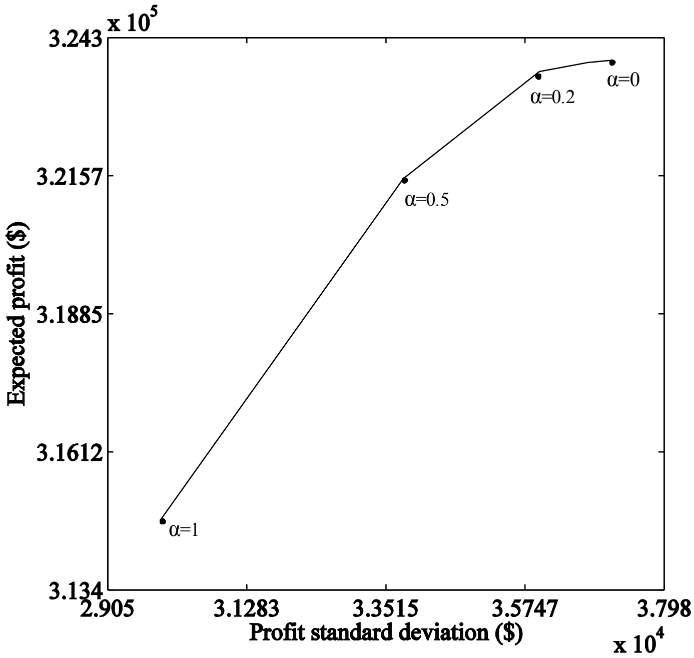

Fig. 4. Expected profit versus profit standard deviation.

