Probabilistic Planning of Electric Vehicles Charging Stations in an Integrated Electricity-Transport System

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Abstract—One of the most important aspects of the development of Electric Vehicles (EVs) is the optimal sizing and allocation of charging stations. Due to the interactions between the electricity and transportation systems, the key features of these systems (such as traffic network characteristics, charging demands and power system constraints) should be taken into account for the optimal planning. This paper addressed the optimal sizing and allocation of the fast-charging stations in a distribution network. The traffic flow of EVs is modeled using the User Equilibrium-based Traffic Assignment Model (UETAM). Moreover, a stochastic framework is developed based on the Queuing Theory (QT) to model the load levels (EVs' charging demand). The objective function of the problem is to minimize the annual investment cost, as well as the energy losses that are optimized through chance-constrained programming. The probabilistic aspects of the proposed problem are modeled by using the point estimation method and Gram-Charlier expansion. Furthermore, the probabilistic dominance criteria are employed in order to compare the uncertain alternatives. Finally, the simulation results are provided for both the distribution and traffic systems to illustrate the performance of the proposed problem.

Index Terms--Fast charging station, queuing theory, traffic assignment, chance-constrained programming, point estimation, probabilistic dominance criteria.

I. INTRODUCTION

Electric Vehicles (EVs) gain increasing importance in transportation due to their role in reducing the production of greenhouse gases [1]. Taking into account the limited All Electric Range (AER) of these vehicles, using them on a wider scale and enabling long-distance travels would necessitate the development of EV charging infrastructures such as Fast Charging Stations (FCSs).

Furthermore, the uncontrolled nature of charging may have adverse effects on the distribution system (such as losses and voltage drops) [2]. Hence, the optimal planning of the FCSs is essential to achieve the benefits of EVs. Estimating the charging demands of EVs is considered as the first and main step in the planning of the FCSs. Charging demands of EVs depend on uncertain conditions such as the time and place of charging the batteries or their State of Charge (SOC). Nevertheless, it is possible to estimate the charging behavior of the EVs' by evaluating the overall pattern of their mobility (obtainable from the traffic flows) [3].

Reference [1] has presented a method based on the Queuing Theory (QT) for modeling the 24-hour charging profile of the Plug-in Electric Vehicles (PEVs). Reference [4] has predicted the required power for charging EVs in a real urban traffic network in two stages. At the first stage, the rate of EVs arrival to the charging station is formulated based on the Markov chain traffic model and the teleportation approach using surveillance camera data. Then, the charging demand of EVs is determined using the Queue model and the data from the first stage. Bae et al. [5] have employed spatial and temporal modeling of the charging demand of an FCS located at a highway exit and have presented a mathematical model based on a fluid dynamic model and the QT. Recently, extensive research has been conducted to determine the optimal location and size of Electric Vehicle Charging Stations (EVCSs). In [6], a spatial and temporal model based on the Shared Nearest Neighbor (SNN) clustering algorithm and QT has been introduced for determining the location and capacity of EVCSs.

Reference [7] introduced a method for optimal allocation of charging stations containing two steps. The first step estimates the spatially continuous charging demand on the basis of the locations of the existing FCSs, point of interest, population and traffic data. The optimal allocation of the charging station is determined by the maximum spatial coverage model, in Step 2. Reference [8] has simultaneously considered the network topology and traffic limitations for the optimal planning of charging stations. The effect of the Time-of-Use (TOU) cost on the charging behavior of EVs has been studied in [9]. Reference [10] has presented a multi-objective programming model for the simultaneous expansion planning of the distribution network and the optimal allocation of the FCSs. The equilibrium traffic flow model and QT have been developed to estimate the charging load of charging stations.

The aforementioned studies have utilized different deterministic methods to specify the optimal location and size of the FCS. However, such problems depend on numerous unknown parameters (such as the EVs' position, SOC, etc.) that will directly affect the planning of the FCSs. Faridimehr et al. [11], proposed a robust two-stage stochastic programming model for determining the optimal network of charging stations, where different uncertain resources (such as charging patterns, demand, and drivers' behavior) have been considered.

Reference [12] investigated the planning of the FCSs by considering the reconfiguration of a distribution system, in which a scenario-based approach is employed to model the uncertainties. Cui et al. [13], utilized a method to allocate the EVs' charging station for urban areas.

