

# Exploitation of Microgrid Flexibility in Distribution System Hosting Prosumers

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**Abstract**—Increasing the penetration of renewables in prosumers' side brings about operational challenges in the distribution grid due to their variable and uncertain behavior. In fact, these resources have increased the distribution grid net load fluctuation during recent years. In this paper, the flexibility-oriented stochastic scheduling of a microgrid is suggested to capture the net load variability at the distribution grid level. In this scheduling, the flexibility limits are set to manage the net load fluctuation at a desirable level for the main grid operator. The uncertainties of load and renewables are considered, and their uncertainties are under control by the risk-averse strategy. Moreover, multi-period islanding constraints are added to the problem, preparing the microgrid for a resilient response to disturbances. The model is examined on a typical distribution feeder consisting of prosumers and a microgrid. The numerical results are compared for both flexibility-oriented and traditional scheduling of a microgrid in the distribution level. The proposed model reduces the net load ramping of the distribution grid using an efficient dispatch of resources in the microgrid. A sensitivity analysis is also carried out to show the effectiveness of the model.

**Index Terms**—Flexibility, Islanding, Microgrids, Prosumers, Renewables, Risk constraints.

## NOMENCALTURE

### Acronyms

<i>MG</i>	Microgrid
<i>ESS</i>	Energy storage system
<i>DR</i>	Demand response
<i>OF</i>	Objective function
<i>MILP</i>	Mixed integer linear programming
<i>VoLL</i>	Value of lost load
<i>SOC</i>	State of charge
<i>TOU</i>	Time of use

### Variables

<i>I</i>	Indicator of dispatchable units; commitment (1/0 for committed/otherwise)
<i>v</i>	Indicator of ESS charging (1 for charging states, 0 otherwise)
<i>u</i>	Indicator of ESS discharging (1 for discharging states, 0 otherwise)
<i>Um</i>	Main grid availability status (1/0 means available/unavailable main grid)
<i>p<sup>U</sup></i>	Power provided from main grid
<i>p<sup>M</sup></i>	Power transferred to microgrid
<i>p<sup>P</sup></i>	Power transferred to prosumers
<i>p<sup>Wind</sup></i>	Generated power of wind turbine

<i>P<sup>DU</sup></i>	Output generated power of dispatchable unit
<i>P<sup>load</sup></i>	Demand of consumers in MG
<i>P<sup>LS</sup></i>	load shedding power
<i>IP<sup>DR</sup></i>	Diminished DR consumption
<i>T<sup>ch</sup>, T<sup>dch</sup></i>	Number of successive charging/discharging period for ESS
<i>T<sup>on</sup>, T<sup>off</sup></i>	Number of Successive on/off period for dispatchable unit

### Parameters

<i>UR, DR</i>	Ramp up/down rate in dispatchable unit
<i>UT, DT</i>	Minimum up/down time in dispatchable unit
<i>MC, MD</i>	Minimum charging/discharging hours of ESS
<i>l</i>	Accepted amount of DR reduction
<i>ρ</i>	Electricity price
<i>IC<sup>DR</sup></i>	Encouraging cost for DR
<i>o</i>	Monetary offered by DR
<i>R</i>	Main grid flexibility limit
<i>R<sup>up</sup>, R<sup>low</sup></i>	MG upper/lower flexibility limit
<i>η</i>	Efficiency of ESS in operation
<i>μ</i>	Occurrence probability of scenarios
<i>θ</i>	Moving average parameter
<i>Φ</i>	Autoregressive parameter
<i>d</i>	White noise
<i>y</i>	Number of moving average terms
<i>x</i>	Number of autoregressive terms
<i>β, α</i>	Risk parameters
<i>σ</i>	Mean value
<i>λ</i>	Standard deviation

### Subscripts

<i>t</i>	Index for the scheduling period ( $1 \leq t \leq T$ )
<i>s</i>	Index for scenarios ( $1 \leq s \leq S$ )
<i>g</i>	Index for dispatchable units ( $1 \leq g \leq G$ )
<i>b</i>	Index for DR ( $1 \leq b \leq B$ )
<i>e</i>	Index for energy storage units ( $1 \leq e \leq E$ )
<i>w</i>	Index for wind turbines ( $1 \leq w \leq W$ )
<i>c</i>	Index for DR steps ( $1 \leq c \leq C$ )

## I. INTRODUCTION

### A. Literature review

Due to the global concern for high fuel costs and increased air pollution, over recent years, the application of renewable resources of energy has increased, which has significantly reduced greenhouse gas emissions and operating costs in energy systems [1].

However, inherent volatility and intermittency of renewable generation units have caused complexities in power systems operation and changed the typical load profiles [2]. According to difference between aggregated solar generation and consumption of residential sectors, drops at noon and peaks sharply in the late afternoon because of the sunset and residential consumption increment [3].

Hence, an abrupt change in the duck curve of load profile can be observed by the electric companies, which causes a challenging situation in the operation and control of power grids. As the penetration of renewables increases, grid operator requires more fast ramping units to cope with the challenge of sharp changes in net load to meet the supply-demand balance in the system. One inefficient and time-consuming solution for maintaining this balance is to construct and implement bulk generation units, such as thermal and hydro units, which can be ramped up/down and dispatched quickly [4]. Due to the high penetration of renewables with variable behavior in the power system, the spinning reserve of these fast units should be increased, which reduces the system efficiency and increases operational costs [5]. Moreover, in the power network, there are supply and demand management resources, such as energy storage and demand response (DR) that can mitigate the effect of variability in renewables [6].

However, in order to implement DR resources, advanced metering infrastructures should be developed in power systems, and the willingness of consumers for participating in this program is necessary. Deployment of energy storages as expensive systems needs remarkable financial investments and their application in large scale level of the power system is still not economical [7]. Using the potential flexibility in existing microgrids (MGs) at the distribution system level can offer a viable and local solution to alleviate the net load fluctuations caused by renewable generation [8]-[9]. Moreover, references [10]-[12] also address the application of MGs' potential for flexibility enhancement in the power grid. A market-oriented flexibility provision considering the ramp rate of generation units has been addressed in [10].

