A Control Technique for Operation of Single-Phase Converters in Stand-alone Operating Mode

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***Abstract*—A droop-Lyapunov based control technique using direct-quadrature (d-q) rotating frame dynamic model is presented in this paper for the frequency and voltage magnitude regulation of a stand-alone single-phase voltage-source inverter (SPVSI). Steady-state and dynamic performance of the controller are analyzed based on the d-q frame model and direct Lyapunov method respectively to satisfy control aims and system stability as operation criteria. To further clarify the operation area of the inverter, positive and negative maximum values for d-q components of inverter current are acquired by introducing a capability curve (CC) for entire operating condition. The performance of the proposed control technique is evaluated numerically in the MATLAB/Simulink environment. The simulation results validate the capability of the proposed control method in both steady-state and transient responses.**

Keywords—stand-alone; single-phase voltage-source inverter; direct Lyapunov method; Droop Control; capability curve.

I. INTRODUCTION

To date, various types of power electronic converters with different topologies are used to provide high quality power exchange feasibility in a controlled manner for renewable energy sources based distributed generation (DG) such as photovoltaic (PV) and wind-based generation systems.

The single-phase voltage-source inverters (SPVSIs) that can operate either in grid-connected or stand-alone mode are considered as a proper solution for the integration of renewable energy sources into the grid or local loads. In the stand-alone operation that grid is not present, SPVSI based DG system is responsible to supply local loads by regulation of the frequency and magnitude of output voltage as the main control aims.

Therefore, numerous control methods for SPVSIs aiming to attain fast dynamic response as well as zero steady-state error have been presented in recent years [1-2]. In [3], two different control structures and a straightforward design methodology based on the internal model of the inverter in a microgrid are presented. In another study, a synchronous reference frame, based multi-loop control strategy is designed for single phase inverter-based DG systems operating in island mode [4]. The parabolic current control with fast transient response and constant switching frequency is applied to a single-phase stand-alone inverter in the next study [5]. Hysteresis current control with dynamically adaptive bands, are considered in [6-8] in order to allow SPVSIs to track reference currents with a fixed switching frequency and achieve a higher quality. In [9], an alternative method for the current regulation of single-phase voltage-source converters in direct-quadrature (d-q) synchronous reference frames is proposed.

In this paper, a droop-Lyapunov based control technique [10-12] is proposed for a stand-alone SPVSI, which is designed with respect to the dynamic model of inverter developed in the d-q rotating frame. The dynamic model and direct Lyapunov method are employed to analyze the stability of the controller performance in the steady-state and dynamic operations respectively. Moreover, a capability curve (CC) is introduced to specify positive and negative maximum values for the d-q components of inverter current. Matlab/Simulink is utilized to verify the proper operation of the proposed controller in both steady-state and dynamic operating modes.

II. MATHEMATICAL MODEL OF THE PROPOSED STAND-ALONE SINGLE-PHASE INVERTER

Fig. 1 shows the proposed system in detail. As apparent, it contains a SPVSI with output resistance and inductance, *R* and *L* respectively. Either through dc-dc or ac-dc converters, the renewable energy sources such as PV array or wind turbine system (WTS) generate a dc-link voltage. The dc-link has a current of i_{dc} and a capacitor of C_{dc}. Also a filter capacitor, *C*, is employed to regulate the output ac voltage. Based on Fig.1, the dynamic model of the proposed system is achieved as,

$$
\begin{aligned}
&\frac{di_o}{dt} = -\frac{R}{L}i_o - \frac{1}{L}v_o - \frac{v_{dc}}{L}u \\
&\frac{dv_o}{dt} = \frac{i_o}{C} - \frac{v_o}{Z_l C} \\
&C_{dc}\frac{dv_{dc}}{dt} = i_o u + i_{dc}
\end{aligned}
\quad (1)
$$

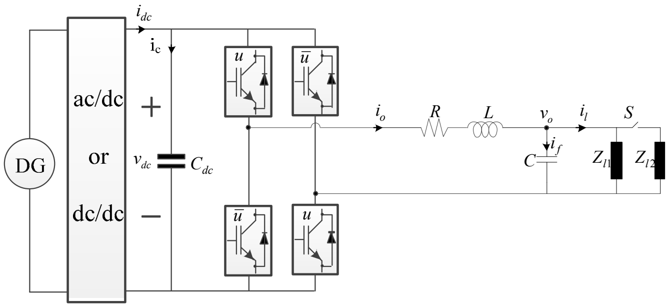

Fig. 1. The proposed stand-alone SPVSI.

