Implementation of Consensus-ADMM Approach for Fast DC-OPF Studies

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This paper proposes a novel method for solving the Optimal Power Flow (OPF) problem in conditions close to real-time. The linearized cost function of the generating units is used to this end. Besides, the presented linear model is solved using the Consensus Alternating Direction Method of Multipliers (C-ADMM) approach. This technique would provide the possibility of modeling the problem both in centralized and decentralized manners. The suggested method exploits the power flow results obtained from the previous iteration to considerably improve the rate of convergence. As the C-ADMM method uses an iterative technique, Lagrange multipliers, and the norm function, the rate of convergence highly depends upon assigning the initial conditions and the optimality gap. Thus, using the operating points of the previous instant due to being close to the operating point of the current instant would enhance the results. The proposed model has been implemented on two case studies including the Pennsylvania-New Jersey-Maryland (PJM) network to verify the results and the 9-bus system to evaluate the performance of the model for the daily operation.

Keywords— Consensus Alternating Direction Method of Multipliers, Lagrangian Relaxation, Linear Programming, Optimal Power Flow.

List of symbols:

i	Index for bus
j	Index for line
NB	Total number of buses
NL	Total number of lines
PG_i	Power generation at bus i
PD_i	Demand at bus i
PL_j	Transmission line flow through line j
$f_i(PG_i)$	Generation cost function of unit i
$\Upsilon_{i,j}$	Incidence matrix
$X_{j,j}^{diag}$	Diagonal reactance matrix of transmission lines
δ_i	Voltage angle of bus i

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I. Introduction

Power flow problem is one of the most important computational tools for power system operation and planning studies. The real-time analysis and the solution time of the Optimal Power Flow (OPF) problem are of high significance in real large-scale power systems. It is noteworthy that the bigger the network, the more considerable solution time of the OPF problem would be [1]. In this regard, the OPF has been utilized by system operators over the past decades to mitigate the operating costs, the losses, and also to improve the voltage profile of the system [2-4]. Since the OPF problem is indeed a non-linear problem and includes the power flow analysis as its sub-problem, fast and flexible solution methods should be used in case of large-scale networks. Decentralized methods are considered one important solution to reduce the computational burden and improve the rate of convergence. Moreover, the decomposition-based techniques have been thus far used to solve the power flow problem [5]. This paper proposes an analytical distributed method which is iteratively solving the OPF problem. Applying this approach, the studied power system is decomposed into different areas interconnected using tie lines. Afterward, the power flow problem is tackled locally by estimating the signals of the power flow of tie-lines to globally solve the problem.

A. Motivation

The rate of convergence of the OPF problem is highly important to real large-scale power systems where there are numerous constraints relating to generating units and transmission lines. It is noted that the computational burden of the problem dramatically increases with the size of the system. Thus, the rate of convergence still remains essential to system operators. This paper solves the DC-OPF problem in a decentralized manner using the Consensus Alternating Direction Method of Multipliers (C-ADMM) technique. Using this method, the power flow problem can be solved in a decentralized fashion. In this respect, the signal estimation approach is based on the load demand prediction, while the power flow results of the previous iteration are used for the next iteration. Accordingly, the computational error is minimized.

B. Literature Review

In recent years there has been a noticeable change in the structure of electric power systems. This change has been

