Optimization of Prosumer's Flexibility Taking Network Constraints into Account

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***Abstract*—The integration of new technologies at the residential level such as energy storage systems, electric vehicles, solar photovoltaic generation and mini wind turbines triggered the appearance of a new agent in the power systems called prosumers. This agent has the potential to provide new forms of flexibility and cost-effective solutions. However, associated with these new solutions there are also a number of problems that affect these solutions, particularly network constraints. This work presents an analysis not only on the benefits of utilizing the prosumer's flexibility but also to the problems associated with the operation and optimization of the network. A new model is presented that considers energy transactions between prosumers in the neighborhood and between them and the network using on a stochastic framework, in order to account for a set of uncertainties in the form of scenarios associated with the availability of various resources and technologies. The results show the economic benefit of energy transactions between prosumers resulting in more flexibility for the system while highlighting the effect of network restrictions and potential problems associated with them.**

Keywords*—*Prosumers flexibility, neighborhood, power losses, network constraints, stochastic optimization

I. NOMENCLATURE

A. Sets/Indices

t, Ω^t	Index/Set of hours of the day
s, Ω^s	Index/Set of scenarios
w, Ω^w	Index/Set of prosumers
c, Ω^c	Index/Set of controllable appliances
f, Ω^f	Index/Set of phases of the controllable appliances

B. Parameters

$\eta_w^{EV,ch}$	Charging efficiency of the wth Electric Vehicle (EV)
$\eta_w^{EV,dch}$	Discharging efficiency of the wth EV
$\eta_w^{ESS,ch}$	Charging efficiency of the wth ESS
$\eta_w^{ESS,dch}$	Discharging efficiency of the wth ESS
$\phi_{w,t}$	Inflexible load of the wth prosumer at time t
$\mathcal{P}_{w,t}^{PV}$	Power produced by the PV panels of the wth prosumer at time t
$\mathcal{P}_{w,t}^{wind}$	Power produced by the Wind turbines of the wth prosumer at time t
$\mu_w^{EV,Ch}$	Charging rate of the wth EV
$\mu_w^{EV,dch}$	Discharging rate of the wth EV
$\mu_w^{ESS,ch}$	Charging rate of the wth Energy Storage System (ESS)
$\mu_w^{ESS,dch}$	Discharging rate of the wth ESS
$SOC_w^{EV,ini}$	Initial SOC of the EV of the wth prosumer
$SOC_w^{EV,max}$	Maximum SOC of the EV of the wth prosumer
$SOC_w^{EV,min}$	Minimum SOC of the EV of the wth prosumer
$SOC_w^{ESS,ini}$	Initial SOC of the ESS of the wth prosumer
$SOC_w^{ESS,max}$	Maximum SOC of the ESS of the wth prosumer
$SOC_w^{ESS,min}$	Minimum SOC of the ESS of the wth prosumer
a_w	Arrival period of the EV of wth prosumer
d_w	Departure period of the EV of wth prosumer
λ_t^{buy}	Price at which power is bought at time t
λ_t^{sell}	Price at which power is sold at time t
$\mathcal{P}_{w,t,s,c}^{appliance}$	Power consumed by controllable appliance c of the wth prosumer while in period t in scenario s
δ_{w_1,w_2}	the loses of the transfer between prosumer w_1 and prosumer w_2
$\bar{\delta}_{w_1,w_2}$	Limits of the transfer between prosumer w_1 and prosumer w_2
Δt	Time interval

C. Variables

$\mathcal{J}_{w,t}$	Total input power of the wth prosumer at time t
$\mathcal{L}_{w,t}$	Total load of the wth prosumer at time t
$\mathcal{O}_{w,t}$	Total output power of the wth prosumer at time t
$\mathcal{S}_{w,t}$	Total sold power of the wth prosumer at time t
$PS_{w,t}^{PV}$	Portion of the PV-generated power sold back to the grid
$PU_{w,t}^{PV}$	Portion of the PV-generated power used by the prosumer
$PS_{w,t}^{wind}$	Portion of the wind-generated power sold back to the grid
$PU_{w,t}^{wind}$	Portion of the wind-generated power used by the prosumer
$\psi_{w,t}^{pros,in}$	Total power input to the wth prosumer
$\psi_{w,t}^{pros,out}$	Total power output of the wth prosumer
$PC_{w,t}^{EV}$	Power charged by the EV
$PD_{w,t}^{EV}$	Power discharged by the EV
$PS_{w,t}^{EV}$	Portion of the EV-generated power sold back to the grid
$PU_{w,t}^{EV}$	Portion of the EV-generated power used by the prosumer
$SOC_{w,t}^{EV}$	State of charge of the wth EV at time t

