

# Offering Strategies for a Wind Power Producer considering Uncertainty through a Stochastic Model

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**Abstract**—This paper deals with the development of offering strategies for a wind power producer considering a stochastic model. Several scenarios are analyzed and two kinds of uncertainty are simultaneously handled: wind power and electricity market prices. The proposed approach allows evaluating production and offering strategies to be submitted to the electricity market with the goal of maximizing profits. An application to realistic case study is presented and conclusions are duly drawn.

**Keywords**—offering strategy; wind power producer; uncertainty; stochastic model

## I. INTRODUCTION

Nowadays, renewable energy sources play an increasingly important role in electricity production [1], [2], since they produce clean energy, respecting the compromise established by the Kyoto protocol. These renewable energy sources can partly replace carbon emitting fossil based electricity generation, and thereby reduce CO<sub>2</sub> emissions [3].

Wind, as a renewable energy source, has been generally applied as a means to reach emission reduction goals as a result of increasing concern regarding environmental protection [4]. Actually, wind power is the world's fastest growing renewable energy source [5].

Portugal is a country of the European Union that has highlighted this area of energy innovation, giving a strong stimulus to the national economy by creating new jobs and scientific development based on the area of electrical power systems. In Portugal, the wind power goal foreseen for 2010 was established by the government as 3750 MW, representing about 25% of the total installed capacity in 2010 [6]. This value has recently been raised to 5100 MW, by the most recent governmental goals for the wind sector.

In deregulated markets, wind power producers are entities owning generation resources and participating in the market with the ultimate goal of maximizing profits [7]. The challenges for wind power producers are related to two kinds of uncertainty: wind power and electricity market prices. The uncertain behavior of wind power [8], [9], and the large variability of electricity market prices, means a large variability in profit [10].

Thus, the decision makers have to consider these two kinds of uncertainty, as well as the several technical constraints associated to the operation of wind farms.

The offer decisions to submit for the electricity market have to be done in each hour, without knowing exactly what will be the value of power generation. The differences between the produced energy and supplied energy constitute the energy imbalances. The imbalances should be penalized by the market balance [11], [12].

A wind power producer needs to know how much to produce in order to make realistic bids, because in case of excessive or moderate bids, other producers must reduce or increase production to fill the so-called deviation, causing economic losses. These economic losses are reflected in so-called costs for deviation or costs of the imbalances. To take into account these uncertain measures, multiple scenarios can be built using wind power forecasting [13]–[15] and electricity price forecasting [16]–[18] tools. A scenario tree represents the different stages that can take the random parameters, i.e., different realizations of uncertainty. The tree is a natural and explicit way of representing nonanticipativity decisions.

The stochastic nature of the uncertain measures can be modeled through a stochastic programming approach [19]–[21]. In this approach, the set of decisions inherent to the problem can be divided into two distinct stages: first-stage decisions, which must be taken before resolving the uncertainty; second-stage decisions, which are made after the uncertainty occurs and are influenced by decisions taken in first stage. The first-stage decisions correspond to the hourly bids to be submitted to the day-ahead market, while the second-stage decisions correspond to the operation of the wind farm for each possible realization of the random variables (the electricity market price, the wind power generation, and the price for imbalance).

According with [22], the random parameters can be merged into single scenarios because there are no decision variables between the market-clearing (i.e., when price uncertainty is unveiled) and the moment in which a better wind production forecast becomes available.

Fig. 1 shows the scenario tree that will be used to represent the decisions to be taken in the two stages mentioned.