where i_o and v_o are the inverter output current and voltage, respectively. Also, u and Z_l are the inverter switching function and load impedance, respectively. Assuming the real value of system state variable to be,

$$x_r = x_m \sin(\omega t + \varphi) \tag{2}$$

where x_m, is the peak value of the state variable, φ is the initial phase and ω is the angular frequency. Ideally corresponding imaginary orthogonal system state variable can be equal to,

$$x_i = x_m \cos(\omega t + \varphi) \tag{3}$$

Based on (2) and (3), the d-q frame transformation matrix is stated as,

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = \begin{bmatrix} \sin(\omega t) & \cos(\omega t) \\ \cos(\omega t) & -\sin(\omega t) \end{bmatrix} \begin{bmatrix} x_r \\ x_i \end{bmatrix} \tag{4}$$

Initially, the differential equation of Eq. (1) can be written in the real-imaginary coordinate as,

$$\frac{d}{dt}\begin{bmatrix} i_{or} \\ i_{oi} \\ v_{or} \\ v_{oi} \\ v_{dcr} \\ v_{dci} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ i_{dcr}/C_{dc} \\ i_{dci}/C_{dc} \end{bmatrix} + \begin{bmatrix} -\frac{R}{L} & 0 & -\frac{1}{L} & 0 & -\frac{u_r}{L} & 0 \\ 0 & -\frac{R}{L} & 0 & -\frac{1}{L} & 0 & -\frac{u_i}{L} \\ \frac{1}{C} & 0 & -\frac{1}{Z_l C} & 0 & 0 & 0 \\ 0 & \frac{1}{C} & 0 & -\frac{1}{Z_l C} & 0 & 0 \\ \frac{u_r}{C_{dc}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{u_i}{C_{dc}} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{or} \\ i_{oi} \\ v_{or} \\ v_{oi} \\ v_{dcr} \\ v_{dci} \end{bmatrix} \tag{5}$$

where $v_{dc(ri)}$ and $i_{dc(ir)}$ are due to the effects of real and imaginary components on the proposed system dc-link voltage and current respectively. As dc link voltage and current have to be maintained constant in both states, therefore $v_{dc(ri)} = v_{dc}$ and $i_{dc(ri)} = i_{dc}$.

As a result, the mathematical model extracted from the inverter shown in Fig.1 in a d-q frame can be expressed as:

$$\frac{d}{dt}\begin{bmatrix} i_d \\ i_q \\ v_d \\ v_q \\ v_{dc} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ i_{dc}/C_{dc} \end{bmatrix} + \begin{bmatrix} -\frac{R}{L} & -\omega & -\frac{1}{L} & 0 & -\frac{u_d}{L} \\ \omega & -\frac{R}{L} & 0 & -\frac{1}{L} & -\frac{u_q}{L} \\ \frac{1}{C} & 0 & -\frac{1}{CZ_l} & -\omega & 0 \\ 0 & \frac{1}{C} & \omega & -\frac{1}{CZ_l} & 0 \\ \frac{u_d}{2C_{dc}} & \frac{u_q}{2C_{dc}} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ v_d \\ v_q \\ v_{dc} \end{bmatrix} \tag{6}$$

The above-mentioned relations are used to design the controller which is presented in the following sections.

III. Control Technique

In this section, the proposed control method is reviewed and analyzed in details in order to demonstrate its capability in appropriate regulation of the inverter frequency and voltage. Also, a stability assessment of designed controller is accomplished in order to guarantee its accurate performance in steady-state and dynamic transitions.