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$PC_{w,t}^{ESS}$	Power charged by the ESS
$PD_{w,t}^{ESS}$	Power discharged by the ESS
$PS_{w,t}^{ESS}$	Portion of the ESS-generated power sold back to the grid
$PU_{w,t}^{ESS}$	Portion of the ESS-generated power used by the prosumer
$SOC_{w,t}^{ESS}$	State of charge of the wth ESS at time t
$PU_{w,t,c}^{applianc}$	Power consumed by controllable appliance c of the wth prosumer while in period t
$\psi_{w,t}^{from\ grid}$	Power bought from the grid
$\psi_{w,t}^{to\ grid}$	Power sold to the grid
$\psi_{w_1,w_2,t}^{pros,to,pros}$	Power exchanged from prosumer w_1 to prosumer w_2 at time t
$x_{w,t}^{EV}$	Binary variable; 1 if the wth EV is charging at time t, otherwise 0
$x_{w,t}^{ESS}$	Binary variable; 1 if the wth ESS is charging at time t, otherwise 0
$x_{s}^{appliance}$	Binary variable; 1 if the sth scenario is the optimal one, otherwise 0

II. INTRODUCTION

Electric power systems are undergoing a significant transition towards systems that are more decarbonized, decentralized and digitalized [1]. This shift is occurring for a number of reasons including measures to reduce the carbon intensity of the sector, cost reductions in renewable energy technologies (such as solar photovoltaic and wind), increase in the computing ability and connectedness of various devices and the desire for consumers to take a more active role in the energy sector [2].

The shift to an electric power system which is heavily reliant on renewable energy sources (some of which have variable power output for example PV and wind) requires that the system becomes more flexible in order to account for the fluctuations in power output while operating in a safe, secure and reliable manner.

Traditionally, flexibility services have been provided by the supply side of the power system by changing the electricity produced by different generators. However, there has been a major increase in attention being paid to the concept of flexibility services being delivered by the demand side of the power system which also includes flexibility services provided by prosumers (those consumers who also produce electricity) [2].

As the electric power system evolves, a rise in active consumer participation is being seen. This change has seen consumers move from passive customers to active consumers (and in some cases prosumers). The increased consumer participation is key if future energy systems are to be designed with the consumer at the heart of the system. Harnessing the flexibility of prosumers can increase the level of participation in consumer-centric energy markets [1]. Sources of flexibility help to balance supply and demand of electricity but they can also contribute to various ancillary services such as voltage profile rectification and frequency control and this can assist distribution system operators as it may allow them to delay costly investments in upgrading network infrastructure [3].

All these factors mean that it will be crucial to correctly quantify the magnitude and timing of flexibility services that can be offered to the electric grid with a major focus on quantifying prosumer generated flexibility services.

A. *Literature review*

There is a growing body of research dedicated to unlocking the potential of prosumers to provide flexibility services to the local distribution grid. In this section, some of them are introduced and discussed.

A model which uses an aggregator to group a number of small prosumers together in order to participate in the energy and ancillary services market is developed in [2]. A model proposed by [4] uses metaheuristic techniques to forecast a multi-period flexibility schedule for prosumers in low voltage networks.

A system to allow prosumers to schedule, bid for and share energy is developed by [5]. This model used a number of different types of Distributed Energy Resources (DERs) such as PV panels, smart appliances, battery energy storage systems. The model used a Mixed-Integer Nonlinear programming model to develop an optimal solution.

A tecno-economic study of prosumer energy storage systems (ESS) was conducted by [6]. The paper used a MILP model to minimize the costs of importing electricity from the grid. Results show that significant savings were achieved for the prosumers, mainly through the use of PV panels for self-generation. Implementation of ESS reduced costs by between 22 and 30% and increased self-consumption by up to 30%.

A Peer-2-Peer energy trading system was developed by [7] which used a double auction market to promote trading amongst peers to unlock small-scale flexibility. Another prosumer energy trading model is proposed by [8] where a double auction method is used in conjunction with a fully decentralized model to help coordinate DERs within a microgrid in order to lower costs, increase self-consumption while preserving the privacy of the participants.