The complex mathematical optimization problem has been converted into mixed integer linear programming using the strong duality theorem. A model predictive control technique has been adopted in [11] for flexibility provision from grid side using cooperative optimization framework. In [12] a clustering approach for utilizing the flexibility from demand-side has been investigated. In this strategy, the individual penalty signals have been considered for activating the flexibility provision by the end-users. This paper extends the previous studies to propose an efficient method for improving the flexibility in distribution systems through MG scheduling. MG, defined as a controllable autonomous entity with a group of local power generation resources and loads, is able to operate in both grid-connected and islanded modes [13].

MGs offer significant advantages such as energy efficiency improvement, enhanced power quality, air pollution reduction and ensuring resiliency as well as reliability in power systems [14]. These small-scale energy systems have attracted the attention of power system developers over the years and encouraged them to consider MGs as a key element in the operational solutions even more in upcoming years. Therefore, it is pivotal to use an efficient control strategy for taking

advantages of installed units in MGs during the scheduling horizon. Three hierarchical control levels containing primary, secondary, and tertiary are introduced in [15].

The primary and secondary control levels focus on droop control as well as frequency/voltage restoration and adjustment [16]. In the third level, MG operator economically dispatches the generation resources to attain optimal scheduling in MG operation [17]. This paper would investigate the optimal scheduling for MGs concerning the flexibility improvement in distribution systems. There have been a significant number of studies that explore different methods in optimal scheduling of MGs, considering benefits that can be guaranteed for power systems. In [18]-[19], resiliency-oriented stochastic scheduling of MG is proposed to minimize the operation cost and reduce unintentional load shedding under weather-related incidents. Stochastic and robust coordination of DR with other generation units in MGs is investigated in [20] and [21], respectively.

An adjustable robust optimization model is presented in [22] for the operation of multi-carrier energy MGs. In order to cope with the uncertainties of complex multi-carrier energy systems, an extensive analysis has been performed in [23]. An optimal control strategy for energy storage systems within microgrids considering CVaR is developed by [24]. The authors considered two methods based on the online rolling horizon control strategy and considered the uncertainty relating to electricity pricing and demand profiles. Considering environmental concerns, a new class of MGs, named provisional MG, is introduced in [25], making MGs more compatible with the high penetration of renewable energy sources in the distribution grid.

In [26], optimal energy management of a renewable-based MG is proposed, and it is discussed that well-management of DR and electric vehicle (EVs) enhances the operator's profit. A new strategy for swapping the batteries of EVs has been investigated to enhance the flexibility of MGs. The participation of MGs in markets has been discussed in different references. In [27], market-based energy management of MGs considering the risk of uncertainties is addressed to reduce the operation costs and peak loads of MGs in power systems. Stochastic operation of multi-carrier energy MG considering the participation of the operator in day-ahead and real-time markets for minimizing operation cost is developed in [28]. An optimum strategy for renewable-based energy management system in the presence of EVs and DR was developed in [29].

## B. Objectives and contributions

Several studies have investigated the optimal scheduling of MG to improve the operator's profit, reduce operation cost, and compensate power shortages in islanding hours. Just a few studies [8], [9] and [30], however, have considered the role of MG in increasing the flexibility of the distribution system by reducing the renewables fluctuation. In [30], MG scheduling is proposed to mitigate the renewables' fluctuation just inside the MG. However, the role of MG in reducing the fluctuation of adjacent prosumers/resources has been neglected. Also, the possibility of main grid failure has not been considered. In the flexibility-related works, uncertainties, flexibility management for adjacent prosumers, islanding constraints, and an efficient methodology to capture the risk of uncertainties have not been modeled together. Table I represents the detailed contributions and novelties of this paper.

TABLE I. COMPARISON BETWEEN THE PROPOSED STUDY AND THE MOST RELEVANT STUDIES

Features	[8]	[9]	[20]	[25]	[29]	[30]	Proposed
Islanded operation	✓	×	✓	✓	×	×	✓
Demand response	✓	✓	✓	✓	✓	✓	✓
Novel risk constraint	×	×	×	×	×	✓	✓
Uncertainties	×	✓	✓	✓	✓	✓	✓
Flexibility constraints	×	✓	×	×	×	✓	✓
Prosumers demand	✓	✓	×	×	×	×	✓

This paper focuses on reducing operational costs by optimal coordination of DR and ESS under uncertainties, as well as considering a real-world situation for MG by adding islanding criteria. In this paper, a flexibility-oriented methodology is investigated, through which MG can reduce fluctuation in the distribution grid hosting high penetration of prosumers. The proposed solution can be easily deployed in the distribution systems without any needs for complex infrastructure nor significant investment costs for metering and market designing. Hence, the MG operator is able to optimally schedule the resources in MG to act as a large controllable fast-response unit for improving flexibility and mitigating sharp ramping in distribution systems. From the mathematical modeling perspective, the uncertainties associated with loads and renewables are considered with the islanding constraints. Moreover, a risk-constrained stochastic scheduling framework is developed for MG operator to mitigate the adverse effects of worst scenarios during the scheduling horizon.

Hence, the main contributions of this work are as follows:

- Developing a risk-constrained stochastic scheduling model for utilizing the potential of resources such as ESS, dispatchable units and DR in the existing MG to enhance the flexibility of distribution systems.
- Investigating the role of ramping limit and risk parameters as well as islanding constraints on the techno-economic operation.

The rest of this paper is organized as follows. Section II deals with the problem outlines. In section III and IV, problem description and formulations are described, respectively. Case studies and results are given in section V. The discussion and conclusion of this study are presented in section VI and VII, respectively.

## II. PROBLEM OUTLINE

In this section, the main parts of the MG scheduling are explained and the concept of flexibility-oriented scheduling is described.

### A. Islanding consideration

Islanding operation mode is the most salient feature of MGs when a disturbance happens in the upstream grid. However, the disturbance occurring time and period are not predetermined for the MG operator. Therefore, online generation adequacy of MG's resources should be ensured by the operator to supply the loads without interruption. A realistic islanding constraint should be implemented in scheduling problems to consider all probable disturbances at any time.  $T$ - $k$  islanding criterion is defined to respond to time-varying islanding events [30], in which  $T$  shows the total number of hours in scheduling horizon, and  $k$  denotes the number of consecutive hours that the MG can operate in islanded mode.