A. Steady and dynamic states analysis

Reaching zero state variable errors is the main objective of the proposed controller in presence of dynamic changes. The state variable errors in the proposed system are defined as,

$$x_1 = i_d - i_d^*, x_2 = i_q - i_q^*, x_3 = v_d - v_d^* \\ , x_4 = v_q - v_q^*, x_5 = v_{dc} - v_{dc}^* \tag{7}$$

To achieve globally asymptotical stability for the proposed system in the dynamic operating conditions, direct Lyapunov method (DLM) is used. Thus, total saved energy of the proposed system due to the state variable errors can be expressed as,

$$H(x_1, x_2, x_3, x_4, x_5) = \frac{1}{2}Lx_1^2 + \frac{1}{2}Lx_2^2 + \\ \frac{1}{2}Cx_3^2 + \frac{1}{2}Cx_4^2 + C_{dc}x_5^2 \tag{8}$$

By the time derivative of (8),

$$\dot{H}(x_1, x_2, x_3, x_4, x_5) = L\dot{x}_1 x_1 + L\dot{x}_2 x_2 + C\dot{x}_3 x_3 \\ + C\dot{x}_4 x_4 + 2C_{dc}\dot{x}_5 x_5 \tag{9}$$

The switching function of SPVSI include two steady and dynamic parts as,

$$u_{dq} = u_{dq}^* + \Delta u_{dq} \quad (10)$$

Using (6), the steady-state parts of the inverter switching functions are derived as,

$$u_d^* = \left(-L\dot{i}_d^* - Ri_d^* - \omega L i_q^* - v_d^*\right) / v_{dc}^* \quad (11)$$

$$u_q^* = \left(-L\dot{i}_q^* - Ri_q^* + \omega L i_d^* - v_q^*\right) / v_{dc}^* \quad (12)$$

Considering (6) and the state variable errors definition in (7), various terms of (9) are obtained as:

$$\begin{aligned}
L\dot{x}_1 &= -Rx_1 - \omega L x_2 - x_3 - u_d^* x_5 - \Delta u_d v_{dc} \\
L\dot{x}_2 &= -Rx_2 + \omega L x_1 - x_4 - u_q^* x_5 - \Delta u_q v_{dc} \\
C\dot{x}_3 &= -\omega C x_4 + x_1 - x_3 / Z_l \\
C\dot{x}_4 &= \omega C x_3 + x_2 - x_4 / Z_l \\
2C_{dc}\dot{x}_5 &= u_d^* x_1 + \Delta u_d i_d + u_q^* x_2 + \Delta u_q i_q + 2\left(i_{dc} - i_{dc}^*\right)
\end{aligned} \quad (13)$$

By substituting (13) into (9), (14) is obtained as,

$$\begin{aligned}
\dot{H}(x_1, x_2, x_3, x_4, x_5) &= -Rx_1^2 - Rx_2^2 - x_3^2 / Z_l \\
&- x_4^2 / Z_l + \Delta u_d \left(i_d x_5 - v_{dc} x_1\right) \\
&+ \Delta u_q \left(i_q x_5 - v_{dc} x_2\right) + 2x_5 \left(i_{dc} - i_{dc}^*\right)
\end{aligned} \quad (14)$$

As per direct Lyapunov method the proposed system can meet a stable performance during both dynamic and steady-state operating conditions, if the total energy of the system and its time derivative become definitely positive and negative respectively. Thus, based on DLM and (14), the inverter dynamic switching functions can be presented as:

$$\Delta u_d = \alpha\left(i_d x_5 - v_{dc} x_1\right) \quad (15)$$

$$\Delta u_q = \beta\left(i_q x_5 - v_{dc} x_2\right) \quad (16)$$

where α and β are negative constant values. In addition, in a relatively appropriate dc-link voltage generation process, the dc link current is close to its reference and consequently the value of $2x_5\left(i_{dc} - i_{dc}^*\right)$ in (14) is negligible.

B. Calculation of reference currents

In order to avoid the increase of instability margins of the proposed system and divergence of steady-state errors, a process for precise reference current calculation is specified in this section. As can be seen in Fig.1, the inverter output current can be written as,

$$i_o = i_f + i_l \quad (17)$$

Eq. (17) can be rewritten in d-q frame as,

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} i_{fd} \\ i_{fq} \end{bmatrix} + \begin{bmatrix} i_{ld} \\ i_{lq} \end{bmatrix} = \begin{bmatrix} C\dfrac{dv_d}{dt} - \omega C v_q \\ C\dfrac{dv_q}{dt} + \omega C v_d \end{bmatrix} + \begin{bmatrix} i_{ld} \\ i_{lq} \end{bmatrix} \quad (18)$$