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seen in the systems becoming progressively more distributed [6]. This is in contrast with the traditional design and operation of these systems which have followed a centralized, top-down approach. This new paradigm of distributed operation and control of electric power grids necessitates a move away from the traditional methods used which followed a centralized approach as such approaches may not be suitable for a distributed electric power system. One of the main methods used to determine the least cost operation of electric power systems is the OPF which has been used since the early 1960s [7]. OPF includes a set of equations determining the optimal sequence of dispatching generators to minimize the cost of operation while satisfying a given load profile and other system constraints [8]. The OPF model has been traditionally implemented by a centralized authority, but as the electric power system undergoes a transition towards a more decentralized future, it is treated differently. The traditional optimization algorithms, including the OPF, are typically carried out in a centralized process. These centralized processes will encounter several issues as the electric power system undergoes a transition to being a more decentralized system. The first issue relates to the amount of data that the Distributed Energy Resources (DERs) will bring. Increasing the number of agents interacting with the electric grid will increase the amount of data that a centralized authority will need to analyze and likewise, the amount of communication between the centralized authority and the DERs will also increase dramatically. The second issue that central authorities may deal with is the type of data that the owners of the DERs are willing to share. The owners of the DERs may not want to share sensitive data related to their cost structure or other data for security concerns [9]. In order to overcome these two challenges, decentralized OPF optimization strategies have been developed. Decentralized algorithms may also be easier to scale and are more flexible to changes to the system as well as being more robust than centralized systems [8]. The decentralized optimization has been successfully used in other multi-agent systems such as aircraft and vehicle networks, wireless sensor networks, cognitive radio network spectrum sensing, state estimation and optimization of smart grids [10]. A number of decentralized optimization techniques have been discussed and used in the electric power system literature [10], [11].

A thorough review of distributed and decentralized approaches to OPF is presented in [6] where the authors discuss six decentralized/distributed algorithms which have been widely used for OPF problems. In [8] an Alternating Direction Method of Multipliers (ADMM) approach was used to solve a dynamic DC-OPF problem including the demand response programs. The ADMM approach was also followed in [10] in order to develop a general decentralized consensus optimization problem. A decentralized Economic Dispatch (EDC) problem for isolated distributed generation systems was solved in [11]. Key to this solution was a penalty function which helped the solution to converge to the global optimum using solely local interactions between the various smart grids. A decentralized DC-OPF algorithm was used in [12] to manage large interconnected power systems. This method splits the original large problem up into several smaller sub-problems and the solutions to these sub-problems are coordinated through a pricing mechanism until there is consensus.

C. *Contribution*

This paper employs the C-ADMM technique to solve the DC-OPF problem over a 24-hour period with one minute time intervals. The load demand signal in each area is used as the estimation signal and the results of the previous five-minute period are utilized to improve the rate of convergence. Furthermore, the consensus method is used for different areas. Consequently, the solution of the previous iteration is the initial point of the ADMM technique.

D. *Paper Organization*

The paper is organized as follows: Section II models the DC-OPF problem using Linear Programming (LP) and Section III proposes the modeling method of linear problems by the ADMM technique while the algorithm to solve the daily power flow has been included in Section IV. Section V represents the simulation results and some relevant conclusions are drawn in the last section.

II. DC OPTIMAL POWER FLOW PROBLEM

The OPF problem seeks to find the optimal operating points of the generating units in the power system. In this problem, it is assumed that all units are online and the binary status of the units is neglected. The decision variables of the problem are the optimal operating points of generating units and the power flow in the lines. The dependent variables are the voltage angles of buses, while the power flow of lines depends on the angles. Thus, the power flow of lines is calculated once the voltage angle of the buses is determined. To solve the problem in a decentralized manner, this paper assigns the operating points of generating units and the voltage angle of buses as the decision variables to the problem. The objective function (1) of the DC-OPF problem is the total operating costs of the system modeled as either a linear function or a non-linear function [13]. The most important constraint of the problem is the power balance constraint. The net power injection of each bus is transmitted to other buses through transmission lines as stated in (2). Eq. (3) indicates that the power flow of lines depends upon the bus voltage angle of the sending and receiving buses in addition to the reactance of the line. Moreover, constraints (5) and (6) assure that the power output of units and power flow of lines are in the permitted interval, respectively. It is worth mentioning that one of the buses is selected as the slack bus and the voltage angles of other buses are determined, accordingly as shown in (6).