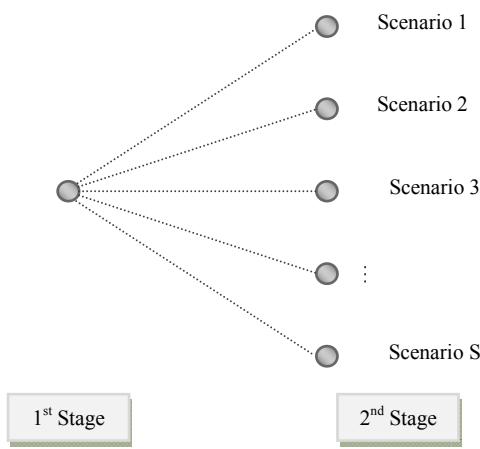


Figure 1. Scenario tree.

The root of the tree represents the first-stage decisions where the hourly bids are shared, i.e., the bids are the same for all scenarios, since they are independent of them. Therefore, they represent a robust solution to face uncertainty. Still, this solution does not have to be optimal in any particular scenario if it is considered alone, but flexible face to everyone [22]. In second-stage the decisions are made based on scenarios of prices and of wind power, spanning the whole market horizon. This stage is also defined by the materialization of the imbalance prices and the wind power generated in the time periods spanning the whole market horizon. So, the deviation incurred by the wind power producer in each one of these periods is known and the consequent cost for imbalance can be computed [7].

In this paper, a stochastic model is used to generate the optimal offers that should be submitted to the day-ahead market by a wind power producer, in order to maximize its expected profit.

This paper is structured as follows. In Section II, the mathematical formulation of the problem is provided. Section III presents the proposed stochastic model. In Section IV, the proposed stochastic model is applied on a realistic case study, to demonstrate its effectiveness. Finally, Section V outlines the conclusions.

## II. PROBLEM FORMULATION

### A. Nomenclature

The notation used throughout the paper is stated as follows.

$S, s$	set and index of scenarios
$H, h$	set and index of hours in the time horizon
$\rho_s$	probability of occurrence of scenario $s$
$\lambda_{sh}$	expected market price in scenario $s$ in hour $h$
$r_{sh}^+$	ratio between positive imbalance price and day-ahead market price in scenario $s$ in hour $h$
$r_{sh}^-$	ratio between negative imbalance price and day-ahead market price in scenario $s$ in hour $h$

$p_{sh}$	power output of the wind farm in scenario $s$ in hour $h$
$v$	cost factor over the market price for energy imbalances
$x_h$	offer by the wind power producer in the day-ahead market for time hour $h$
$dev_{sh}$	deviation for wind production in scenario $s$ in hour $h$
$Pdev_{sh}$	cost for deviation of the wind farm in scenario $s$ in hour $h$
$W_{sh}$	wind generation forecast in scenario $s$ in hour $h$
$\bar{P}$	maximum power of the wind farm
$L_{sh}$	revenue in scenario $s$ in hour $h$
$c$	vector of coefficients for the linear term for first-stage variables
$\mathbf{x}$	vector of decision variables
$A$	matrix of coefficients for the first-stage constraints
$\underline{b}, \bar{b}$	lower and upper bound vectors for the first-stage constraints
$\underline{\mathbf{x}}, \bar{\mathbf{x}}$	lower and upper bound vectors on variables
$\underline{\mathbf{h}}_\omega, \bar{\mathbf{h}}_\omega$	lower and upper bound vectors for the second-stage constraints
$T_\omega$	technology matrix
$W_\omega$	recourse matrix
$q_\omega$	vector of coefficients for the linear term for the second-stage variables
$y_\omega$	second-stage variables that represent decisions to be made after part of the uncertainty is revealed

### B. Objective Function

The objective function to be maximized is expressed as:

$$F = \sum_{s=1}^S \rho_s \sum_{h=1}^H [\lambda_{sh} p_{sh} - Pdev_{sh}] \quad (1)$$

The objective function (1) to be maximized includes the expected profit, where  $S$  is the set of scenarios,  $\rho_s$  is the probability of occurrence of scenario  $s$ ,  $H$  is the set of hours in the time horizon,  $\lambda_{sh}$  is the forecasted electricity market price in scenario  $s$  in period  $h$ ,  $p_{sh}$  is the power output of the wind farm in scenario  $s$  in period  $h$ , and  $Pdev_{sh}$  is the cost for deviation of the wind farm in scenario  $s$  in period  $h$ .