This paper addresses an integrated planning model for the optimal sizing and allocation of the fast-charging stations in the distribution system. A stochastic framework is developed based on the QT and User Equilibrium Based Traffic Assignment (UETA) model to model the uncertain behavior of EV drivers, as well as the load levels. The objective function of the problem is to minimize the annual investment cost, as well as energy losses that are optimized through chance-constrained programming. The probabilistic aspects of the proposed problem are modelled by using the point estimation method and Gram-Charlier expansion. Furthermore, the probabilistic dominance criteria are employed to compare the uncertain alternatives. Moreover, the effect of charging tariffs on the optimum planning of the charging stations is addressed. The main contributions of this paper are summarized as follows: developing the EV arrival rate profile by considering the effect of different charging tariffs, stochastic modeling of the proposed problem by using the point estimation method and Gram-Charlier expansion and evaluating the risk of a single-objective function by considering probabilistic dominance criteria and creating the list of non-dominant investment schemes.

The paper is organized as follows. In Section II, the proposed model for determining the charging demand and capacity (the number of charging devices) of the FCSs is presented. Then, in Section III, charging station allocation by considering uncertainty is presented. Section IV is dedicated to probabilistic dominance criteria. The simulation results and analyses are provided in section V. Finally, conclusions are provided in Section VI.

II. The Determination of the Capacity and Charging Demand of the FCSs

A. User Equilibrium Based Traffic Assignment Model

Generally, the policies and planning of the transportation system can change the spatial and temporal distribution of EVs and, as a result, the pattern of their charging demand (that will affect the operation of the power distribution systems) [14]. By assuming $G(N^T, \Omega^{TL})$ as a traffic network graph, the UETA problem is formulated for each period as follows [15]:

$$Min \quad f(x) = \sum_{(mn)\in\Omega^{TL}} \int_0^{x_{mn}} p_{mn}(\omega)\, d\omega \tag{1}$$

subject to:

$$\sum_{q\in Q_{rs}} f_q^{rs} = q_{rs} \tag{2}$$

$$f_q^{rs} \geq 0 \tag{3}$$

$$x_{mn} = \sum_{r\in N^T} \sum_{s\in N^T} \sum_{q\in Q_{rs}} f_q^{rs}\, \delta_{mn,q}^{rs} \tag{4}$$

In the above equations, (1) is to minimize the sum of the areas under the performance function of all the links in the network. Equation (2) means that the sum of the flows of all the paths between each origin-destination (O-D) pair equals with the travel demand of that O-D. Equation (3) is a constraint to guarantee the non-negativity of the traffic flow in the path q between the origin r and destination s. According to (4), the flow in the link mn equals the sum of the flows in all the paths containing the link mn.

The UETA problem is a nonlinear optimization problem. In this paper, the Advanced Interactive Microscopic Simulator for Urban and Non-Urban Networks (AIMSUN) has been utilized to solve the UETA problem.

The traffic flow captured by the k^{th} candidate charging station at time t can be computed using the results obtained from the traffic assignment as (5) [10].

$$f_{k,t} = \sum_{r\in N^T} \sum_{s\in N^T} \sum_{q\in Q_{rs}} f_{q,t}^{rs}\, \delta_{k,q}^{rs}\, u_k \tag{5}$$

Here, $f_{k,t}$ is the sum of the traffic flows of all the paths passing through the traffic node *k* at time *t*.

B. Determination of the capacity of the FCS using the QT

It is assumed that the daily travel pattern and driving behavior of the EV drivers are similar to the conventional drivers. As mentioned before, $f_{k,t}$ represents the number of vehicles passing through the traffic node *k*; therefore, the average number of vehicles arriving at the k^{th} candidate FCS at time *t* can be expressed as (6) (more details are represented in appendix II):

$$\lambda_{k,t} = C \frac{f_t^{trip}}{\sum_{t\in T} f_t^{trip}} \frac{f_{k,t}}{\sum_{k\in\Omega^K} f_{k,t}} \times (1 + \sum_{t'\in T} E(t,t').\frac{c^E(t') - c^{E0}}{c^{E0}}) \quad \forall t,t' \in T, \forall k \in \Omega^K \tag{6}$$

$E(t,t')$ can be classified as :

$$\begin{cases} self\ elasticity: \ E(t,t') \le 0 \ \ if \ t = t' \\ cross\ elasticity: E(t,t') \ge 0 \ \ if \ t \ne t' \end{cases} \tag{7}$$

The arrival rate of vehicles at the k^{th} candidate FCS during the rush hour is formulated by (8).

$$\lambda_{k,t_{RH}} = \max\left(\lambda_{k,t}\right) \quad \forall k \in \Omega^K \tag{8}$$