For example,  $T$ -1 means that the MG operator should schedule the local resources to adequately supply the loads for all probable 1-hour islanding period during  $T$  hours. With this method,  $T$  different islanding scenarios are obtained, which cover all probable islanding events with 1-hour duration. Therefore, a resilient operation with adequate online generation resources would be performed for all  $T$  scenarios. Further explanations about this multi-period islanding constraint can be found in [31].

### B. Uncertainty modeling

Different uncertainties associated with the renewables and load should be considered in a realistic scheduling model. As stated in (1), the Gaussian probability distribution function is used to simulate the uncertainty of load [20].

$$f_d(p^{load}) = \left( \frac{1}{\sigma\sqrt{2\pi}} \right) \exp \left( -\frac{(p^{load} - \lambda)^2}{2\sigma^2} \right) \quad (1)$$

The ARMA model, showing the forecasting value of a parameter as a linear function of historical data, is employed for modeling the uncertainties associated with renewables [27]. The mathematical description of this time series model is as (2).

$$r_t = d_t - \sum_{i=1}^y \theta_i d_{t-i} + \sum_{j=1}^x \phi_j r_{t-j} \quad (2)$$

By applying the scenario generation algorithm [27], a vector including all the scenarios with the same probability is obtained. To satisfy the accuracy of results and to diminish the computational times of the scheduling, a reduced number of scenarios is acquired using SCENRED tool in GAMS [32].

## III. PROBLEM DESCRIPTION

Various generation units and residential consumers are commonly connected to a distribution feeder of the power system. Nowadays, a significant percentage of the residential consumers can generate electricity using the local installed generation units, especially rooftop solar panels, and these groups of consumers are known as prosumers [8]. In this regard, consider a typical distribution feeder to which an MG and aggregated prosumers are connected. The power that the upstream grid should supply to this typical feeder is equal to the MG and prosumers' net loads, as shown in (3). The net load is defined as the difference of power generation and consumption in these small energy systems.

$$P_{t,s}^J = P_{t,s}^M + P_{t,s}^P \quad (3)$$

The net load of prosumers is extremely variable and uncontrollable from the upstream grid perspective due to the high penetration of solar-based generation units. Although the MG's net load is also variable, it can be controlled as its operator has access to the dispatchable units, DR and ESSs [30].

To restrict the intense ramping of the prosumers' net load, the power supplied to the consumers at the distribution level should be limited as (4). This constraint has just been considered for the transferred power of MG in [30], but in this study, it is added not only for MG, but also for the prosumers connected to the same distribution feeder. The main grid entity considers this constraint based on the desired level of flexibility and the forecasting data [8].

To reach this target, MG operator should contribute to providing a suitable level of flexibility by rescheduling its resources and modifying the transferred power with the main grid. As this service is provided for the main grid through the flexibility-oriented scheduling of MG, the main grid entity can compensate it by incentivizing the MG operator. In fact, this flexibility-oriented scheduling is not the most economical scheduling for MG [9] because the grid ramping limitation represented in (4) should be used in MG power transferred constraint, which imposes additional costs in daily operation. The explanations on this constraint will be presented in the mathematical formulation section.

$$-R \leq P_{t,s}^U - P_{t-1,s}^U \leq R \quad (4)$$

Fig. 1 illustrates the flexibility-oriented stochastic scheduling framework of MG for the day-ahead market. In this scheduling process, the MG operator seeks to attain the minimum operation cost for MG and to ensure the flexibility for the distribution grid. First of all, the obtained historical data of loads and wind generation are collected, as shown in box 1. These data are given to the scenario generation and reduction algorithms, which are shown in box 2. To be more specific, we use ARMA and normal distribution function to generate the set of renewables and load data, respectively. The set of scenarios are processed using fast forward scenario reduction method [27]. The reduced scenarios are combined with islanding scenarios to provide the completed set of uncertain inputs. Moreover, the certain inputs presented in box 3 are the other set of inputs, known as deterministic inputs. These data are related to VoLL, contract of DR, technical information of MG's components, the flexibility limit and day-ahead market price. The flexibility level (R) is determined by the main grid operator.

These certain and uncertain inputs are simultaneously given to the optimization scheduling tool. Before running the scheduling problem, the risk preference level is set on the desired value as depicted in box 5 to manage the uncertainties. Then, the scheduling is executed as illustrated in box 6, and during the scheduling process seeks to attain the minimum operation cost for MG and to ensure the desired flexibility level specified by operator in the distribution grid. The outcomes of this problem are classified as the first stage and second stage decisions, which are shown in box 7 and 8, respectively.

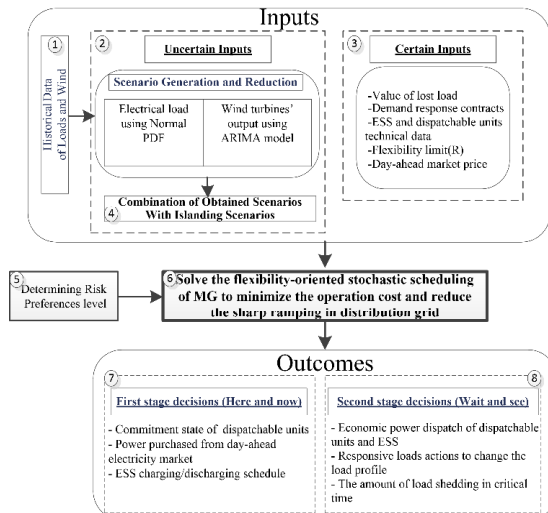


Fig. 1. Presented framework for flexibility-oriented scheduling of MG

The first stage decisions are related to the commitment program of ESS charging/discharging and dispatchable units for a day-ahead horizon. Also, the purchased power from the electricity market is determined at this stage. Decisions at the second stage are associated with the economic dispatch of ESS and the dispatchable unit, DR implementation and unintentional load shedding. The power dispatch of resources and purchased power are adjusted in a way to capture the variability of prosumers' net load.