The deviations can be generated by excess or deficit of energy:

$$dev_{sh} = p_{sh} - x_h \quad (2)$$

The cost for deviation corresponds to the product of the price for the shifted power in absolute value:

$$Pdev_{sh} = \begin{cases} \lambda_{sh} r_{sh}^+ dev_{sh}, & dev_{sh} \geq 0 \\ \lambda_{sh} r_{sh}^- dev_{sh}, & dev_{sh} < 0 \end{cases} \quad (3)$$

The revenue is given by the product of the expected energy market price by the power output of the wind farm:

$$L_{sh} = \lambda_{sh} p_{sh} \quad (4)$$

The expected profit is calculated as the difference between the revenue of the wind farm and the cost for deviation.

Substituting (3) into (1) gives:

$$F = \sum_{s=1}^S \rho_s \sum_{h=1}^H [\lambda_{sh} p_{sh} - \lambda_{sh} r_{sh}^+ d_{sh}^+ - \lambda_{sh} r_{sh}^- d_{sh}^-] \quad (5)$$

### C. Constraints

The total deviation (imbalance)  $dev_{sh}$  is decomposed as the sum of positive and negative imbalances,  $d_{sh}^+$  and  $d_{sh}^-$ , respectively.

For a total deviation  $dev_{sh} = d_{sh}^+ - d_{sh}^-$ , the optimal solution is guaranteed to be achieved with one of the variables  $d_{sh}^+$  or  $d_{sh}^-$  equal to zero, due to the fact that  $r_{sh}^+ \leq 1$  and  $r_{sh}^- \geq 1$ :

$$p_{sh} - x_h - d_{sh}^+ + d_{sh}^- = 0 \quad (6)$$

In order to make the offers to the market, it is required to satisfy the technical limitations of the wind farm. So, the optimal value of the objective function is determined subject to inequality constraints or simple bounds on the variables.

The constraints are indicated as follows:

$$0 \leq d_{sh}^+ \leq W_{sh} \quad (7)$$

$$0 \leq d_{sh}^- \leq \bar{P} \quad (8)$$

Constraints (7) and (8) impose caps on the positive and negative deviations, respectively. Wind power is limited superiorly by the value of the forecasted wind power production,  $W_{sh}$ , in scenario  $s$  in period  $h$ .

In (7) the maximum positive deviations occur in scenarios where the wind power producer does not offer any amount of wind power in the day-ahead market for period  $h$ , but it eventually produces wind power  $W_{sh}$  during that period.

In (8) maximum negative deviations occur in scenarios where the wind power producer offers its full capacity in the day-ahead market,  $x_h = \bar{P}$ , for period  $h$ , but its final production in that period is  $W_{sh}$ .

The offers are limited by the maximum power installed in the wind farm  $\bar{P}$ :

$$0 \leq x_h \leq \bar{P} \quad (9)$$

### D. Linearization of the Objective Function

The objective function, presented in the previous subsection, is characterized by nonlinearity due to the existence of the absolute value. So, it is required to use a mathematical process that allows reformulating the linear problem.

In this subsection, the problem involving absolute value terms is transformed into a standard linear programming formulation.