EVs stochastic arrival in and departure from FCS are modeled using a queuing system [5]. In this system, EV drivers are considered as customers who need to refuel and if all charging devices are busy, they will have to wait in a queue. In the M/M/s queuing model, *s* stands for the number of identical charging devices, the first *M* denotes the arrival of the vehicles to the FCS that is based on the Poisson process with the parameter $\lambda_{k,t}$, and the second *M* is the service time for each charging device that is independent and has the exponential distribution with the average μ_q. By using the M/M/s, it is possible to model the capacity of the FCSs as the following optimization problem. The number of charging devices of the k^{th} FCS is minimized based on the patience of the customers (waiting charging time) as Equations (9)-(11) [10].

$$Objective: \ Min(z_k) \tag{9}$$

subject to:

$$W_{k,t_{RH}} < W^{permissibe} \tag{10}$$

$$W_{k,t_{RH}} = \frac{(z_k \rho_{k,t_{RH}})^{z_k} \rho_{k,t_{RH}}}{\lambda_{k,t_{RH}} (z_k !)(1-\rho_{k,t_{RH}})^2} \pi_{k,o}(t_{RH}) \tag{11}$$

Equation (12) is the probability of being no customer in the k^{th} FCS at time *t* [5].

$$\pi_{k,0}(t) = \left[\sum_{n=0}^{z_k - 1} \frac{(z_k \rho_{k,t})^n}{n!} + \frac{(z_k \rho_{k,t})^{z_k}}{(z_k !)(1-\rho_{k,t})} \right]^{-1} \tag{12}$$

$\rho_{k,t}$ is defined according to (13) [5]:

$$\rho_{k,t} = \frac{\lambda_{k,t}}{z_k \mu_q} \tag{13}$$

The values of $\pi_{k,o}(t_{RH})$ and $\rho_{k,t_{RH}}$ could be computed by substituting *t* as rush hour in (12) and (13), respectively. It is worth mentioning that, since it is difficult to find the inverse functions of equation (11), an enumeration technique is directly employed to solve the minimization problem. Based on the M/M/s queuing theory, this system is stable if and only if: $\rho_{k,t} < 1$, such that the queue lengths do not grow to infinity [5]. By combining the necessary and sufficient condition (the above inequality) for the queuing system stability and equation (13), the lowest number of charging devices should satisfy equation (14).

$$z_k > \frac{\lambda_{k,t}}{\mu_q} \tag{14}$$

By substituting $\lambda_{k,t}$ in $\lambda_{k,t_{RH}}$, the initializing value of z_k is computed. Then, $W_{k,t_{RH}}$ is calculated and compared with the $W^{permissibe}$. Then, z_k should be increased (1 unit in each step) until $W_{k,t_{RH}} < W^{permissibe}$. The corresponding value of z_k would be the optimal number of charging devices.

In the process of determining the capacity of the FCS, the number of charging devices should satisfy the constraint (15):

$$z_k^{\min} \le z_k \le z_k^{\max} \tag{15}$$

C. The determination of the charging demand of the FCS using the M/M/s queuing system

If the charging rate of the charging devices during the charging process is taken into account to be constant, the EVs' charging demand in the queue system of the k^{th} FCS in time *t* can be expressed by (16) [16]:

$$P_{k,t}^{FCS} = B_{k,t}\, p^{FCS} \tag{16}$$

Equation (17) shows the steady-state probability $\pi_{k,n}(t)$ in the M/M/s queuing system [5].

$$\pi_{k,n}(t)=\begin{cases}(\frac{\lambda_{k,t}}{\mu_q})^n\frac{\pi_{k,0}(t)}{n!} & n<z_k\\ (\frac{\lambda_{k,t}}{\mu_q})^n\frac{\pi_{k,0}(t)}{z_k!}z_k^{\,z_k-n} & n\ge z_k\end{cases} \quad (17)$$

where n is the number of vehicles requiring charging. Hence, the number of charging devices at service could be obtained as $\min(z_k, n)$ [5]. Therefore, in the M/M/s queuing model, $B_{k,t}$ could be computed as (18) [17].

$$B_{k,t}=\begin{cases}(\frac{\lambda_{k,t}}{\mu_q})^n\frac{\pi_{k,0}(t)}{n!} & n<z_k\\ (\frac{\lambda_{k,t}}{\mu_q})^{z_k}\frac{\pi_{k,0}(t)}{z_k!(1-\rho_{k,t})} & n=z_k\end{cases} \quad (18)$$

III. Optimal Allocation of FCSs

A. Objective function

The objective function consists of the following parts:

- The investment cost for building the FCS

The investment cost is formulated as (19).

$$\min f_{inv}=\frac{\varepsilon(1+\varepsilon)^{n^{FCS}}}{(1+\varepsilon)^{n^{FCS}}-1}\sum_{k\in\Omega^K}\{u_k[c^{CH}z_k+c_k^{other}z_k+c_k^F]\} \quad (19)$$

In which, c^{CH} is the building cost of FCSs (such as the cost of fast charging devices, transformers, etc.). This part of the cost is independent of geographical location. c_k^{other} is the cost of land use that is proportional to the number of charging devices at each FCS, and c_k^F is the fixed investment cost (independent of the capacity of the FCSs).