In the next section, the mathematical formulation of objective function and constraints are presented.

#### IV. PROBLEM FORMULATION

The flexibility-oriented scheduling problem of MG is formulated as a mixed-integer linear programming (MILP). The objective function and the relevant constraints are presented as follows:

The goal of the scheduling program is to minimize the objective function (OF) proposed in (5) over the uncertain set of variables and time period. This function includes the cost for purchased power from the main grid, operation costs of dispatchable units, unintentional load shedding costs and financial incentives for DR's action.

$$OF = \sum_{t \in T} \left( \rho_t P_t^M + \sum_{s \in S} \sum_{g \in G} \mu_s F(P_{g,t,s}^{DU}) + \sum_{s \in S} \mu_s P_{t,s}^{LS} VOLL + \sum_{s \in S} \sum_{b \in B} \mu_s IC_{b,t,s}^{DR} \right) \quad (5)$$

##### A. Constraints

The summation of power purchased from the electricity market and generation units as well as the reduced consumption of consumers using DR and load shedding programs should be equal to the MG load. This power and supply balance is presented in (6).

$$P_t^M + \sum_{g \in G} P_{g,t,s}^{DU} + \sum_{e \in E} P_{e,t,s}^{ESS} + \sum_{w \in W} P_{w,t,s}^{wind} + \sum_{b \in B} IP_{b,t,s}^{DR} + P_{t,s}^{LS} = P_{t,s}^{Load} \quad (6)$$

In (7), the power purchased from the main grid is restricted between the minimum and maximum values. These values are considered based on the power capacity of the main grid distribution feeder. The binary parameter, i.e.,  $\zeta$  whose value is 1 in grid-connected condition and 0 in islanded mode, is added to ensure that the MG can provide power only in grid-connected mode, while MG should rely on its local resources in islanded mode.

$$P_{\min}^M \zeta_{t,s} \leq P_t^M \leq P_{\max}^M \zeta_{t,s} \quad (7)$$

The flexibility limit of the main grid shown in (4) is converted to a new constraint on the net load of MG, which is presented as (8a). The upper and lower limits of this constraint are obtained based on the net load of prosumers, as shown in (8b) – (8c). The upper and lower values are dependent on the scenario and time in this scheduling framework.

$$R_{t,s}^{low} \leq P_t^M - P_{t-1}^M \leq R_{t,s}^{up} \quad (8a)$$

$$R_{t,s}^{low} = -R - (P_{t,s}^P - P_{t-1,s}^P) \quad (8b)$$

$$R_{t,s}^{\eta} = R - (P_{t,s}^P - P_{t-1,s}^P) \quad (8c)$$

The operational constraints for dispatchable units are defined as (9a) – (9e) [30]. The generated power of these units meets the constraint (9a), in which the binary variable  $I_{g,t}$  denotes the commitment status of the units. The ramp-up/down rate, limiting the provided power in two consecutive hours, follows (9b) – (9c). Moreover, the minimum up/down hours, showing the consecutive on and off operation hours of the units are presented in (9d) – (9e).

$$P_g^{DU,\min} I_{g,t} \leq P_{g,t,s}^{DU} \leq P_g^{DU,\max} I_{g,t} \quad (9a)$$

$$P_{g,t,s}^{DU} - P_{g,t-1,s}^{DU} \leq UR_g \quad (9b)$$

$$P_{g,t-1,s}^{DU} - P_{g,t,s}^{DU} \leq DR_g \quad (9c)$$

$$UT_g (I_{g,t} - I_{g,t-1}) \leq T_{g,t}^{on} \quad (9d)$$

$$DT_g (I_{g,t-1} - I_{g,t}) \leq T_{g,t}^{off} \quad (9e)$$

The ESS follows several operational constraints presented in [20]. The supplied power based on the charging and discharging of ESS in each time slot is calculated by (10a). The discharging and charging process of ESS follows constraints (10b) and (10c), and these limitations are confined to the minimum and maximum capacity in each hour.

Constraint (10d) should be added to the list of constraints to show that the charging and discharging procedures do not coincide. The state of charge (SOC) in ESS is calculated with (10e). This constraint provides the mathematical relation between the SOC in the previous hour, ESS efficiency, and ESS power in charging and discharging cycles. The SOC should be kept between the maximum and minimum ranges, as stated in (10f). Constraints (10g) and (10i) denotes the minimum consecutive hours during which ESS keeps charging and discharging states. Indeed, to avoid the sea-side operation of the ESS, the dynamic behavior of the storage unit in charging and discharging modes are modeled. In this case, when the ESS starts working in charging mode, the associated counter will start. The corresponding binary variables, ( $u_t - u_{t-1}$ ), used for this purpose. After passing the minimum charging period, MC, the ESS can move to another state, i.e. idle or discharge mode. The same conditions have been considered for discharging mode operation. The minimum charging/discharging periods can be different; therefore, two different dynamic equations have been considered in the mathematical model.

$$P_{e,t,s}^{ESS} = P_{e,t,s}^{ESS,dch} - P_{e,t,s}^{ESS,ch} \quad (10a)$$

$$P_e^{dch,\min} u_t \leq P_{e,t,s}^{ESS,dch} \leq P_e^{dch,\max} u_t \quad (10b)$$

$$P_e^{ch,\min} v_t \leq P_{e,t,s}^{ESS,ch} \leq P_e^{ch,\max} v_t \quad (10c)$$

$$u_t + v_t \leq 1 \quad (10d)$$

$$C_{e,t,s} = C_{e,t-1,s} + \eta P_{e,t,s}^{ESS,ch} - (1/\eta) P_{e,t,s}^{ESS,dch} \quad (10e)$$

$$C_e^{\min} \leq C_{e,t,s} \leq C_e^{\max} \quad (10f)$$

$$MC_e (u_t - u_{t-1}) \leq T_{e,t}^{ch} \quad (10g)$$

$$MD_e (v_t - v_{t-1}) \leq T_{e,t}^{dch} \quad (10i)$$

Constraint (11) guarantees that the unintentional load shedding should be less than the total load in MG.