Initially, it is considered:

$$\text{Max } F = c^T x - |x| \quad (10)$$

$$\text{subject to } \underline{x} \leq x \leq \bar{x} \quad (11)$$

$$x \in R^n \quad (12)$$

In (10),  $F(\cdot)$  is the objective function of decision variables, where  $c$  is the vector of coefficients for the linear term. In (11),  $\underline{x}$  and  $\bar{x}$  are the lower and upper bound vectors on variables. The variable  $x$  is a set of decisions variables. Subsequently, absolute-valued variables are replaced with two strictly positive variables:

$$|x| = x^+ + x^- \quad (13)$$

In addition, each variable is substituted by the difference of the same two positive variables, as:

$$x = x^+ - x^- \quad (14)$$

The equivalent linear programming problem is given by:

$$\text{Max } F = c^T x - (x^+ + x^-) \quad (15)$$

$$\text{subject to } \underline{x} \leq x \leq \bar{x} \quad (16)$$

$$x = x^+ - x^- \quad (17)$$

$$x^+ \geq 0, x^- \geq 0 \quad (18)$$

## III. PROPOSED APPROACH

Stochastic programs are among the most challenging optimization problems. The stochasticity in the parameters appears in this approach due to the uncertainty, which is modeled via a finite set of scenarios.

### A. Stochastic Programming

The stochastic programming model can be formulated as:

$$\text{Max } c^T x + E[\max_{y_\omega} q_\omega^T y_\omega] \quad (19)$$

$$\text{subject to } \underline{b} \leq Ax \leq \bar{b} \quad (20)$$

$$\underline{h}_\omega \leq T_\omega x + W_\omega y_\omega \leq \bar{h}_\omega \quad (21)$$

$$x \geq 0, y_\omega \geq 0. \quad (22)$$

where  $c$  is a known vector of the objective function coefficients for the  $x$  variables in the first-stage,  $\underline{b}, \bar{b}$  are the lower and upper bound vectors for the first-stage constraints, and  $A$  is the known matrix of coefficients for the first-stage constraints. For each  $\omega$ ,  $\underline{h}_\omega, \bar{h}_\omega$  are the vectors for the second-stage constraints, and  $q_\omega$  is the vector of the objective function coefficients for the  $y$  variables, while  $T_\omega$  is the technology matrix and  $W_\omega$  is the recourse matrix under scenario  $\omega$ . Finally,  $E_{y_\omega}$  represents the expectation with respect to  $\omega$  over the set of scenarios  $S$ . In the second-stage, the constraints and right-hand sides are permitted to be random. The matrix that links the random parameters related to first-stage variables is called technology matrix.

In the first-stage, the “here-and-now” decision should be taken, before the uncertainties represented by  $x$  are known. In the second-stage, where the information  $x$  is already available, the decision is made about the value of the vector  $y$ . The first-stage decision of  $x$  depends only on the information available until that time; this principle is called nonanticipativity constraint. The problem of two stages means that the decision  $x$  is independent of the achievements of the second-stage, and thus the vector  $x$  is the same for all possible events that may occur in the second-stage of the problem. The first-stage decisions, corresponding to  $x$ , are the hourly bids to be submitted to the day-ahead market and the second-stage decisions, corresponding to  $y$ , are related to the operation of the wind farm for each possible realization of the random variables.

### B. Deterministic Equivalent Programming

The stochastic model is usually a difficult computational problem, so it is common to choose the deterministic model solution using the average of random variables or solving a deterministic problem for each scenario.

The problem shown in the previous subsection is equivalent to the so-called deterministic equivalent programming that in the splitting variable representation is as follows:

$$\text{Max}_{x,y_s} \quad c^T x + \sum_{s=1}^S \rho_s q_s^T y_s \quad (23)$$

subject to

$$\underline{b} \leq Ax \leq \bar{b} \quad (24)$$

$$\underline{h}_s \leq T_s x + W_s y_s \leq \bar{h}_s \quad \text{for } s = 1, \dots, S \quad (25)$$

$$x \geq 0, \quad y_s \geq 0. \quad \text{for } s = 1, \dots, S \quad (26)$$

## IV. CASE STUDY

The proposed approach has been developed and implemented in MATLAB and solved using the optimization solver package CPLEX. The numerical simulation has been performed on a 2-GHz based processor with 2GB of RAM.

The proposed approach has been applied on a realistic case study, based on a wind farm in Portugal located in the Vila Real region. The total installed wind power capacity of the plant is 270 MW.