- The annual cost of energy losses

$$Min\ f_{lost}=c^{E0}d^{year}\{\sum_{(i,j)\in\Omega^{DL}}\sum_{t\in T}[g_{i,j}(U_{i,t}^2+U_{j,t}^2-2U_{i,t}U_{j,t}\cos\theta_{ij,t})]\} \quad (20)$$

B. Problem constraints

1) Deterministic constraints

$$P_{i,t}=P_{i,t}^D+\sum_{k\in\Omega_i}P_{k,t}^{FCS}+U_{i,t}\sum_{j\in N^D}U_{j,t}[G_{ij}\cos\theta_{ij,t}+B_{ij}\sin\theta_{ij,t}]\forall i\in N^D,\forall t\in T \quad (21)$$

$$Q_{i,t}=Q_{i,t}^D+U_{i,t}\sum_{j\in N^D}U_{j,t}[G_{ij}\sin\theta_{ij,t}-B_{ij}\cos\theta_{ij,t}]\quad \forall i\in N^D,\forall t\in T \quad (22)$$

$P_{i,t}^D$ and $Q_{i,t}^D$ are considered to have a normal distribution.

In the proposed demand model of the FCSs, $P_{k,t}^{FCS}$ has been considered as a *PQ* bus with random characteristics. By considering the load factor equal to 1 for the vehicle batteries [1, 18], the demand for the vehicles could be expressed as follows:

$$P_{k,t}^{FCS}=B_{k,t}\,p^{FCS} \quad (23)$$

$$Q_{k,t}^{FCS}=0 \quad (24)$$

2) Chance constraints

$$\mathrm{Prob}(S_{ij}\le S_{\max})\ge\alpha \qquad \forall(ij)\in\Omega^{DL} \quad (25)$$

$$\mathrm{Prob}(U_{\min}\le U_i\le U_{\max})\ge\alpha \qquad \forall i\in\Omega^{DL} \quad (26)$$

In the above equations $\mathrm{Prob}(.)$ presents the events' probabilities. It should be noted that the chance constraint ensures that constraints are satisfied with a specified confidence interval of α. However, it requires the Probability Distribution Function (PDF) of bus voltages and line flows. In this regard, the probabilistic power flow is performed using the combined Point Estimation Method and Gram-Charlier Expansion (PEM-GSE) method. The proposed model employs the "2m+1 scheme" model of the Point Estimation Method to find the bus voltage and line current moments (where *m* is the number of input random variables). *2m+1* scheme evaluates two deterministic power flows (Equations (21) and (22)), once above and once below the mean value for each input variable, while other variables are held at their mean values. Then, it needs to solve one additional deterministic evaluation assuming that all input variables are at their mean values [19]. According to [20], Gram-Charlier series expansion can approximate the PDF of bus voltages and line flows in terms of their first few moments. Based on the PEM-GCE method, the confidence interval of uncertain variables can be obtained.

IV. Probability dominance criteria

By considering the probabilistic characteristic of the system load, the energy losses will also be probabilistic.

A. Mean-variance method

This first method is the most common method of decision-making under uncertainty. Assume two plans *A* and *B* with mean values μ_A and μ_B as well as standard deviations σ_A and σ_B. According to the mean-variance criterion, in a maximization problem, plan *A* dominates plan *B* if [21]:

$$\mu_A\ge\mu_B \quad (27)$$

$$\sigma_A\le\sigma_B \quad (28)$$

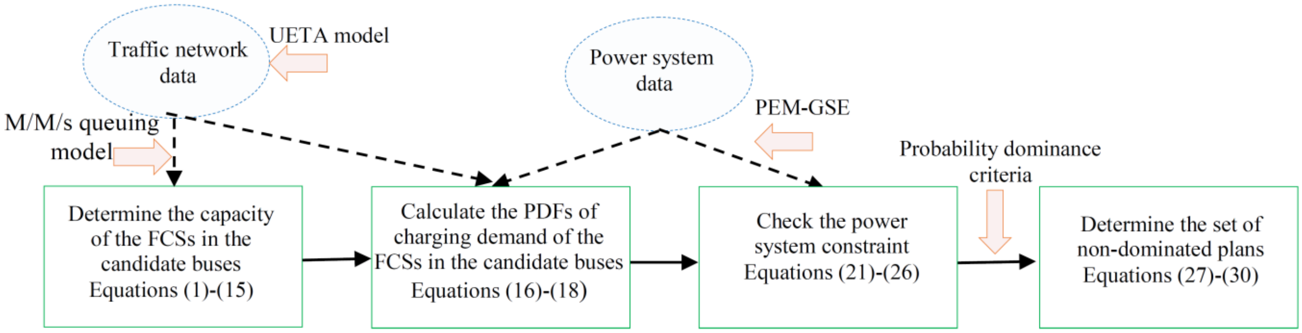

Figure 1. The stages of the optimization procedure

In the mean-variance model, the mean is an indicator of fitness, and variance is an indicator of risk. In the above equations, if (27) is satisfied with inequality, then (28) just could be equality and vice versa. If each of the plans satisfies only one of these equations, plans *A* and *B* will be non-dominated with respect to each other.

B. Stochastic dominance method

In this method, the PDF of the fitness is used instead of the mean and variance values; therefore, this method is a more comprehensive criterion for determining the risk and fitness [21]. Let assume two arbitrary cumulative distribution functions (CDFs), $F_A(x)$ and $F_B(x)$. Plan *A* dominates plan *B* if and only if:

$$F_A(x) \le F_B(x) \quad (29)$$

C. Probabilistic Exceedance Measure

Assume two random variables *A* and *B*. To select the greater variable, assume the difference between the two random variables $(\Delta_{AB} = A - B)$. According to this method, the probability that *A* is greater than *B* is computed using (30) [22].

$$r_{AB} = 1 - F_{\Delta_{AB}}(0) \quad (30)$$

where, $F_{\Delta_{AB}}$ is the CDF of Δ_{AB}. *A* and *B* could be compared with each other by using the following conditions in the symmetric threshold range $[T_l, 1-T_l]$.

- If $r_{AB} \ge 1 - T_l$, then *A* is greater than *B*.
- If $r_{AB} \le T_l$, then *B* is greater than *A*.
- If $T_l < r_{AB} < 1 - T_l$, then *A* and *B* are equal.

The threshold of $T_l \in [0, 0.5]$ should be determined to

select the dominant plan. In practice, T_l = 0.3 provides acceptable results [23].

Figure 1 schematically illustrates the stages of the proposed optimization procedure

V. Numerical Study

A. Input data and assumptions

The Sioux Falls traffic network and the corresponding 33-bus distribution test system are used here [24], [25]. As shown in Figure 2, common nodes between the traffic and distribution systems are considered as the candidate locations for the construction of the FCSs. O-D matrix and trip ratio data are available in [24]. According to the IEC 61851-1 standard [12] p^{FCS} is assumed as 44 kW. The maximum voltage deviation is 10% [3]. Other data and parameters are shown in Tables I-III.

TABLE I. The construction cost of fast charging stations.

Candidate FCSs	**1**	**2**	**3**	**4**	**5**
Traffic network nodes	3	10	12	15	18
Distribution system buses	10	20	5	25	32
c^{CH} **[10⁴ USD]** ([10])	8	8	8	8	8
c^{other} **[10⁴ USD]**	2	4.2	3.8	3	3.55
c_k^F **[10⁴ USD]**	25	45	44	32	49

TABLE II. Modeling parameters values

Parameter	**Value**	**Parameter**	**Value**
z^{max} [10]	10	z^{min} [10]	6
C	850	$W^{alloweed}$ [10]	5 min
ε [10]	0.1	n^{FCS} [10]	20 year
c^{E0} [12]	0.2 USD/kWh	α [26]	99.87%

TABLE III. Electricity price and elasticity coefficients [12],[27]

Time of day	**Self and cross elasticities**			**Price [USD/kWh]**
	Peak	***Off-peak***	***Low load***	
Peak (7-8 & 14-19)	-0.1	0.016	0.012	0.5
Off-peak (1-6 & 22-24)	0.016	-0.1	0.01	0.2
Low load (other times)	0.012	0.01	-0.1	0.15

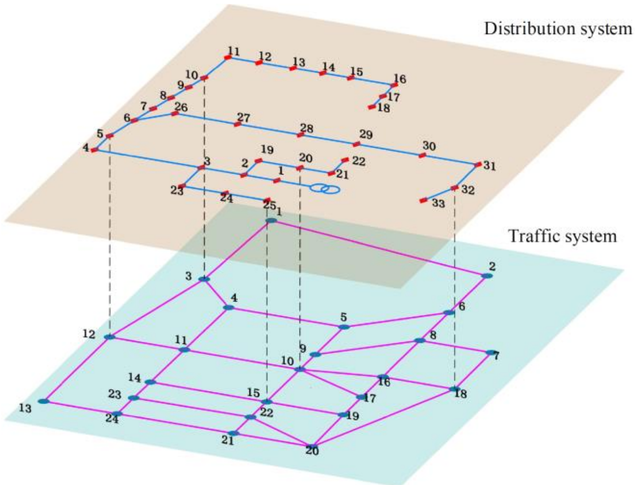

Figure 2. The graphical topology of the coupled distribution and transportation system (Gray links illustrate corresponding geographical nodes of the traffic and distribution system)

B. *Simulation results and analyses*

According to the traffic data of the Sioux Falls network in each period (one hour), a traffic flow distribution based on UETA model is computed by using the AIMSUN software.