$$\min_{PG_i} \sum_{i=1}^{NB} f_i(PG_i) \tag{1}$$

$$PG_i - PD_i = \sum_{j=1}^{NL} \Upsilon_{i,j} PL_j \tag{2}$$

$$\sum_{i=1}^{NB} \Upsilon_{i,j}^{T} \delta_i = \sum_{j=1}^{NL} \mathrm{X}_{j,j}^{diag} PL_j \tag{3}$$

$$-PL_j^{\max} \le PL_j \le +PL_j^{\max} \tag{4}$$

$$PG_i^{\min} \le PG_i \le PG_i^{\max} \tag{5}$$

$$\delta_{ref} = 0 \tag{6}$$

III. MATHEMATICAL MODELING OF LP-ADMM

The distributed optimization problem in the ADMM framework can be stated as (7):

$$\min_{x_1,x_2} \quad c_1(x_1)+c_2(x_2)$$
$$Subject\,to: \; A_1x_1 + A_2x_2 = b \tag{7}$$

The objective function of the problem is set to be minimized to find the optimal values of variables, x_1 and x_2. As the problem is proposed in the framework of the LP-ADMM, the global optimum can be found. The ADMM technique updates the decision variables and the Lagrange multipliers of the optimization problem.

$$\min_{x_1,x_2} \quad L_\rho = c_1(x_1)+c_2(x_2)+ \lambda^T(A_1x_1+A_2x_2-b)+\frac{\rho}{2}\|A_1x_1+A_2x_2-b\|_2^2 \tag{8}$$

$$x_1^{k+1} = \arg\min_{x_1} L_\rho(x_1,x_2^k,\lambda^k) \tag{9}$$

$$x_2^{k+1} = \arg\min_{x_2} L_\rho(x_1^{k+1},x_2,\lambda^k) \tag{10}$$

$$\lambda^{k+1} = \lambda^k + \rho\left(A_1x_1^{k+1}+A_2x_2^{k+1}-b\right) \tag{11}$$

Where λ^k are Lagrange multipliers of (8), ρ >0 is a predefined parameter and $\|\bullet\|_2$ represents the ℓ_2-norm of a vector. ADMM consists of an iterative procedure, where k is the index of ADMM iterations. Using the ADMM approach, the decision variables are separately taken into account and the problem is iteratively solved to obtain the optimal solution with the desired amount of accuracy. A specific ADMM method known as the C-ADMM technique is developed in this paper, using which a common global variable is considered for all decision variables of the problem. The common global variable, z, should be simultaneously optimized. The mathematical representation of the C-ADMM approach is as follows:

$$\min_{x_i} \quad f_i(x_i)$$
$$Subject\,to: \; x_i - z = 0 \tag{12}$$

$$L_\rho(x_1,x_2,...,x_N,z) = \sum_{i=1}^{N} f_i(x_i) - \lambda_i^T(x_i-z)+\frac{\rho}{2}\|x_i-z\|_2^2 \tag{13}$$

$$x_i^{k+1} = \arg\min_{x_i}\left\{f_i(x_i)-\left(\lambda_i^k\right)^T(x_i-z^k)+\frac{\rho}{2}\left\|x_i-z^k\right\|_2^2\right\} \tag{14}$$

$$z_i^{k+1} = \frac{1}{N}\sum_{i=1}^{N}\left(x_i^{k+1}-\frac{1}{\rho}\lambda_i^k\right) \tag{15}$$

$$\lambda_i^{k+1} = \lambda_i^k + \rho\left(x_i^{k+1}-z_i^{k+1}\right) \tag{16}$$

Where the enhanced Lagrange function is proposed in (13) and Eqs. (14)-(16) indicate the iterative process of the method to find the optimal solution with the desired accuracy. This paper utilizes the C-ADMM method to solve the DC-OPF problem where the decision variables, x_i, are the operating points of the generating units and the global common variable, z, is the voltage angle of the buses. This global common variable can be defined as the angles of the buses interconnected through tie-lines while the power generation of each area is simulated in a centralized way. The next section provides the DC-OPT problem in the C-ADMM framework.

IV. C-ADMM Approach for Solving DC-OPF

As it has been previously mentioned, the proposed DC-OPF problem is modeled in a LP convex framework. Although the fuel cost function of generating units is mostly modeled as a quadratic function, using the piecewise linearization, the fuel cost function of units can be linearized.

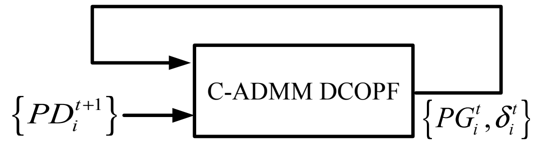

Fig. 1. The proposed method to solve the daily OPF.