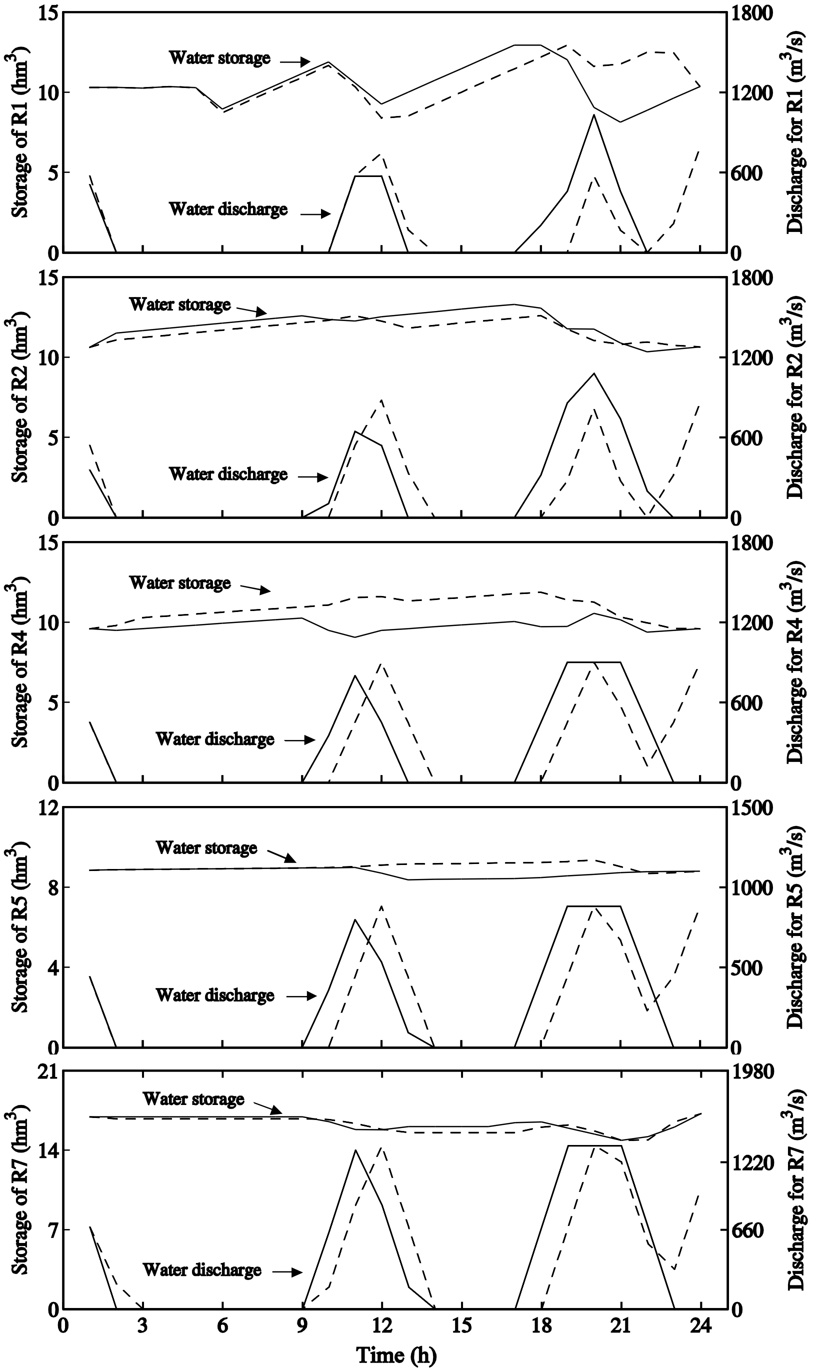

Fig. 5. Optimal storage trajectories and discharge profiles for the reservoirs. The solid line denotes the results obtained using a level of risk $\alpha = 0$, while the dashed line denotes the results obtained using a level of risk $\alpha = 1$.

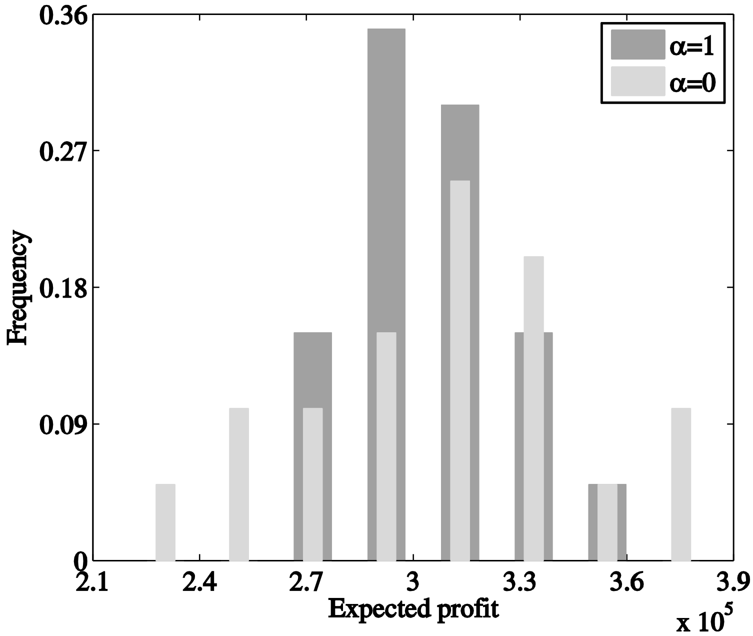

Fig. 6. Histogram of the obtained profits for levels of risk $\alpha = 0$ and $\alpha = 1$.

Tables

Table 1

Problem size

Continuous variables	$3 \cdot I \cdot K + N + 1$
Binary variables	$I \cdot K$
Constraints	$4 \cdot I \cdot K + N$

Table 2

Comparison of the increase in profit for several levels of risk

Level of risk	Profit standard deviation ($)	Expected Profit ($)	% Increase	CPU Time (s)
1.0	29,885	314,764	-	7.06
0.5	33,792	321,516	2.14	6.43
0.2	35,976	323,630	2.82	5.27
0.0	37,105	323,848	2.89	4.42

Table 3

Scheduling results for two levels of risk – Plant 2

Hour	Power generation per water discharge [MWh/hm^3]		Hour	Power generation per water discharge [MWh/hm^3]	
	$\alpha = 0$	$\alpha = 1$		$\alpha = 0$	$\alpha = 1$
1	79.46	79.06	13	0	78.86
2	0	0	14	0	0
3	0	0	15	0	0
4	0	0	16	0	0
5	0	0	17	0	0
6	0	0	18	80.04	0
7	0	0	19	79.49	78.80
8	0	0	20	79.14	78.55
9	0	0	21	78.95	78.84
10	79.84	0	22	79.03	0
11	79.97	79.09	23	0	79.11
12	79.90	78.92	24	0	79.07

Table 4

Comparison of the increase in profit considering head-dependency and $\alpha = 0$

Optimization method	Expected Profit ($)	% Increase
MILP	309,330	-
MIQP	323,848	4.69

Table 5

Linearization errors

Linearization	Average Error (%)	Std. Error (%)
Efficiency vs. Head	0.54	0.16
Water Level vs. Water Storage	0.12	0.08