Arrival rates of EVs in each FCS in each time period are obtained based on the candidate nodes, for each possible plan (2^5=32), according to (6).

Then the capacities of the FCSs in all possible plans are determined. The problem constraints are checked in two parts. In the first part, the minimum and the maximum number of charging devices constraint is examined. Then, according to the procedure described in part C of section II, the probabilistic charging load of FCSs in each possible plan is determined and the power system constraints are checked by considering the effects of the increased load. Afterwards, the feasible solutions for the problem can be obtained.

At the first scenario, the deterministic planning model is investigated. The second scenario develops the probabilistic model. In the third scenario, the effects of different electricity tariffs have been modeled.

1) *First Scenario: deterministic planning*

In this scenario, the mean values of charging demands and conventional demands are considered as input data. The investment cost of this case is 635245.8 USD. The locations of the FCSs, as well as the number of charging devices at each FCS, are summarized in table IV.

2) *Second scenario: probabilistic planning without considering electricity tariffs*

In this case, the chance-constrained programming framework is employed to consider the uncertainty of the proposed problem. Table V shows the results of this scenario. The confidence interval of the bus voltages and line flows are obtained using the PEM-GSE. In order to demonstrate the accuracy of the PEM-GSE method, it is compared with Monte-Carlo Simulation (MCS) in figure 3. This figure shows the CDF of the voltage of bus #5 at 10 AM of the feasible plan #2 in the second scenario. As it can be observed in this figure, the proposed model can precisely approximate the CDF of the bus voltage. Comparing the results of Tables IV and V, shows that the optimum plan of the first scenario is not feasible according to the second scenario. Furthermore, the planning cost is increased as a consequence of considering the uncertainties.

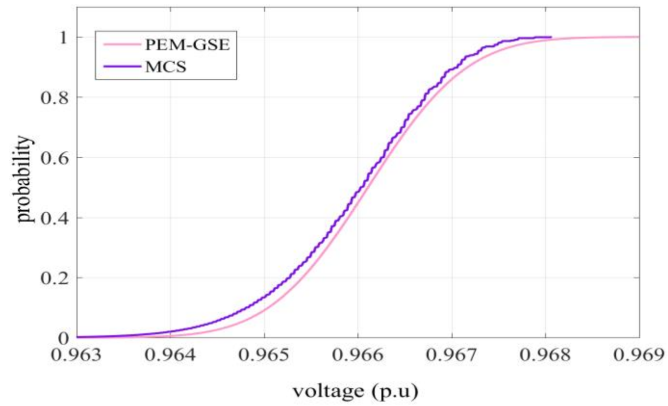

Figure 3. CDF of the voltage of bus #5 at 10 AM of the feasible plan #2 in the second scenario

TABLE IV. PLANNING RESULTS OF THE FIRST SCENARIO

Locations (bus number)	Number of charging devices
10	6
5	9
32	10
Total cost=635245.8 USD	

It should be noted that, as it could be seen in Table V, the third plan has the minimum cost mean, while its variance is higher than plans 1 and 5. In order to select the optimum plan among these feasible plans (which all have their arbitrary PDF), probability dominance criteria methods are utilized. As displayed in Table VI, the third and fifth plans are non-dominant in terms of mean cost and risk.

After forming the non-dominant list, a decision-maker can select the final plan based on its interests confidently. Although this paper investigated the propagating of the uncertainties in the objective functions to form a list of the non-dominant plans, the selection of the final solution is not focused here.