The other constraints of the DC-OPF problem are as follows:

$$\left\{PG_i^k,\delta_i^{k+1}\right\} = \min\left\{C_i(PG_i)+(\lambda_i^k)^T(\delta_i-z^k)+\frac{\rho}{2}\left\|\delta_i-z^k\right\|_2^2\right\} \tag{17}$$

Subject to: (2-6)

$$z^{k+1} = \frac{1}{N}\sum_{i=1}^{N}\delta_i^{k+1} \tag{18}$$

$$\lambda_i^{k+1} = \lambda_i^k + \rho\left(\delta_i^{k+1}-z^{k+1}\right) \tag{19}$$

$$\left\|\lambda_i^{k+1}-\lambda_i^k\right\|_2^2 \le \varepsilon \tag{20}$$

$$\rho\left\|\delta_i^{k+1}-\delta_i^k\right\|_2^2 \le \varepsilon \tag{21}$$

The stopping criterion is considered to be 1e-5 in this case study. The objective function for the DC-OPF problem is solved by the C-ADMM, while set ρ=20 denotes all δ_i as the global variable z. For each area i, δ_i=z is the constraint. The updating procedure for z, δ_i, PG_i and λ_i is given as follows. Note that λ_i is the dual variable vector related to the δ_i=z constraint. It is not the same as the Local Marginal Price (LMP) price vector. It is noteworthy that the inequality constraints can be easily modeled taking into account the continuity condition of the search space. In each iteration, if the value of the decision variable is smaller than the lower bound of that variable, the value is fixed at the lower bound. In case the value is greater than the upper bound, it is fixed at the upper bound. The main aim of this paper is to propose a fast method for the DC-OPF problem applicable to large-scale power systems in conditions close to real-time. The problem is modeled and solved using a day-ahead framework with one-minute resolution. The C-ADMM method is developed to solve the problem, in which the solution of the previous iteration is used to improve the rate of convergence. Accordingly, the problem is dynamically studied. The required parameters for the estimation are the voltage angle of buses and the power output of generating units. Fig. 1 shows the procedure of solving the problem using the developed method in this paper.

V. Simulation Results

This section includes two case studies to analyze the DC-OPF problem using the C-ADMM approach. The first case study is the PJM power network presented to verify the obtained results and the second case study is a 9-bus standard test system for the daily operation study.

A. PJM Test System

The PJM system comprises five buses and six transmission lines. The results obtained for this case study are exactly the same as those reported in [14]. The amount of the load demand at buses B, C, and D is 300 MW each, and it should be noted the modified system has been utilized in [15]. The generation cost of the unit located at Sundance is changed from 30 $/MWh to 35 $/MWh.

TABLE I. TRANSMISSION LINE'S DATA -PJM TEST SYSTEM [14]

Line	1	2	3	4	5	6
Connection	**AB**	**AD**	**AE**	**BC**	**CD**	**DE**
R%	0.281	0.304	0.064	0.108	0.297	0.297
X%	2.81	3.04	0.64	1.08	2.97	2.97
Limit(MW)	999	999	999	999	999	240

TABLE II. GENERATION UNIT'S DATA-PJM TEST SYSTEM [14]

Unit	Location	Indication	PG^{max}	PG^{min}	Offer
Alta	A	1.1	110	0	14
Park City	A	1.2	100	0	15
Solitude	C	3.1	520	0	30
Sundance	D	4.1	200	0	35
Brighton	E	5.1	600	0	10

This is done so that the generation of cost of the unit would be different from that of Solitude for the sake of more suitable representation. Table I and Table II show the data of the system and Fig. 2 depicts the single-line diagram of the PJM system. Fig. 3 indicates the convergence trend of the problem while the optimality gap trend of the problem is 0.001 and ρ=1. Table III represents the comparison results.