The relation of (18) shows that the SPVSI currents in d-q frame can be regulated by appropriately setting inverter output voltage and also considering the load currents in d-q frame. Thus, the proposed reference currents for SPVSI are achieved as,

$$i_{dq}^* = \left(k_{pdq} + \frac{k_{idq}}{s}\right) x_{(34)} + \left(1 - LPF\right) i_{ldq} + I_{dq1} \quad (19)$$

k_{pdq} and k_{idq} are the proportional and integral coefficients of PI controller related to inverter output voltage. A low pass filter (LPF) is employed to extract available harmonic components of the load as demonstrated in (19). In addition, I_{dq1} is associated with the fundamental frequency component of the inverter output current in d-q frame. The inverter output current at fundamental frequency can be given as,

$$I_{dq1} = \gamma I_{ldq1} \quad (20)$$

I_{ldq1} are d-q components of the load current at fundamental frequency. Equation (20) shows that SPVSI can generate a fraction (γ) of load current at fundamental frequency. Because of the stand-alone operation mode of SPVSI, the inverter output current capacity is considered as such to generate the whole load current at fundamental frequency that is $\gamma = 1$.

C. Droop controller

In order to obtain the instantaneous values of SPVSI output frequency and voltage magnitude, a new droop controller based on SPVSI d-q component currents is presented in this section. The conventional droop controller equation can be written as,

$$\begin{aligned}
\omega &= \omega^* - m_p P \\
E &= E^* - m_q Q
\end{aligned} \quad (21)$$

ω^* and E^* are the desired values of angular frequency and voltage magnitude of the inverter, respectively. m_p and m_q are the conventional droop coefficients of frequency and voltage magnitude respectively. In the d-q reference frame, the instantaneous SPVSI active and reactive power can be given as,

$$\begin{aligned}
p &= v_d i_d + v_q i_q \\
q &= v_q i_d - v_d i_q
\end{aligned} \quad (22)$$

Considering stable operation of the proposed system and also the fundamental frequency of the SPVSI currents, P and Q can be written as,

$$P = v_m i_{d1}, \mathrm{Q} = -v_m i_{q1} \quad (23)$$

where v_m and i_{dq1} are the desired magnitude of the inverter voltage and the fundamental frequency components of SPVSI currents, respectively. By substituting (23) into (21), the new droop equation can be obtained as,

$$\omega = \omega^* - m_p' i_{d1} \quad (24)$$
$$E = E^* + m_q' i_{q1}$$

where $m_p' = m_p v_m$ and $m_q' = m_q v_m$.

IV. Capability Curve of SPVSI

The capability of SPVSI in generation of d-q component currents is limited and this is discussed in this section in details.

By obtaining the SPVSI switching functions from the two first terms of (6), they can be presented as

$$u_d = -\left(LI_{avd} + Ri_d + \omega Li_q + v_d\right)/v_{dc} \quad (25)$$
$$u_q = -\left(LI_{avq} + Ri_q - \omega Li_d + v_q\right)/v_{dc}$$

By substituting (25) into the last term of (6) and also after some simplifications, (26) can be achieved as,

$$\left(i_d + \frac{LI_{avd} + v_d}{2R}\right)^2 + \left(i_q + \frac{LI_{avq} + v_q}{2R}\right)^2 = \frac{\left(LI_{avd} + v_d\right)^2 + \left(LI_{avq} + v_q\right)^2 + 8Ri_{dc}v_{dc} - 8RC_{dc}\bar{v}_{dc}v_{dc}}{4R^2} \quad (26)$$

I_{avdq} and $\bar{v}_{dc}$ are average values of the inverter currents and dc-link voltages, respectively. Equation (26) shows that the operation area of SPVSI currents are dependent of the inverter parameters, dc-link characteristics and also the average value of d-q components of the inverter current. Based on (26), positive and negative maximum values of d-q components of inverter currents are specified in Fig.2.