TABLE V. PLANNING RESULTS OF THE SECOND SCENARIO

Feasible plans	Capacities and locations of the FCSs					Cost mean	Cost variance
	Bus 10	*Bus 20*	*Bus 5*	*Bus 25*	*Bus 32*		
1	0	8	6	7	6	715828.3	61755.7
2	0	0	8	9	8	639150.1	106657.8
3	0	10	0	8	7	636268.5	67454.7
4	0	10	7	0	7	644183.9	70898.8
5	0	10	7	8	0	636294.1	24039.6

TABLE VI. RESULTS OF THE PROBABILISTIC ASSESSMENT IN THE SECOND SCENARIO

	Mean-variance	Stochastic Dominance	Probabilistic Exceedance Measure
*Non dominate plans**	3, 5	3, 5	3, 5

*= These numbers indicate plan numbers taking from the first column of Table V

3) Third scenario: probabilistic planning considering electricity tariffs

In this scenario, the rush hours of the traffic network, as well as the peak hours of the distribution system is considered. Also, the impact of electricity tariffs on the arrival rate of vehicles in the FCSs is considered. It is assumed that if the charging durations of EVs falls between two periods, the price of the first period will be taken into account as their charging tariff. The results are shown in Tables VII and VIII. According to these tables, both the mean and variance of the cost of feasible plans of the third scenario are lower than the second one that proves the performance and effectiveness of a proper electricity tariff design.

VI. CONCLUSION

In this paper, the probabilistic planning model has been developed for the optimum allocation and sizing of the FCSs, by taking into account both the electrical distribution system and the coupled traffic network. The UETA method has been utilized in order to model the traffic flows, while the QT is employed to handle the uncertainties of EVs' charging demand. Moreover, the chance-constrained method has been used to model the probabilistic constraints of the problem. The Point Estimation method, as well as the Gram-Charlier expansion, has been employed to specify the PDFs of the uncertain variables. Furthermore, the probabilistic dominance criteria have been employed in order to compare the uncertain alternatives. Three scenarios have been introduced to investigate the performance of the proposed problem. On the one hand, the effect of considering the uncertainties has been compared with a deterministic model. On the other hand, the effect of different electricity tariffs on the behavior of EV drivers has been investigated. The results showed that the presence of uncertainties may significantly change the planning decisions. Also, a proper design of the electricity tariffs could change the behavior of EV drivers in a way to decrease their charging demand in the rush and peak times. Indeed, applying higher prices at rush times (when the traffic flow is high) or at peak periods (when the electrical load is high) will change the behavior of responsive EV drivers in an effective way. Therefore, not only the investment risk could be decreased, but also the planning of the FCSs could be managed more economically.

TABLE VII. PLANNING RESULTS OF THE THIRD SCENARIO

Feasible plans	Capacities and locations of the FCSs					Cost mean	Cost variance
	Bus 10	*Bus 20*	*Bus 5*	*Bus 25*	*Bus 32*		
1	0	0	7	8	7	597765.7	101898
2	0	8	0	7	6	580330.6	63949.2
3	0	9	6	0	6	601623.4	67228.8
4	0	8	6	7	0	580367.7	22742.3

TABLE VIII. RESULTS OF THE PROBABILISTIC ASSESSMENT IN THE THIRD SCENARIO.

	Mean-variance	Stochastic Dominance	Probabilistic Exceedance Measure
*Non dominate plans**	2,4	2,4	2,4

*= These numbers indicate plan numbers taking from the first column of Table VII

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APPENDICES

A. Appendix I

Notations

<u>*Sets*</u>

N^T	Set of nodes of the traffic network.
Ω^{TL}	Set of links of the traffic network.
Q_{rs}	Set of paths connecting the origin *r* and the destination *s*.
Ω^K	Set of candidate FCSs.
T	Set of hours.
Ω^{DL}	Set of distribution system feeders.
Ω_i	Set of candidate FCSs at bus *i*.
N^D	Set distribution system buses.

<u>*Parameters*</u>

q_{rs}	Travel demand between the origin *r* and the destination *s*.
f_t^{trip}	Trip ratio at time *t*.

C	The total number of charging EVs during the time horizon *T*.
c^{E0}	The initial cost of electric energy.
$c^{E}(t)$	Electric energy cost in time *t*.
$E(t,t')$	Price electricity of time *t* versus time t'.
$W^{permissibe}$	Permissible waiting time.
μ_q	Mean service rate of charging devices.
$z_k^{\min}$, $z_k^{\max}$	Capacity limits of FCSs.
p^{FCS}	Charging rate of the charging devices.
ε	Interest rate.
n^{FCS}	Planning horizon.
c^{CH}	The building cost of FCSs.
c_k^{other}	Cost of land use.
c_k^{F}	Fixed investment cost.
d^{year}	The number of days in a year.
$g_{i,j}$	The conductance of feeder *ij*.
$P_{i,t}^{D}$, $Q_{i,t}^{D}$	Conventional active and reactive load levels at bus *i* at time *t*.
α	The confidence interval of the random constraints.
$U_{\min}, U_{\max}$	Upper and lower thresholds of the bus voltages.
$S_{\max}$	The upper threshold of the flow passing through feeder *ij*.