Two different approaches have been implemented:

- Approach 1. Considering the cost factor over the market price for energy imbalances;
- Approach 2. Considering the ratio between imbalance price and day-ahead market price.

In the first approach, the deviation cost has been fixed at 30% of the daily market price,  $\nu = 0.3$ . The number of scenarios generated for the day-ahead market in the optimization problem is  $N = 100$ . The second approach takes into account the uncertainty not only regarding wind power and electricity market prices, but also regarding the imbalance price ratio scenarios. Thus, the total number of scenarios generated in the second approach is  $N = 1000$ .

Imbalance pricing provides an incentive for market participants to execute their programs as planned, while at the same time it provides an incentive for market participants to respond to the secondary control signal received from the system operator.

The time horizon chosen is one day divided into 24 hourly periods. The electricity market prices scenarios are computed by the approach proposed in [16], Fig. 2, while the wind power scenarios are computed by the approach proposed in [15], Fig. 3.

The optimal bids for the two approaches are shown in Fig. 4. The dashed lines denote the results for the first approach, while the solid lines denote the results for the second approach.

According to Fig. 4, it can be seen that the optimal offers for the first approach are higher than for the second approach, since imbalance prices are simply modeled as proportional to the spot market price.

Choosing one scenario of the problem, it can be verified in Figs. 5 and 6 that the wind farm adjusts its production in order to reduce its own imbalance costs which may in turn lead to an increased system balancing efficiency.

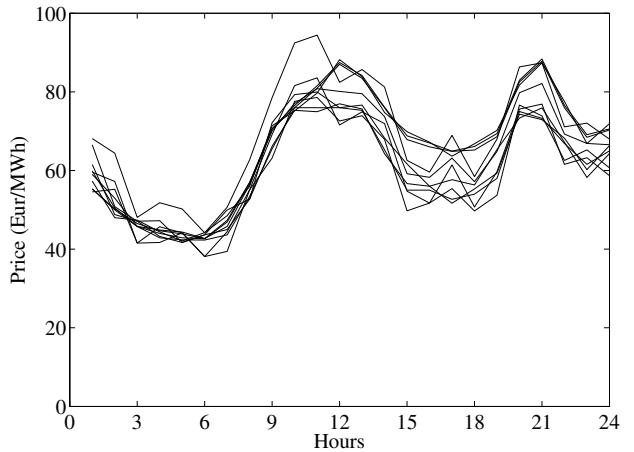


Figure 2. Electricity market price scenarios.

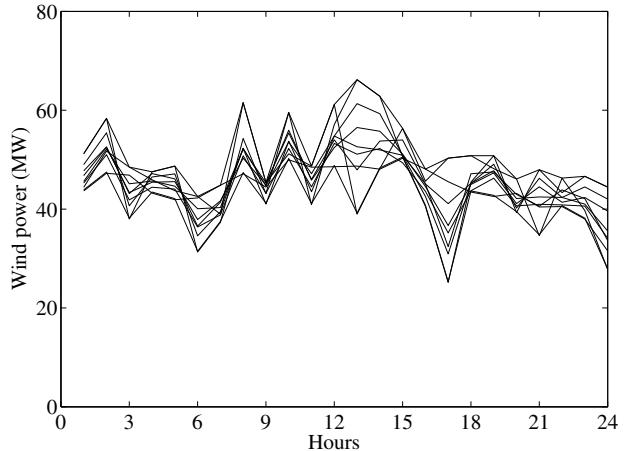


Figure 3. Wind power scenarios.