A system which incorporates a two-stage framework for the sharing of energy between prosumers and an energy retailer using a combination of renewable energy sources, energy storage systems and shiftable loads was proposed by [9].

The first stage of the framework made use of a bilevel scheduling model in order to develop a robust schedule for energy sharing while the second stage optimizes the energy use schedule of the prosumers through online optimization while taking the state of the system into account.

A control system using transactive energy concepts is developed by [10] and the authors evaluate a mixture of tariff regimes. Findings from the paper show that transactive energy control can help to extend the life of transformers and reduce active power losses in the system.

B. *Contributions*

The main contributions of this paper are the following:

- An improved mixed-integer linear programming (MILP) operational model considering prosumers flexibility jointly with grid constraints;
- An extensive operational analysis of a system considering energy transactions between prosumers in the neighborhood and between them and the network which aims to assess the benefits from harnessing prosumer's flexibility but also addressing problems associated with the operation and optimization of the network.

III. MATHEMATICAL FORMULATION

A. Objective Function

This work minimizes the power costs for all prosumers and maximizes their profit. The total cost for prosumer *w* at time *t* can be calculated by the product of the buying price λ_t^{buy} and the amount of power bought externally (neither generated, nor bought by any other prosumer in the network) $\vec{\psi}_{w,t}^{grid}$. This treats the power received from the other prosumers $\sum_{\tilde{w}} \psi_{w,\tilde{w},t}^{pros,to,pros}$ as a fixed price. Thus, the market structure followed in this paper prioritizes the energy exchange among the prosumers over buying power externally from the rest of the network. Similarly, the total profit can be calculated by the product of λ_t^{sell} and $\psi_{w,t}^{to\,grid}$.

$$\min \sum_{w,t} \Big(\big(\lambda_t^{buy} * \psi_{w,t}^{from\,grid} * \Delta t\big) - \big(\lambda_t^{sell} * \psi_{w,t}^{to\,grid} * \Delta t\big)\Big) \quad \forall w \in \Omega^w, \forall t \in \Omega^t \tag{1}$$

B. Constraints

1) Power Balance

Constraint (2) ensures that at all times and for all prosumers, the total input power $\mathcal{J}_{w,t}$ is equal to the total load $\mathcal{L}_{w,t}$. The total input power consists of the input from the grid (prosumers on the network and externally) $\psi_{w,t}^{pros,in}$ plus the power received from the PV panels, wind turbines, EVs and ESSs (3). The total load consists of the inflexible load, the charging of both EVs and ESSs and the entire load of the home appliances (4).

$$\mathcal{J}_{w,t} = \mathcal{L}_{w,t} \quad \forall w \in \Omega^w, \forall t \in \Omega^t \tag{2}$$

$$\mathcal{J}_{w,t} = \psi_{w,t}^{pros,in} + PU_{w,t}^{PV} + PU_{w,t}^{wind} + PU_{w,t}^{EV} + PU_{w,t}^{ESS} \quad \forall w \in \Omega^w, \forall t \in \Omega^t \tag{3}$$

$$\mathcal{L}_{w,t} = \phi_{w,t} + PC_{w,t}^{EV} + PC_{w,t}^{ESS} + \sum\nolimits_c PU_{w,t,c}^{appliance}, \quad \forall w \in \Omega^w, \forall t \in \Omega^t \tag{4}$$

Equation (5) ensures that for all periods and for all prosumers the total output power $\mathcal{O}_{w,t}$ is equal to the total sold $\mathcal{S}_{w,t}$. The total output power is the power sold to the grid (prosumers on the network and externally) for profit (6). The total power sold is the summation of the power sold of the excess generation of the PV panels and wind turbines and the excess power stored in EVs and ESSs (7). Excess power is either sold to the grid or stored in the ESS for later use.

$$\mathcal{O}_{w,t} = \mathcal{S}_{w,t}, \quad \forall w \in \Omega^w, \forall t \in \Omega^t \tag{5}$$

$$\mathcal{O}_{w,t} = \psi_{w,t}^{to\,grid}, \quad \forall w \in \Omega^w, \forall t \in \Omega^t \tag{6}$$

$$\mathcal{S}_{w,t} = PS_{w,t}^{PV} + PS_{w,t}^{wind} + PS_{w,t}^{EV} + PS_{w,t}^{ESS}, \quad \forall w \in \Omega^w, \forall t \in \Omega^t \tag{7}$$