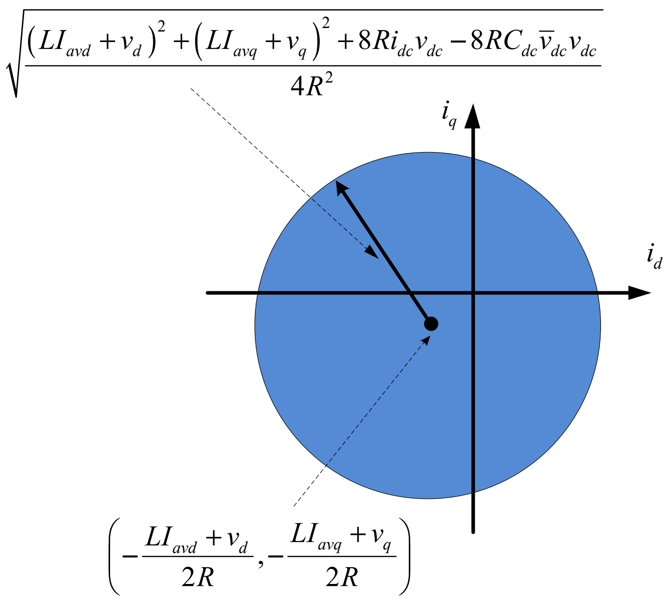

Fig. 2. The capability curve of SPVSI.

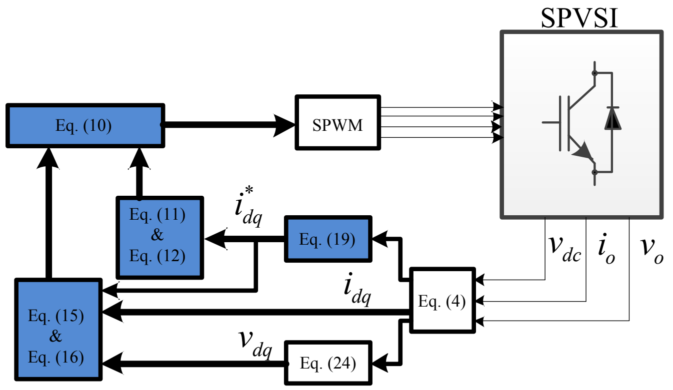

Fig. 3. The overall structure of the proposed controller.

V. Simulation and Results

To verify the effectiveness of the proposed control technique for the stand-alone SPVS, a detailed controller model controller is implemented in the MATLAB/Simulink as depicted in Fig. 3. The model parameters are given in Table I.

In order to evaluate the performance of the SPVSI through the proposed controller in dynamic and steady-state operation, a load step change is applied to the PCC. Initially, the load at the PCC is adjusted. Then, at t=0.3s it is stepped up by adding an additional load.

TABLE I
SIMULATION MODEL PARAMETERS

Parameter	*Value*
Load 1	20+j0.3
Load 2	30+j0.15
dc-link voltage set-point (v_{dc})	110 V
ac voltage	90 V
Fundamental frequency	50 Hz
Switching/Sampling frequency	10 kHz
SPVSI resistance	0.1 mΩ
SPVSI inductance	10 mH
α	0.001
β	0.0001

A. Active and reactive power sharing

Active and reactive power sharing of SPVSI, loads and AC filter are shown in Fig.4. When SPVSI acts in the first step, the total active power of load is generated by the inverter and AC filter reactive power is used to maintain the PCC voltage at its desired value. Tracking the required active and reactive power of integrated load at t=0.3s is entirely accomplished by both SPVSI and AC filter as illustrated in Fig.4. As evident in Fig.4, the oscillations and deviations of responses during transitions are negligible that verify the stable operation of the proposed control system during both steady and dynamic states.

B. Regulation of SPVSI voltage and current

In the proposed system shown in Fig.1, the frequency and magnitude of the PCC voltage have to be set to their reference values. Fig.5 illustrates the PCC voltage and its frequency and voltage magnitude during considered operation scenario. As can be seen in this figure, in the steady-state period, which is occurring after a short transient time, the proper performance of SPVSI leads to a desired regulation of PCC voltage. After augmentation of load at t=0.3s, the proposed control method is able to keep the PCC voltage at relatively reference values as depicted in Fig.5.

Although, a slight overshoot happens in the time of load change, this is not a noticeable incident. The load, ac filter and SPVSI current are shown in Fig.6. As evident in this figure, the SPVSI current is able to suitably follow the load current in both steady and dynamic states. To sum up this section, it can be said that the simulation results show the accuracy of the achieved dynamic model and ability of the proposed controller at reaching stable operation for SPVSI in both steady-state and dynamic condition.