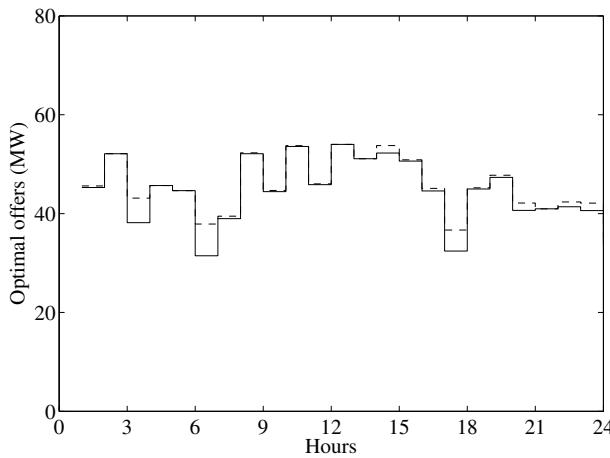


Figure 4. Optimal hourly bids. The dashed lines denote the results for the first approach, while the solid lines denote the results for the second approach.

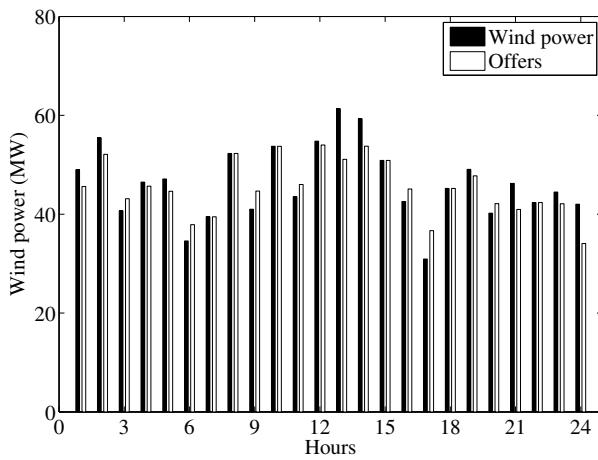


Figure 5. Optimal offers to be submitted to the day-ahead market and power produced, for the first approach.

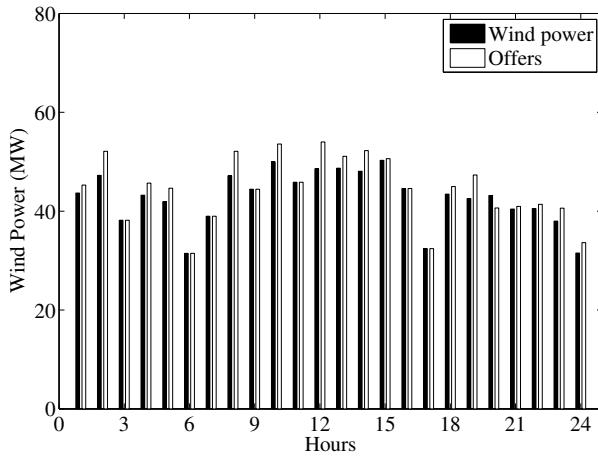


Figure 6. Optimal offers to be submitted to the day-ahead market and power produced, for the second approach.

Nevertheless, in almost every hour there are differences between the offers and the power output of the wind farm.

The consideration of uncertain information on wind power forecasts, to estimate the imbalance costs, reveals more realistic results considering the rules of the electricity market.

The deviations from generated power, for this scenario, are shown in Figs. 7 and 8 for the first and second approaches, respectively.

Table I provides the expected value of profit for both approaches.

Although the profit may be higher for the first approach, it should be noted that the second approach provides more realistic results. Hence, the profit is overestimated if we consider the first approach.