$$0 \leq P_{t,s}^{LS} \leq P_{t,s}^{Load} \quad (11)$$

Implementing the Interruptible/Curtailable (I/C) method as a DR plan, the incentive signal is sent to the responsive loads by the MG operator. Once the responsive loads receive this signal, they make the decision whether to take part in I/C plan or not. Industrial loads commonly follow this plan, for which consumption reduction steps are defined. Different steps are considered with specific prices based on the financial agreement between the industry and utilities. The industrial loads response as (12a)–(12d) [21].  $L_{b,c}$  and  $L_{b,\min}$  are associated with the maximum and minimum reductions of consumption in steps  $c$  and one, respectively. In addition, the aggregation of the reduced consumption should not be greater than  $L_{b,c}$  at each period of time [27].

$$L_{b,\min} \leq l_{b,1,t,s} \leq L_{b,1} \quad (12a)$$

$$0 \leq l_{b,c,t,s} \leq (L_{b,c+1} - L_{b,c}); \forall c = 2, 3, \dots \quad (12b)$$

$$IP_{b,t,s}^{DR} = \sum_{c \in C} l_{b,c,t,s} \quad (12c)$$

$$IC_{b,t,s}^{DR} = \sum_{c \in C} o_{b,c} l_{b,c,t,s} \quad (12d)$$

Since the flexibility challenge in the distribution system is highly related to the intense penetration of intermittent renewables and various consumers, it is crucial to use an applicable methodology to handle the risk of their uncertainties. Risk management constraints as presented in (13a) – (13d) are used to mitigate the effect of their uncertainties. In the presented formulations, CVaR is defined as (13a), showing the expected cost of  $\alpha\%$  of all scenarios [27]. This percentage of scenarios is selected as costly or worst scenarios [19]. Using (13b) enables the operator to determine the risk preferences level by adjusting the risk parameter, i.e.  $\beta$  to compromise between the risk aversion and expected cost [19]. Closer values to 1.0 address a risk-averse decision-maker, while a risk-taker person can be modeled by a value of  $\beta$ , which is greater than one [30].

$$CVaR = \frac{1}{(1-\alpha)} \sum_{s \in S} \mu_s \psi_s + VaR \quad (13a)$$

$$CVaR - \beta \times OF \leq 0 \quad (13b)$$

$$\sum_{t \in T} \rho_t P_t^M + \sum_{g \in G} F(P_{g,t,s}^{DU}) + \quad (13c)$$

$$+ P_{t,s}^{LS} VOLL + \sum_{b \in B} IC_{b,t,s}^{DR} - VaR - \psi_s \leq 0 \quad (13d)$$

$$\psi_s \geq 0$$

## V. CASE STUDIES AND RESULTS

### A. Test system

The flexibility-oriented schedule of the understudy MG is performed considering the interaction with prosumers on a typical distribution feeder, as shown in Fig. 2. Table II represents the technical characteristics of the installed units in MG. The trends associated with the demand of MG and prosumers and the renewable generation outputs are given in Figs. 3 and 4. The data related to the wind and market price is obtained from [27], [30].

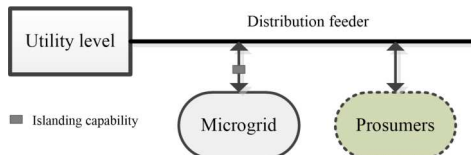


Fig. 2. A typical distribution feeder consisting of an MG and prosumers

TABLE II. SPECIFICATION OF DISPATCHABLE UNITS, ESS AND WIND TURBINES

Generation units	Operation Cost (€/MWh)	Capacity (MW)	Min Up/Down hours	Ramp Up/Down rate (MW/h)
DU 1	39.1	0.8-3	2	1.5
DU 2	67	0.5-3	1	2
DU 3	75	0.4-2	1	1.6
ESS	-	0.4-2	5	-
WTs	-	0-1	-	-

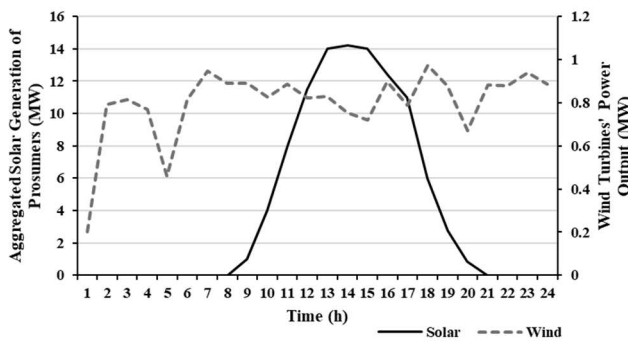


Fig. 3. Generated power by wind turbines and residential rooftop solar panels

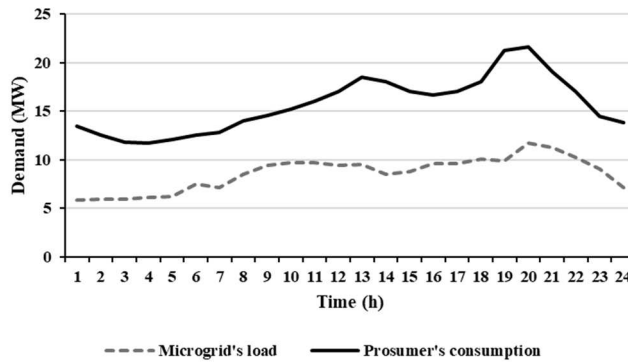


Fig. 4. Integrated demand of MG load and prosumers

The characteristic of responsive loads, known as DR resources is borrowed from [30]. The consumption of these loads can be reduced up to the specific capacity in each step based on the received financial incentives.

In the following section, the scheduling program is executed in CPLEX using min gap of 0% [33] on a computer with 8GB RAM and 2.2 GHz processor.

### B. Numerical studies

A set of uncertainties should be generated in order to use them as the input data of the scheduling problem. By implementing scenario generation methods, explained in II, a vector of 1000 scenarios consisting of three uncertain parameters, i.e. prosumers' load, MG load and renewable generation output, is generated for 24 hours.