2) Grid and Losses

Constraints (8) and (9) define the 2 variables: Total power input $\psi_{w,t}^{pros,in}$ and Total power output $\psi_{w,t}^{pros,out}$. The total power input is the summation of all the power obtained from other prosumers on the network $\sum_{\tilde{w}} \psi_{w,\tilde{w},t}^{pros,to,pros}$, multiplied by $\delta_{w,\tilde{w}}$, to account for line losses, plus the power received externally $\psi_{w,t}^{from\,grid}$. Similarly, the total power output $\psi_{w,t}^{pros,out}$ is the summation of all the power the prosumer sends to other prosumers on the network, $\sum_{\tilde{w}} \psi_{w,\tilde{w},t}^{pros,to,pros}$, multiplied by $\delta_{w,\tilde{w}}$ to account for line loses, plus the power sold externally $\psi_{w,t}^{to\,grid}$. The formulation of the loss functions have been derived from [11].

$$\psi_{w_1,t}^{pros,in} = \sum\nolimits_{w_2}\big(\psi_{w_1,w_2,t}^{pros,to,pros} * \delta_{w_1,w_2}\big) + \psi_{w_1,t}^{from\,grid}, \quad \forall t \in \Omega^t, \forall w_1 \in \Omega^w \tag{8}$$

$$\psi_{w_2,t}^{pros,in} = \sum\nolimits_{w_1}\big(\psi_{w_1,w_2,t}^{pros,to,pros} * \delta_{w_1,w_2}\big) + \psi_{w_2,t}^{to\,grid}, \quad \forall t \in \Omega^t, \forall w_2 \in \Omega^w \tag{9}$$

The approximation of the losses is made through a quadratic function of the power flow on the line (10).

$$\delta_w = b \cdot \big|\psi_{w,\tilde{w},t}^{pros,to,pros}\big| + c \cdot {\psi_{w,\tilde{w},t}^{pros,to,pros}}^2, \quad \forall t \in \Omega^t, \forall w \in \Omega^w \tag{10}$$

The coefficients units *b* and *c* are [.] and [kW^{-1}], respectively. The expression can be linearized using the Special-Order Sets of Type 2 (SOS2) considering 5 segments.

$$\sum\nolimits_{p \in P} Z_{w,\tilde{w},t} = 1 \quad \forall t \in \Omega^t, \forall w \in \Omega^w \tag{11}$$

$$\overleftrightarrow{\psi}_{w,\tilde{w},t}^{grid} = \sum\nolimits_{p \in P} X_p \cdot Z_{w,\tilde{w},t} \quad \forall t \in \Omega^t, \forall w \in \Omega^w \tag{12}$$

$$\overleftrightarrow{\phi}_{w,\tilde{w},t}^{grid} = \sum\nolimits_{p \in P} Y_p \cdot Z_{w,\tilde{w},t} \quad \forall t \in \Omega^t, \forall w \in \Omega^w \tag{13}$$

Constraint (14) bounds the power exchange between two prosumers by the line limits between them $\bar{\delta}_{w,\tilde{w}}$.

$$\psi_{w_1,w_2,t}^{pros,to,pros} \le \bar{\delta}_{w_1,w_2} \; \forall t \in \Omega^t, \quad \forall w_1, w_2 \in \Omega^w \tag{14}$$

3) Electric Vehicles

Equation (15) states that the power discharged by the EV multiplied by the efficiency ($0 \le \eta_w^{EV,dch} \le 1$) is the sum of the power used and power sold.

$$PU_{w,t}^{EV} + PS_{w,t}^{EV} = \eta_w^{EV,dch} * PD_{w,t}^{EV}, \quad \forall w \in \Omega^w, \forall t \in \Omega^t \tag{15}$$

The binary variable, $x_{w,t}^{EV}$, is 1 if the EV is charging and 0 otherwise, so (16) and (17) set the upper limit for the charging $PC_{w,t}^{EV}$ and discharging $PD_{w,t}^{EV}$ variables; if $x_{w,t}^{EV}$ is 1 then $PC_{w,t}^{EV} \le \mu_w^{EV,ch}$ and $PD_{w,t}^{EV} = 0$, and if $x_{w,t}^{EV}$ is 0 then $PC_{w,t}^{EV} = 0$ and $PD_{w,t}^{EV} \le \mu_w^{EV,dch}$. Where $\mu_w^{EV,ch}$ is the maximum rate of charging, and $\tilde{\mu}_w^{EV}$ is the maximum rate of discharge.