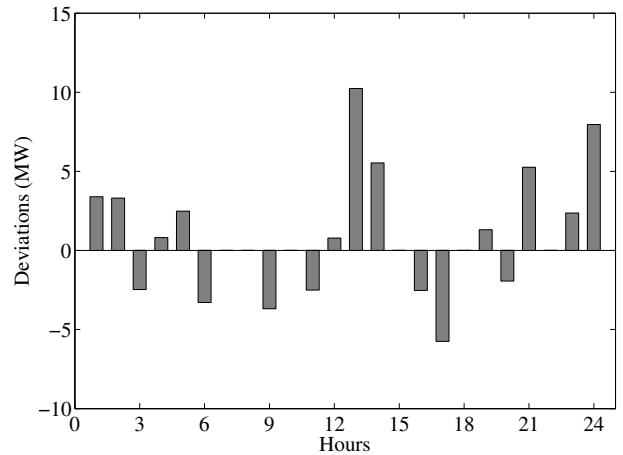


Figure 7. Deviations resulting from the difference between the power produced and the offers, for the first approach.

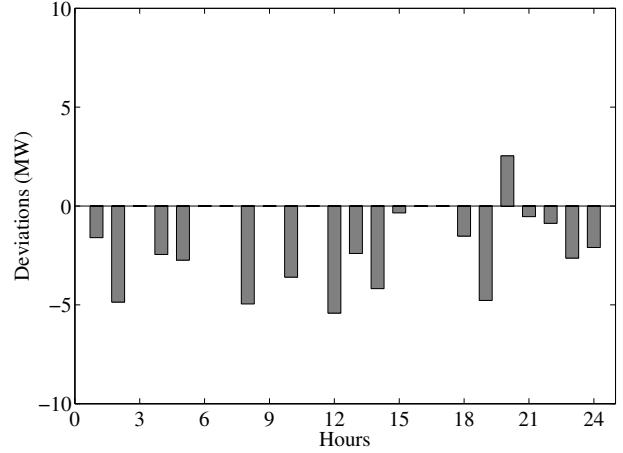


Figure 8. Deviations resulting from the difference between the power produced and the offers, for the second approach.

TABLE I. COMPARISON OF RESULTS

Proposed Approach	Expected profit (Eur)	CPU Time (s)
Approach 1	68598	0.78
Approach 2	65438	1.43

## V. CONCLUSIONS

The goal in this paper is to maximize the profit of a wind power producer reducing deviations and taking into account the uncertainty associated with wind power production and electricity market prices, using a stochastic model. A realistic case study is provided based on a wind farm in Portugal. Also, two different approaches have been implemented and compared, considering the cost factor over the market price for energy imbalances, or considering the ratio between imbalance price and day-ahead market price. This second approach provides more realistic results, while the profit can be overestimated using the first approach. Hence, it has been shown that the correct modeling of imbalance prices is very important for the optimal participation of wind power producers in electricity markets.