Indicators

$\delta_{mn,q}^{rs}$	Indicator of links (1: if the link *mn* is a part of the path *q* between the origin *r* and the destination *s*, 0: otherwise).
$\delta_{k,q}^{rs}$	Indicator of paths (1: if the traffic flow between the origin *r* and the destination *s* is captured by the k^{th} charging station; 0: otherwise).

Variables

$p_{mn}(\omega)$	Performance function of the link *mn*.
f_q^{rs}	Traffic flow on the path *q* between the origin *r* and the destination *s*.
x_{mn}	Traffic flow on link *mn*.
$f_{k,t}$	Traffic flow captured by the k^{th} FCS at time *t*.
u_k	Binary variable (1: if the k^{th} FCS is built; 0: otherwise).
$\lambda_{k,t}$, $\lambda_{k,t_{RH}}$	The average number of vehicles arriving in the k^{th} FCS at time *t* and t_{RH} (rush hour), respectively.
z_k	The capacity of the k^{th} FCS.
$W_{k,t_{RH}}$	Average waiting time for refuelling in k^{th} FCS at rush hour.
$\pi_{k,0}(t)$, $\pi_{k,o}(t_{RH})$	Probability of being no customer in the k^{th} FCS at time *t* and t_{RH}, respectively.
$\rho_{k,t}$, $\rho_{k,t_{RH}}$	Occupation rate of each charging device in the k^{th} FCS at time *t* and t_{RH}, respectively.
$B_{k,t}$	Number of charging devices at service at time *t*.
$\pi_{k,n}(t)$	Probability of being *n* number of EVs requiring charging in the k^{th} FCS at time *t*.
$P_{k,t}^{FCS}$, $Q_{k,t}^{FCS}$	Active and reactive probabilistic demands of the k^{th} FCS at time *t*, respectively.
$U_{i,t}$, $U_{j,t}$	Magnitudes of the voltage of buses *i* and *j* at time *t*.
$\theta_{ij,t}$	The angle of the voltage of feeder *ij* at time *t*.
S_{ij}	The upper threshold of the flow passing through feeder *ij*.

B. *Appendix II*

In this appendix, the supplementary explanations have been provided to clarify the equation (6). Assume that the probabilities for the EV drivers to charge at any FCS on the path *q* are the same. So, it could be estimated that the more numbers of passing EVs, will result in the more probability for going to be charged. So, the mean arrival rate at the k^{th} candidate FCS would be $f_k / \sum_{k\in\Omega^K} f_k$. Therefore, the initial average number of vehicles arriving in the k^{th} candidate FCS at time t can be expressed as:

$$\lambda_{0k,t} = C\,\frac{f_t^{trip}}{\sum_{t\in T} f_t^{trip}}\,\frac{f_{k,t}}{\sum_{k\in\Omega^K} f_{k,t}},\ \forall t\in T,\ \forall k\in\Omega^K \qquad \text{(A–1)}$$

Also, it is considered that the electricity tariffs will affect the decisions of the EV drivers. Therefore, EV drivers will change their charging behavior in response to electricity tariffs. Consequently, the average number of EVs entering the FCS would change from $\lambda_{0k,t}$ (initial value) to $\lambda_{k,t}$. Hence, $\Delta\lambda_{k,t}$ could be expressed as (A-2):

$$\Delta\lambda_{k,t} = \lambda_{k,t} - \lambda_{0k,t} \qquad \text{(A–2)}$$

According to reference [27], self and cross elasticity can be defined as (A-3).

$$E(t,t') = \frac{c^{E0}}{\lambda_{0k,t}}\frac{\partial\lambda_{k,t}}{\partial c^{E}(t)} \qquad \text{(A–3)}$$

With the linearity assumption of $\frac{\partial\lambda_{k,t}}{\partial c^{E}(t)}$, Equation (A-3) could be rewritten as (A-4):

$$E(t,t^{'}) = \frac{c^{E0}}{\lambda_{0k,t}} \frac{\Delta\lambda_{k,t}}{\Delta c^{E}(t)} \quad \text{(A–4)}$$

By substituting Equations (A-4) and (A-1) in Equation (A-2), the responsive average number of vehicles arriving in the k^{th} candidate FCS at time t could be formulated as:

$$\lambda_{k,t} = C \frac{f_t^{trip}}{\sum_{t\in T} f_t^{trip}} \frac{f_{k,t}}{\sum_{k\in\Omega^K} f_{k,t}} \times (1 + \sum_{t^{'}\in T} E(t,t^{'}). \frac{c^{E}(t^{'}) - c^{E0}}{c^{E0}}) \quad \text{(A–5)}$$
$$\forall t, t^{'} \in T, \ \forall k \in \Omega^{K}$$

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