$$PC_{w,t}^{EV} \le \mu_w^{EV,ch} * x_{w,t}^{EV} \quad \forall w \in \Omega^w, \forall t \in \Omega^t \tag{16}$$

$$PD_{w,t}^{EV} \le \mu_w^{EV,dch} * \big(1 - x_{w,t}^{EV}\big) \quad \forall w \in \Omega^w, \forall t \in \Omega^t \tag{17}$$

Equation (18) defines how the SOC (State of Charge) of the EV evolves with respect to time. The current hour's SOC is the previous hour's SOC plus the power charged times the charging efficiency minus the power discharged time the discharging efficiency.

$$SOC_{w,t}^{EV} = SOC_{w,t-1}^{EV} + \big(\eta_w^{EV,ch} * PC_{w,t}^{EV} * \Delta t\big) - \big(\eta_w^{EV,dch} * PD_{w,t}^{EV} * \Delta t\big) \quad \forall t \in]a_w, d_w], \forall w \in \Omega^w \tag{18}$$

Constraints (19) – (23) define how the SOC is bounded with respect to the arrival a_w and departure d_w time of the EV. Firstly, the arrival time should be before departure (20). Moreover, between the arrival and departure time the SOC is bounded between the maximum and minimum value of the wth EV (16). At departure time the SOC should be at the maximum allowed (19). After the departure time and before the arrival time there will be no charging or discharging (17).

$$SOC_w^{EV,min} \le SOC_{w,t}^{EV} \le SOC_w^{EV,max} \quad \forall t \in]a_w, d_w[, \forall w \in \Omega^w \tag{19}$$

$$PC_{w,t}^{EV} = PD_{w,t}^{EV} = 0 \quad \forall t \in \{\Omega^t - [a_w, d_w]\}, \forall w \in \Omega^w \tag{20}$$

$$SOC_{w,t}^{EV} = SOC_w^{EV,ini} \quad \forall t = a_w, \forall w \in \Omega^w \tag{21}$$

$$SOC_{w,t}^{EV} = SOC_w^{EV,max} \quad \forall t = d_w, \forall w \in \Omega^w \tag{22}$$

$$a_w \le d_w \quad \forall w \in \Omega^w \tag{23}$$

4) *Electrical Storage System*

Constraint (24) states that the power discharged by the ESS multiplied by the efficiency ($0 \le \eta_w^{ESS,dch} \le 1$) is the sum of the power used and power sold.

$$PU_{w,t}^{ESS} + PS_{w,t}^{ESS} = \eta_w^{ESS,dch} * PD_{w,t}^{ESS} \quad \forall w \in \Omega^w, \forall t \in \Omega^t \tag{24}$$

The binary variable, $x_{w,t}^{ESS}$, is 1 if the ESS is charging and 0 otherwise, so (25) and (26) set the upper limit for the charging $PC_{w,t}^{ESS}$ and discharging $PD_{w,t}^{ESS}$ variables; if $x_{w,t}^{ESS}$is 1 then $PC_{w,t}^{ESS} \le \mu_w^{ESS,ch}$ and $PD_{w,t}^{ESS} = 0$, and if $x_{w,t}^{ESS}$is 0 then $PC_{w,t}^{ESS} = 0$ and $PD_{w,t}^{ESS} \le \mu_w^{ESS,dch}$. Where $\mu_w^{ESS,ch}$ is the maximum rate of charging, and $\tilde{\mu}_w^{ESS}$ is the maximum rate of discharge.

$$PC_{w,t}^{ESS} \le \mu_w^{ESS,ch} * x_{w,t}^{ESS} \quad \forall w \in \Omega^w, \forall t \in \Omega^t \tag{25}$$

$$PD_{w,t}^{ESS} \le \mu_w^{ESS,dch} * \left(1 - x_{w,t}^{ESS}\right) \quad \forall w \in \Omega^w, \forall t \in \Omega^t \tag{26}$$

Equation (23) defines how the SOC of the ESS evolves with respect to time. The current hour's SOC is the previous hour's SOC plus the power charged multiplied by the charging efficiency minus the power discharged multiplied by the discharging efficiency.

$$SOC_{w,t}^{ESS} = SOC_{w,t-1}^{ESS} + \left(\eta_w^{ESS,ch} * PC_{w,t}^{ESS} * \Delta t\right) - \left(\eta_w^{ESS,dch} * PD_{w,t}^{ESS} * \Delta t\right) \quad \forall t > 1, \forall w \in \Omega^w \tag{27}$$