In order to hedge the complexity of computation, scenario reduction technique is used through the *SCENRED tool* in the *GAMS software* to acquire three scenarios, each consisting of a vector of demand profiles of MG and prosumers as well as renewable generation outputs. These scenarios are multiplied with 24 islanding scenarios, so 72 scenarios are obtained for the scheduling program.

In the following, the advantages of the presented scheduling model over the traditional scheduling model are investigated. The sensitivity analysis will be presented to show the competence of the proposed model.

**Case 1 (base case):** MG optimal scheduling without considering flexibility constraints

In this case study, the traditional scheduling model for MG is executed without any concern for improving the flexibility of the distribution grid. Hence, the flexibility constraints presented in (8a)-(8c) are omitted from the mathematical formulations shown in this paper. The risk parameters,  $\beta$  and  $\alpha$ , are adjusted to 1.05 and 0.95, respectively in order to manage the set of uncertainties considered in the day-ahead decisions. The line capacity between the MG and the main grid is 10 MW.

**Case 2:** MG optimal scheduling considering flexibility constraints

This scheduling model delves into the flexibility improvement in distribution grid by reducing the variability of the grid net load caused by prosumers. Hence, the desirable level of flexibility for main grid will be translated to the constraints that are used in MG scheduling. Using these constraints, represented in (8a)-(8c), provides the flexibility-oriented scheduling for MG, and enables the MG operator to use the potential of installed resources for supporting the distribution grid flexibility. The risk parameters and the line capacity are the same as case 1, and the desired ramping (flexibility limit) for the main grid is set to 3 MW/h.

The MG scheduling executed in case 1 is to minimize the operation cost without the contribution of MG in flexibility improvement of the distribution grid. The operation cost is calculated as € 8544.6 in this case, while the obtained operation cost is € 9282.3 for case 2. In case 2, the operator reschedules the resources not only for minimizing operation cost, but also for improving system flexibility considering the ramping limit imposed by the main grid entity. It is shown that the flexibility-oriented scheduling (Case 2) resulted in a 6.64% increase in operating cost in comparison with the scheduling performed in case 1. This additional cost imposed on the MG is the cost of ensuring flexibility, which means that the MG operator changes the power transferred with the upstream grid and modifies the generation dispatches of resources in MG to meet the grid flexibility requirement.

Therefore, the main grid entity should compensate for this additional cost by suggesting some incentives to the MG operators. Practically, this entity should offer financial support equal or more than the imposed cost to the MG operators in order to encourage them to participate in this flexibility-oriented scheduling. If the main grid entity does not pay this additional cost, which is imposed in flexibility-oriented scheduling, the MG operator would not be encouraged to take part in providing such flexibility services for the distribution grid.



Fig. 5 demonstrates the expected value of the net load for distribution feeder in both cases. As illustrated in this figure, it is evident that the presence of MG in distribution feeder would increase the variability of the distribution feeder net load in case 1. For instance, the grid operator can observe a maximum of 6.84 MW/h net load change between the hours 17 and 18. This sharp ramping, which can also be seen in hours 18 and 19, results in an undesirable average peak of 23.64 MW between the hours 19 and 21 for the main grid. On the contrary, observing the net load of distribution feeder in case 2, it is realized that the flexibility-oriented scheduling of MG modifies the net load in a way to reduce the intensity of fluctuations. The ramping between the hours 17 and 18 is 1.96 MW/h, and the average peak attains to 19.77 MW between hours 19 and 21. This average peak is 3.87 MW fewer in comparison with the average peak in case 1. As stated before, these advantages would not be seen in the distribution grid without changing the purchased power (MG net load) and the scheduling of installed resources in MG in a flexible way.

To be more specific, the profiles of purchased power, known as MG net load, are shown in Fig. 6 for both cases. The MG operator buys power to supply its loads in case 1, where there is no collaboration among the grid and MG to support flexibility after the solar generation falls at noon. However, in case 2, the MG operator reduces the purchased power once the generation of solar units in prosumers' side becomes fewer after 17 pm. Therefore, the sharp changes in the net load of prosumers will be controlled by this method of scheduling. To compensate the power shortages in MG for case 2, which is a result of reducing transferred power, the operator commits the generation units in more hours.

Table III tabulates the commitment status of dispatchable units for cases 1 and 2, respectively. The Changes of commitment status in case 2 compared with case 1 are indicated by highlighted squares. In case 2, unit 3 is committed in a longer period of time to support flexibility.

Moreover, the power dispatched of dispatchable units and also ESS is changed in almost all hours to ensure the grid required flexibility.

Fig. 7 compares the ESS power dispatched in both cases. For case 1, the ESS discharges between the hours 8 and 24 to prevent load shedding and keep the scheduling secure in probable islanding hours, while in case 2, it can be seen that the ESS starts discharging once the solar radiation decreases as well as the demand increases, and consequently, the flexibility of grid would be satisfied.