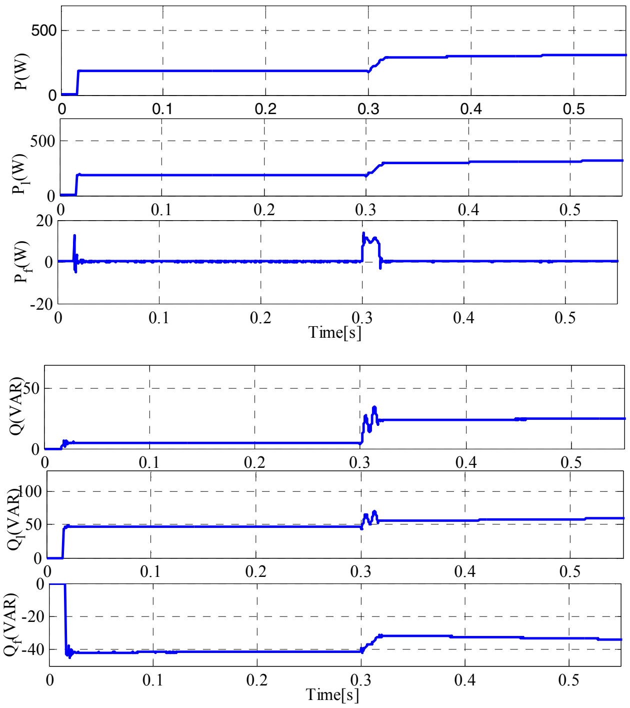

Fig. 4. Active and reactive power of SPVSI, load and AC filter.

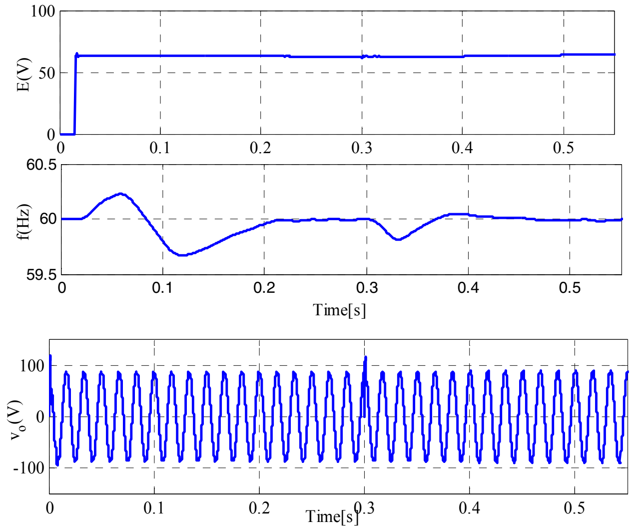

Fig. 5. SPVSI voltage and its magnitude and frequency.

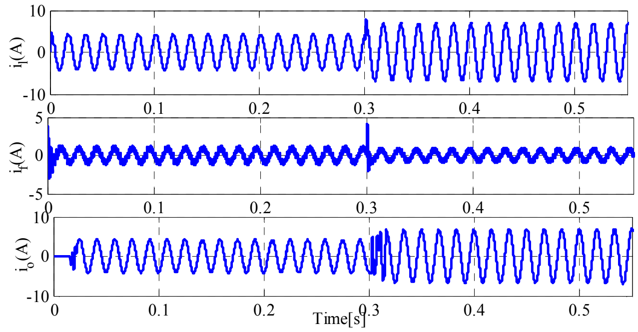

Fig. 6. Load,ac filter and SPVSI currents.

VI. Conclusion

A dynamic model based on Direct-Quadrature (DQ) rotating frame and a droop-Lyapunov based control technique was presented in this paper for a stand-alone single-phase voltage-source inverter (SPVSI). In order to evaluate the performance of the controller versus the main control objectives, including desired frequency and voltage magnitude, the steady-state and dynamic performances were analyzed based on the d-q frame model and direct Lyapunov method, respectively. In addition, the positive and negative maximum values for the d-q components of inverter currents were discussed by introducing a capability curve for the entire operating condition. Finally, to assess the steady-state performance and the transient response of the proposed controller, the MATLAB simulations are carried out under load change conditions. Simulation results indicate that the designed controller based on the proposed dynamic model is able to make stable operation for SPVSI in both steady-state and dynamic operating conditions.

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