Constraints (28) – (30) define how the SOC is bounded over Ω^t . The SOC is bounded between the maximum and minimum value of the w^{th} ESS (28). At the end of the day (*t* = 24) the SOC should hold the maximum allowed value (30).

$$SOC_w^{ESS,min} \le SOC_{w,t}^{ESS} \le SOC_w^{ESS,max} \quad \forall t \in \Omega^t, \forall w \in \Omega^w \tag{28}$$

$$SOC_{w,t}^{ESS} = SOC_w^{ESS,ini} \quad \forall t = 1, \forall w \in \Omega^w \tag{29}$$

$$SOC_{w,t}^{ESS} = SOC_w^{ESS,max} \quad \forall t = 24, \forall w \in \Omega^w \tag{30}$$

5) *PVs*

Equation (31) states that the summation of the power used from the PV panels, $PU_{w,t}^{PV}$, and the power sold from the PV panels $PS_{w,t}^{PV}$ at any time is equal to the power generated by the PV panels, $\mathcal{P}_{w,t}^{PV}$.

$$PU_{w,t}^{PV} + PS_{w,t}^{PV} = \mathcal{P}_{w,t}^{PV}, \quad \forall t \in \Omega^t, \forall w \in \Omega^w \tag{31}$$

6) *Wind Turbines*

Constraint (32) states that the summation of the power used from the wind turbines $PU_{w,t}^{wind}$ and the power sold from the wind turbines $PS_{w,t}^{wind}$ at any time is equal to the power generated by the wind turbines, $\mathcal{P}_{w,t}^{wind}$.

$$PU_{w,t}^{wind} + PS_{w,t}^{wind} = \mathcal{P}_{w,t}^{wind}, \quad \forall t \in \Omega^t, \forall w \in \Omega^w \tag{32}$$

7) *Controllable Appliances*

Equations (33) defines the power used by appliance c of prosumer *w* at time *t* is the summation across all scenarios of the product of $x_s^{appliance}$ and $\mathcal{P}_{w,t,s,c}^{appliance}$. As $x_s^{appliance}$ is a binary variable and the sum of $x_s^{appliance}$ across all scenarios is 1 (34), therefore $x_s^{appliance} = 1$in only one scenario (zero otherwise). This scenario is when $\mathcal{P}_{w,t,s,c}^{appliance}$ is lowest so $PU_{w,t,c}^{appliance}$ becomes the lowest and the total cost of the network is minimized.

$$PU_{w,t,c}^{appliance} = \sum_s x_s^{appliance} * \mathcal{P}_{w,t,s,c}^{appliance} \quad \forall t \in \Omega^t, \forall w \in \Omega^w, \forall c \in \Omega^c \tag{33}$$

$$\sum_s x_s^{appliance} = 1 \tag{34}$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

A. Data and Assumptions

The test system shown in Fig. 1 was used to analyse the model. The system is composed of three prosumers, prosumer one, two and three which were meant to simulate a household of two, one and five persons, respectively. Each variable/parameter is a 24-valued vector representing the performance of that variable/parameter across the 24 hours of the day. In this paper GAMS was used to solve the model using the solver MILP CPLEX.

In Fig. 2 the total load of the prosumers for the 24-hour period is shown. Each prosumer had inflexible loads which are the loads which do not change according to the fluctuations in the electricity price, like a refrigerator and the lighting loads. The inflexible loads ranged from 1.6 kWh to 7.5 kWh.

Each prosumer also has EVs which is modelled after Chevrolet Volt, Volkswagen E-Golf and BMW i3 for prosumer one, two and three, respectively. They have discharging/charging rate ranging from 3.3 kW to 7.2 kW and batter capacity 16 kWh to 24 kWh.

They also have discharging/charging efficiency equal to 95%. The EVs leave between 9-10 AM and return between 4-6 PM. Constraints were added so that for prosumers' convenience, the EVs SOC must be at maximum by the departure time. Values for the data used are derived from [12].

As for the ESSs, they have a capacity of 3-4 kWh, charging/discharging rate 0.6 kW and efficiency 90%. Also, in this technology constraints were added, so that the ESS by the end of the day has reached maximum SOC.

In this study, the prosumers had PV panels which generated energy during the period from 9 AM to 6 PM ranging from 0.0004 kWh to 0.9 kWh, they also have residential wind turbines which generate energy through the whole day ranging from 0.3 kWh to 0.8 kWh.