As it can be observed from Table III, the optimal power outputs of units 4.1 and 5.1 relating to the areas Sundance and Brighton have a different value. As reported in Table II, the generation cost of unit 4.1 is greater than unit 5.1 and as the small reduction in the generation level of unit 4.1 is compensated by unit 5.1, the operating cost is obtained a little bit smaller than Ref. [14]. It is noteworthy that in the Ref. [14] CPLEX solver has been implemented for solving the DC-OPF problem.

TABLE III. GENERATION DISPATCH RESULTS-PJM TEST SYSTEM

Bus	Indication	Power (MW) CPLEX. [14]	Power (MW) C-ADMM
1	1.1	110.00	110.00
	1.2	100.00	100.00
2	-	-	-
3	3.1	0.0	0.0
4	4.1	116.079	**116.0757**
5	5.1	573.921	**573.9243**

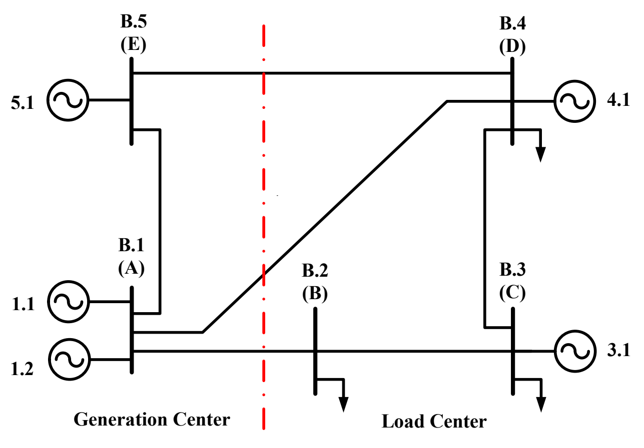

Fig. 2. The single-line diagram of the PJM system [14].

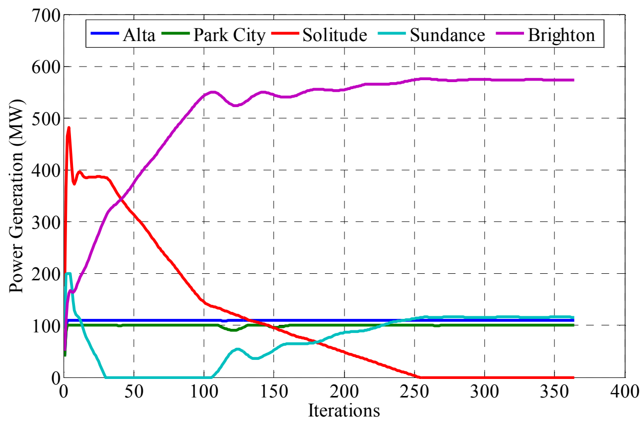

Fig. 3. Power outputs of generating units- PJM network

B. 9-Bus Standard Test System

After verifying the solutions, the proposed model is implemented on the standard 9-bus test system for the 24-hour period analysis with one-minute resolution. This test system has nine buses, nine branches, and three generation units as well as three load buses. The transmission network data of this test system is available in [16]. The generating units' data are provided in Table IV. It is noteworthy that the generation cost function is provided as a quadratic function and in this study, the linearization of the cost function has been adopted. The total segments are considered to be 200 in this study. The linearization method employed in this paper is addressed in Ref. [13]. The forecasted power for the demand buses is provided in Fig. 5. The precession of the load forecasting is one minute. Fig. 6 shows the simulation results over the 24 hours. As this figure illustrates, the power generation of the units at buses B.1 and B.3 are at the upper bound over the peak hours. In addition, the power output of the unit at bus B.3 is at the lower bound at the initial and final hours due to techno-economic reasons. The simulation results verify that the developed technique can be effectively and efficiently used for real-time studies.