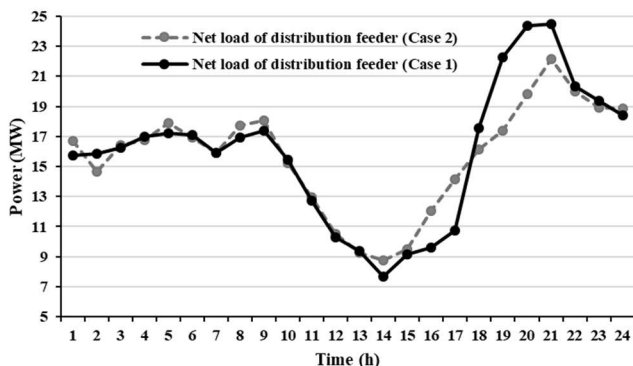


Fig. 5. Net load of distribution feeder for cases 1 and 2

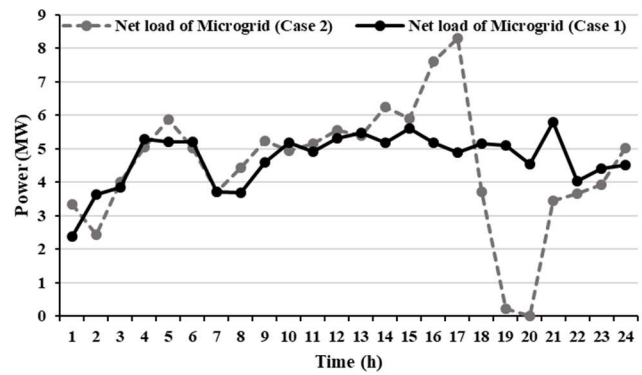


Fig. 6. Net load of MG for cases 1 and 2

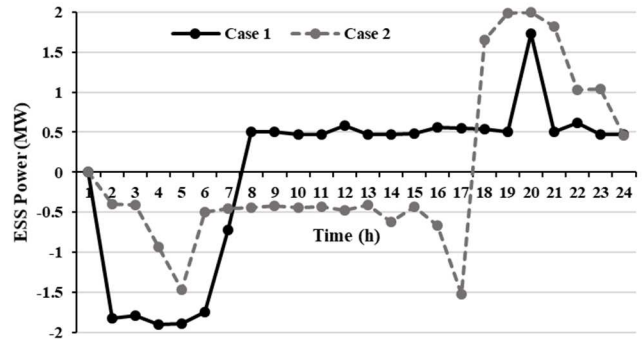


Fig. 7. Power dispatch of ESS for cases 1 and 2

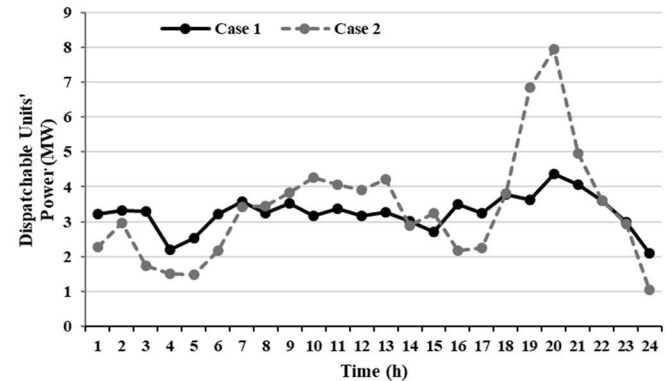


Fig. 8. Power dispatch of dispatchable units for cases 1 and 2

TABLE III. UNIT COMMITMENT STATUS OF THE DISPATCHABLE UNITS

Time	Hours (1-24)																												
DU 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
DU 2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
DU 3	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	0	0	

Similarly, as shown in Fig. 8, dispatchable units generate more power up to the maximum capacity, 8 MW, during the critical hours (i.e. decreasing solar generation and increasing load demand) in case 2 compared with case 1. This flexible response reduces the need for MG for providing power from main grid, so the fluctuations in distribution grid are decreased.

**Case 3:** After validating the competence of the flexibility-oriented scheduling (case 2) through the comprehensive comparisons, the influence of changing the risk parameter, ramping limit, and islanding duration will be investigated in this case.

The chosen value of  $\beta$  specifies the risk preference level in decision-making problems. As mentioned before, risk-averse operators select a value near to one, but risk-taker individuals select a larger value.

The effect of adjusting  $\beta$  on expected operation cost versus CVaR is illustrated in Fig. 9. Setting  $\beta$  on smaller values brings about higher operation cost and lower CVaR value. This risk-averse strategy happening by choosing a smaller value for  $\beta$  makes the MG scheduling robust against the occurrence of worst scenarios in the operation time horizon; hence, it reduces the operation cost of worst scenarios. This explains why the CVaR value, which is the expected operation cost of the worst scenarios, becomes smaller.

However, selecting a larger value diminishes the operation cost and augments the CVaR value. By choosing the amount of  $\beta$  more than 1.15, the operation cost will stay constant and equal to € 8821.1. It means that the risk constraints do not affect mitigating worst scenarios after a specified value, here 1.15. Hence, this point (i.e.,  $\beta = 1.15$ ) is a saturation point of this risk-constrained scheduling, and the program will be risk-neutral for a larger value than 1.15.

The operation costs of MG in flexibility-oriented scheduling are shown in Table IV for different ramping limits of main grid. It is dignified that the operation cost is increased by decreasing the ramping limit. This imposed cost in MG scheduling should be compensated by the main grid entity.

In this table, when the ramping is set to 2.5, the operation cost is increased significantly to meet the distribution grid desirable flexibility. Hence, the increments associated with the smaller value of the ramping limit is happened because of the following reasons:

- MG operator should commit more units and dispatch them in uneconomic hours to support the requested level of flexibility from the main grid operator.
- MG operator should reshape the demand pattern of customers using the DR resources and also load shedding, and these changes will result in additional costs for MG.

Fig. 10 shows the influence of DR and load shedding on the demand profile of MG. The amount of load shedding, which is an undesirable event for consumers, can be reduced by improving the penetration of DR resources and take advantage of the flexibility provided by DR resources.

Choosing an appropriate ramping limit is so crucial because the flexibility-oriented scheduling should satisfy the economic operation for MG and improve the flexibility in the distribution grid. In future work, we will investigate the flexibility studies from the main grid entity's perspective.

Adding islanding constraints to the problem enables the MG operator to perform a resilient operation against probable incidents. The incidents may happen as a result of different reasons such as equipment failure, voltage/frequency fluctuation, and weather-related catastrophes, each of which can result in different islanding hours for MG [19]-[20]. Therefore, the operator should be aware of the history of outages in the region MG located. This is important in choosing an appropriate  $k$  in islanding criteria.

Fig. 11 shows a sensitivity analysis for the islanding duration between the hours of 0 to 4. It is shown that the operation cost is increased by increasing the value of  $k$ . The

additional cost imposed to the MG operation in islanded modes is the cost for improving distribution system resiliency and ensuring reliability for consumers in MG because in islanding situation, the operator should commit more units and implement DR resources to satisfy resiliency and reliability.