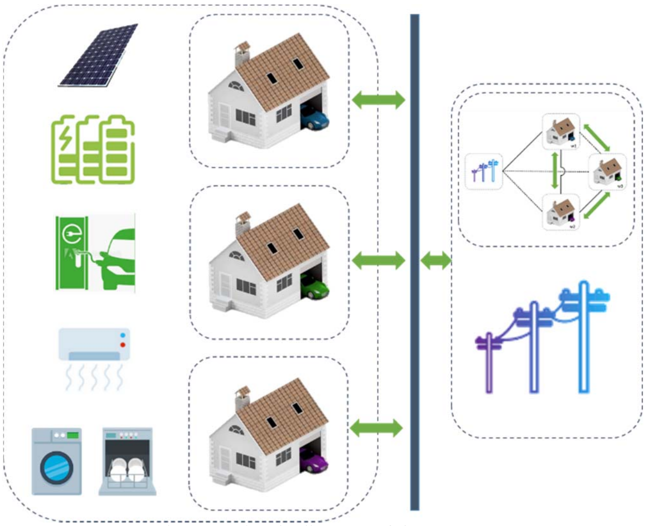

Fig. 1. A simple diagram of the system.

The behavior of the washing machine, dishwasher and HVAC for each prosumer was also modelled. A set of scenarios for the three prosumers for all appliances were considered. A summary of the ranges of the consumption of the appliances can be found in Table I.

The prosumers can exchange energy to help decrease the total electricity costs and the grid constraints were included in the modelling. However, if a prosumer requires more energy and which they cannot generate and there is not enough excess energy in the network (neighborhood), the prosumer may buy energy externally (from the grid). The buying prices range from 0.005 €/kWh to 0.05 €/kWh (prices follow the demand trend). Similarly, prosumers can sell energy externally if all the demand in the network is met. The selling price is flat and equal to 0.03 €/kWh.

B. *Discussion of Numerical Results.*

In this paper 8 Cases (A-H) were used to analyse the model. Case A is the base case where prosumers did not generate any energy, and Case H was the case where all consumers generated and exchanged energy between them and with the grid. All Cases are shown in Table II.

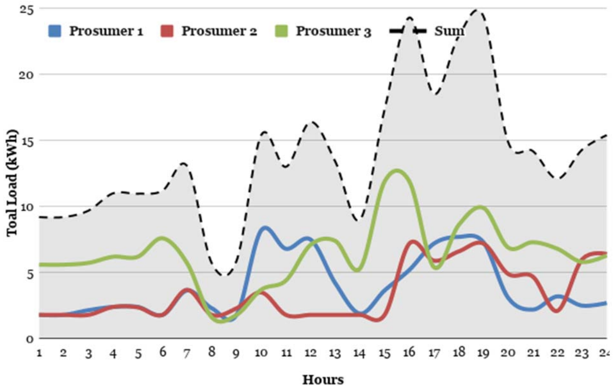

Fig. 2. Total load of the three prosumers

TABLE I: APPLIANCES RANGES (KW)

	Minimum	Maximum	Average
Washing Machine	0.4	0.5	0.45
Dish Washer	1.2	1.5	1.35
HVAC	1	4	2.5

The results from minimizing the total cost of the network were ranked, and are presented in Table III. From this table, it is possible to see that, when all the prosumers are using their generation, the total cost of the network goes down by 16.45%. For cases in which one prosumer generates (D, C, D) and two (E, F, G) we have an average reduction of 5.12% and 11% respectively. Therefore, the increase generation capacity of the various prosumers brings several benefits, in particular, economic benefits for the prosumers (evident by the lower costs) but also to the network in terms of reduced energy losses and increased grid reliability. The possibility to generate, consume, and sell energy among prosumers in the neighborhood and buy from and sell to the grid, considerably increases prosumers flexibility, opening the door for prosumers to participate in ancillary services which may be needed by the grid.

From the grid's perspective, it is possible to see that the losses decrease (Table III) as power is purchased from the other prosumers in the neighborhood, and the losses are higher as the power is purchased from the network. This is because part of the demand is suppressed locally, increasing the reliability of the network. However, this new paradigm increases bidirectional energy flows, which makes network operation much more complex.