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## REFERENCES

- [1] J. P. S. Catalão, H. M. I. Pousinho, and V. M. F. Mendes, "Mixed-integer nonlinear approach for the optimal scheduling of a head-dependent hydro chain," *Electr. Power Syst. Res.*, vol. 80, pp. 935–942, 2010.
- [2] J. L. Angarita, J. Usaola, and J. Martinez-Crespo, "Combined hydro-wind generation bids in a pool-based electricity market," *Electr. Power Syst. Res.*, vol. 79, pp. 1038–1046, 2009.
- [3] E. D. Delarue, P. J. Luickx, and W. D. D'haeseleer, "The actual effect of wind power on overall electricity generation costs and CO<sub>2</sub> emissions," *Energ. Convers. Manage.*, vol. 50, pp. 1450–1456, 2009.
- [4] S. Kamalinia, M. Shahidehpour, and A. Khodaei, "Security-constrained expansion planning of fast-response units for wind integration," *Electr. Power Syst. Res.*, vol. 81, pp. 107–116, 2011.
- [5] M. H. Albadi, and E. F. El-Saadany, "Overview of wind power intermittency impacts on power systems," *Electr. Power Syst. Res.*, vol. 80, pp. 627–632, 2010.
- [6] A. Estanqueiro, R. Castro, P. Flores, J. Ricardo, M. Pinto, R. Rodrigues, and J. P. Lopes, "How to prepare a power system for 15% wind energy penetration: the Portuguese case study," *Wind Energy*, vol. 11, pp. 75–84, 2008.
- [7] J. M. Morales, A.J. Conejo, and J. Pérez-Ruiz, "Short-Term Trading for a Wind Power Producer," *IEEE Trans. Power Syst.*, vol. 25, pp. 554–564, 2010.
- [8] G. K. Toh, and H. B. Gooi, "Incorporating forecast uncertainties into EENS for wind turbine studies," *Electr. Power Syst. Res.*, vol. 81, pp. 430–439, 2011.
- [9] J. Valenzuela, and J. Wang, "A probabilistic model for assessing the long-term economics of wind energy," *Electr. Power Syst. Res.*, vol. 81, pp. 853–861, 2011.
- [10] G. B. Shrestha, B. K. Kokharel, T.T. Lie, and S-E Fleten, "Medium Term Power Planning With Bilateral Contracts," *IEEE Trans. Power Syst.*, vol. 20, pp. 627–633, 2005.
- [11] M. Shahidehpour, H. Yamin, and Z. Li, "Market Operations in Electric Power Systems - Forecasting, Scheduling and Risk Management," Wiley, 2002.
- [12] F. Bourry, L. M. Costa, and G. Kariniotakis, "Risk-Based Strategies for Wind/Pumped-Hydro Coordination under Electricity Markets," *Proc. IEEE Bucharest Power Tech Conf.*, Bucharest, Romania, 2009.
- [13] S. Fan, J.R. Liao, R. Yokoyama, L. N. Chen, and W. J. Lee, "Forecasting the wind generation using a two-stage network based on meteorological information," *IEEE Trans. Energy Convers.*, vol. 24, pp. 474–482, 2009.
- [14] A. Kusiak, H. Y. Zheng, and Z. Song, "Wind farm power prediction: A data-mining approach," *Wind Energy*, vol. 12, pp. 275–293, 2009.
- [15] J. P. S. Catalão, H. M. I. Pousinho, and V. M. F. Mendes, "An artificial neural network approach for short-term wind power forecasting in Portugal," *Eng. Int. Syst.*, vol. 17, pp. 5–11, 2009.
- [16] J. P. S. Catalão, S. J. P. S. Mariano, V. M. F. Mendes, and L. A. F. M. Ferreira, "Short-term electricity prices forecasting in a competitive market: a neural network approach," *Electr. Power Syst. Res.*, vol. 77, pp. 1297–1304, 2007.
- [17] L. D. Coelho, and A. A. P. Santos, "A RBF neural network model with GARCH errors: Application to electricity price forecasting," *Electr. Power Syst. Res.*, vol. 81, pp. 74–83, 2011.
- [18] N. Amjadi, and A. Daraeepour, "Mixed price and load forecasting of electricity markets by a new iterative prediction method," *Electr. Power Syst. Res.*, vol. 79, pp. 1329–1336, 2009.
- [19] F. Bourry, J. Juban, L. M. Costa, and G. Kariniotakis, "Advanced strategies for wind power trading in short-term electricity markets," *Proc. European Wind Energy Conf.*, Brussels, Belgium, 2008.
- [20] A. Tuohy, E. Denny, P. Meibom, and M. O'Malley, "Benefits of Stochastic Scheduling for Power Systems with Significant Installed Wind Power," *Proc. IEEE PMAPS*, Puerto Rico, 2008.
- [21] V. S. Pappala, I. Erlich, K. Rohrig, and J. Dobschinski, "A Stochastic Model for the Optimal Operation of a Wind-Thermal Power System," *IEEE Trans. Power Syst.*, vol. 24, pp. 940–950, 2009.
- [22] J. Garcia-González, R. M. R. Muela, L.M. Santos, and A. M. González, "Stochastic Joint Optimization of Wind Generation and Pumped-Storage Units in an Electricity Market," *IEEE Trans. Power Syst.*, vol. 23, pp. 460–468, 2008.