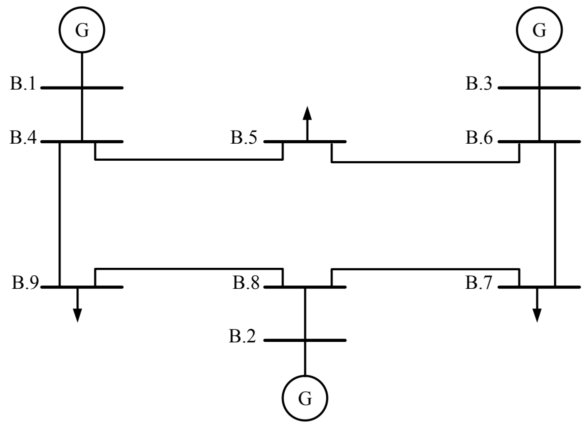

Fig. 4. The single-line diagram of the 9-bus test system.

TABLE IV. GENERATION UNIT'S DATA OF 9-BUS TEST SYSTEM

Unit	PG^{max}	PG^{min}	*a*	*b*	*c*
B.1	200	50	0.10	2.4	150
B.2	300	75	0.12	3.8	600
B.3	150	90	0.15	1.1	335

$f_i(PG_i)=a_i\,PG_i^2+b_i\,PG_i+c_i$

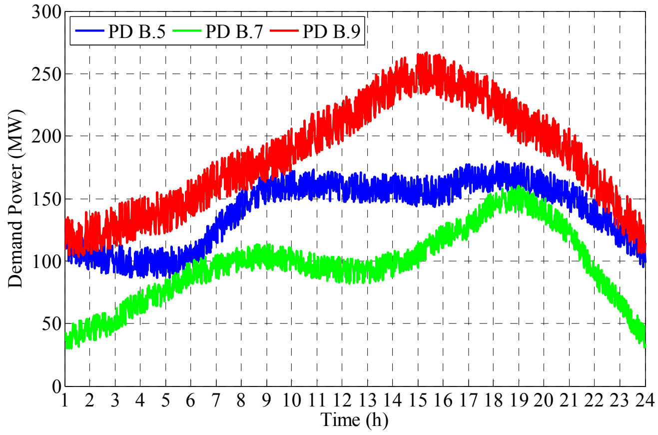

Fig. 5. Daily forecasted load demand of the 9-bus test system.

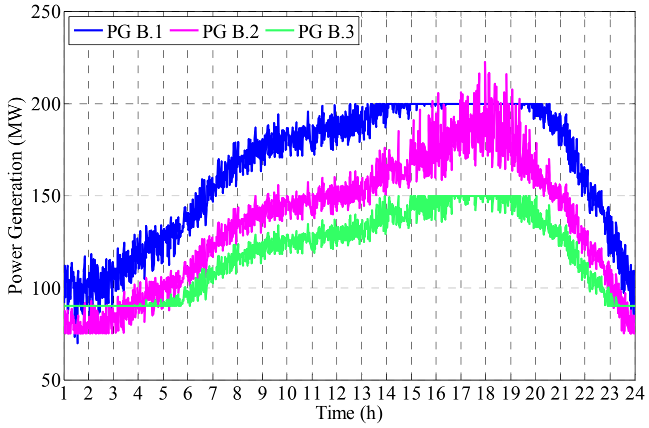

Fig. 6. The optimal generation of units- 9-bus test system.

VI. Conclusion

This paper presented the Consensus Alternating Direction Method of Multipliers (C-ADMM) to solve the DC Optimal Power Flow (DC-OPF) problem in a decentralized way. This problem was implemented in conditions close to real-time utilizing the solution of the previous iteration to improve the rate of convergence. The developed model was simulated on the PJM network to verify the results. The obtained results were more accurate compared to the CPLEX solver. Besides, the model was implemented on the standard 9-bus system over a 24-hour period with one-minute resolution which led to rational results. The proposed model can be used as a sub-problem for other power system operational purposes. For instance, this model can be used for generation scheduling, peer-to-peer transactive energy transactions, as well as unit commitment problem. The main core of the aforementioned problems is for solving the DC-OPF problem, therefore, the proposed method can be effectively incorporated as a sub-problem. Moreover, the proposed model has this potential to be implemented by the end-users with private information participating in an auction-based market. Moreover, by implementing the C-ADMM model for decentralized optimal power flow studies, the end-user private data would remain safe and the end-users can participate in the market without any concerns regarding the cybersecurity issues.

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