Fig. 3 and Fig. 4 show the evolution of the inputs and outputs for a specific prosumer in the case that prosumers generate their own energy (Case H) and in comparison, none of them generates energy (Case A). From these figures, it is possible to see the decrease in the dependence on the network by prosumer 1, and the increase in flexibility, which could be larger but is limited due to the large inflexible load. The greatest optimization occurs at the level of EV and ESS operation. Nevertheless, it can be seen that the prosumer demand response action associated with the increase in flexibility leads to a decrease in peak demand.

In this work, a further analysis was carried out to assess the effect of the restrictions (8) - (13) on the operation costs. The analysis was made considering Case H. The results are presented in Table V, where H_0 and H_1 are the cases where the constraints are active and not active, respectively.

TABLE II: DESCRIPTION OF THE CASES

	Prosumer 1 Generating	Prosumer 2 Generating	Prosumer 3 Generating	Generating Number
A	No	No	No	0
B	No	No	Yes	1
C	No	Yes	No	
D	Yes	No	No	
E	Yes	Yes	No	2
F	No	Yes	Yes	
G	Yes	No	Yes	
H	Yes	Yes	Yes	3

TABLE III: RESULTS OF THE CASES (€)

	Total Cost	Generating Number	Average Cost	Average losses
A	9.18	0	9.18	1.67
B	8.72	1	8.71	1.48
C	8.69			
D	8.72			
E	8.13	2	8.17	1.33
F	8.23			
G	8.15			
H	7.67	3	7.67	1.21

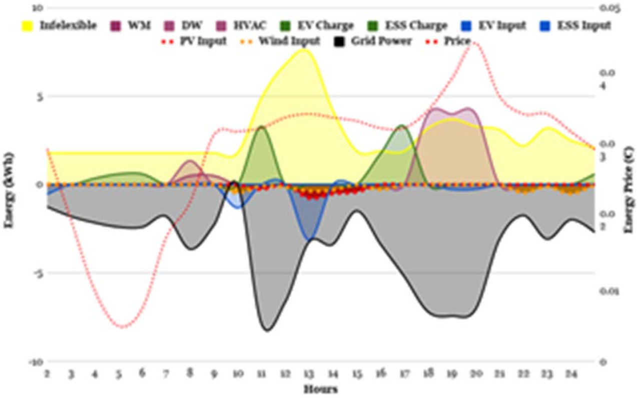

Fig. 3. Diagram of energy flows for Case H (Prosumer 1)

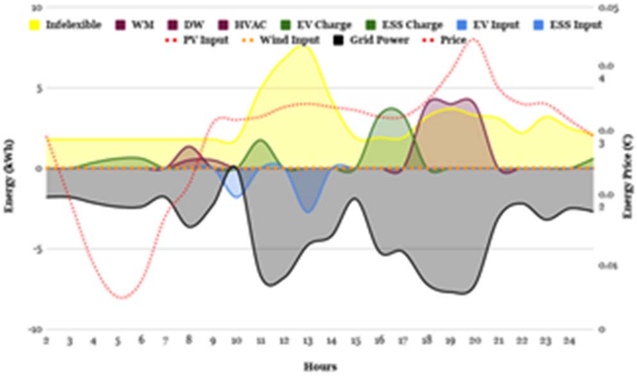

Fig. 4. Diagram of energy flows for Case A (Prosumer 1)

TABLE V: EFFECT OF THE RESTRICTIONS ON COSTS

	Total Cost (€)
Case H_0	7.67
Case H_1	7.58

The results show that when the restrictions are not considered, the cost is lower, since the energy flow of each transaction is not limited, allowing the transaction of higher energy values. However, for this case, the variation of values is not significant, but with more complex systems these values may become significant. This demonstrates the importance of making a realistic representation of the several network operational constraints so that optimization process outputs are as reliable as possible.

V. CONCLUSIONS

In this work an analysis on the benefits from the prosumer's flexibility and also issues associated related to the operation and optimization of the network was presented. This work presents a new model which considers energy transactions between prosumers in the neighborhood as well as transactions between prosumers and the network based on a stochastic tool, in order to consider a set of uncertainties in the form of scenarios associated with the availability of various resources and technologies. Numerical results show that when all consumers are prosumers the network total costs decrease by 16.45% and the network flexibility increases considerably since they can trade energy between each other as well as the network. There is also a 27.5% reduction in energy losses due to local generation, increasing the network reliability. However, if network operation is not well managed, local congestion problems may occur due to bi-directional power flows. It has been shown that it is possibly to verify the added flexibility that prosumers bring to the network. This may open the door for the provision of ancillary services by prosumers.

VI. REFERENCES

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