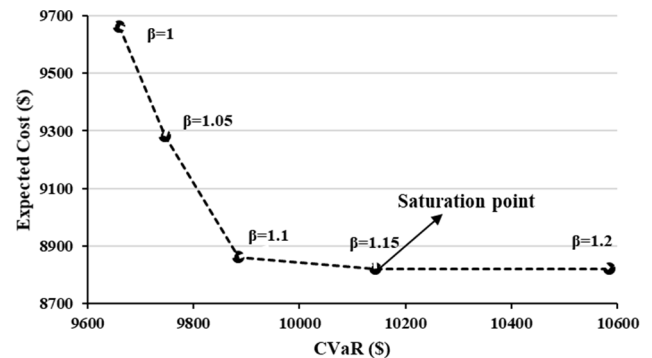


Fig. 9. CVaR versus expected cost trend

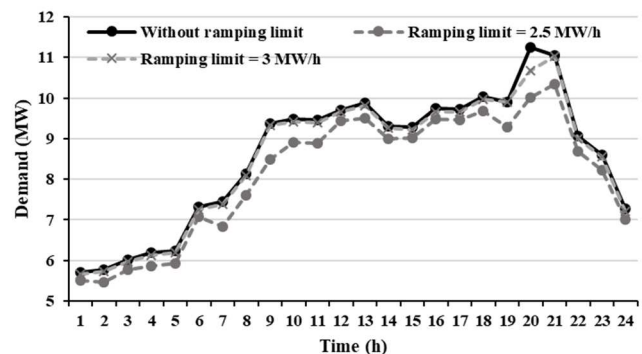


Fig. 10. MG demand profile with different ramping limits

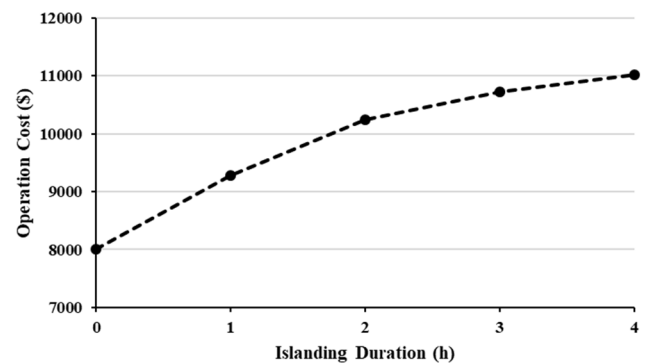


Fig. 11. Operation cost versus islanding duration

TABLE IV. EXPECTED OPERATION COST OF MG WITH DIFFERENT RAMPING LIMITS

Ramping limit (MW/h)	2.5	3	4	5	6	7
Operation cost (€)	16672	9282.3	8651	8588	8562	8548

## VI. DISCUSSION

The presented flexibility-oriented scheduling for MGs has improved the distribution system flexibility and reduced the operational costs simultaneously by utilizing ESS, dispatchable units, DR and purchased energy from main grid. This efficient



scheduling has modified MG operation's economic and technical aspects compared to the traditional schedule and provided significant advantages for the power system. According to the numerical studies, the following features were obtained in the presented model with the ramping limit.

- The flexibility-oriented scheduling led to just 6.64% additional cost in MG operation compared with the traditional scheduling neglected the flexibility limit. However, the net load ramping in the distribution system was reduced to a desirable level for the main grid once the solar generation dropped, and load demand increased. Also, the average peak in net load was 16.37% reduced. The additional operation cost should be paid to the MG operator as an incentive if the main grid entity expects the MG operator to participate in the flexibility-oriented scheduling.
- According to table 3, reducing the value of ramping (flexibility) limit resulted in higher MG operation cost and more flexible operation. This meant that this value had to be adjusted considering the economic aspect for MG operation and the desired flexibility for the distribution system. A predefined contract between MG and the main grid entity needs to be assigned to satisfy them.
- The ESS and dispatchable units were dispatched in a way to support the MG power shortages as the MG operator had to reduce the transferred power with the main grid to prevent the sharp changes in the distribution grid net load. As shown in Figs. 7 and 8, most of the dispatched power was assigned between the hours 19 and 21 when the net load increased.
- It was shown that utilizing DR resources guaranteed the flexibility requirement by reshaping the MG consumption pattern, which was illustrated in Fig. 10. As the smaller value of the ramping limit caused some power deficiency in MG, an appropriate action of I/C DR compensated the generation shortage.
- As shown in Fig. 11, increasing the value of  $\beta$  led to a lower operation cost, but made the risk-averse scheduling ineffective. It was also explained that setting a value near to 1 made the scheduling robust against worst scenarios.
- Islanding criteria enabled the MG operator to switch to islanded operation once an incident happened in the upstream level. This consideration made the scheduling realistic and guaranteed a resilient and reliable operation for the distribution grid and consumers.

Due to the flexibility advantages of the proposed model shown in numerical analysis, and the easy application of it on the real power network, the need for upgrading the lines, generation units, market design, and communicational infrastructures in power system for improving flexibility has been diminished. In fact, upgrading the power system needs high investment costs, and multi-step studies in optimal placement and sizing of equipment by the complex planning programs. Therefore, significant cost will be saved for main grid entity by using the proposed model, and they can compensate for the contribution of MG in flexibility-oriented scheduling by small expense during the days.

## VII. CONCLUSION

A flexibility-oriented stochastic scheduling model for MG was developed in this paper to address the net load ramping

caused by the variable and uncertain behavior of prosumers in the distribution system. Risk constraints were added to the mathematical scheduling formulation to capture the uncertainties associated with loads and renewables. Islanding constraints were considered in the proposed scheduling model to ensure a reliable MG operation for consumers and satisfy the resiliency in the distribution network. To meet these goals besides providing an acceptable level of flexibility in the distribution grid and economic operation, the MG operator implemented DR resources, dispatchable units and ESS in an efficient way through an optimal decision-making approach used in the presented model. The simulation results in this study shows that the flexibility-oriented scheduling can provide a local solution for reducing the net load variability at the distribution level and satisfied a least-cost operation under the uncertainties. In addition, the application of ramping constraints in the problem formulation reduced the need for high investment costs for installing dispatchable units and ESS in the distribution grid.

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