

Optimal Bidding of a Group of Wind Farms in Day-Ahead Markets Through an External Agent

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Abstract—In deregulated electricity markets, producers offer their energy to the day-ahead market. As the subsidies for renewable producers are becoming lower and lower, they have to adapt to market prices. This paper models the energy trading in the day-ahead market for wind power producers. Different strategies are proposed for this purpose: 1) several wind farms offering their energy separately to the day-ahead market; 2) the same strategy as in 1) but compensating the imbalance among different wind farms; and 3) a joint offer involving several wind farms through an external agent in order to minimize the imbalances between the offer and the final power generation. The strategies are modeled with stochastic mixed integer linear programming and Conditional Value at Risk is used to consider the risk assessment. The expected profit including risk aversion is maximized for each wind power producer and for the set of wind power producers in the case of a joint offer. A comparison of the different cases is described in detail in a case study and relevant conclusions are provided.

Index Terms—Conditional Value at Risk (CVaR), day-ahead market, energy trading, external agent, imbalances, stochastic mixed integer linear programming, wind power.

NOMENCLATURE

A. Indices and Numbers

t	Index of time periods from 1 to N_T .
w	Index of scenarios from 1 to N_W .
i, j	Index of wind farms from 1 to N_I .

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B. Parameters

α	Per unit confidence level.
\bar{P}_i	Capacity of wind farm i [MW].
β	Risk aversion of the wind power producers.
c^g	Wind farm operation cost [euros/MWh].
g_{wtwi}	Power produced by wind farm i in period t and scenario w [MW].
λ_{tw}	Day-ahead market price in period t and scenario w [euros/MWh].
$\lambda_{tw}^{DW}/\lambda_{tw}^{UP}$	Downward/upward energy prices resulting from the balancing market in period t and scenario w [euros/MWh].
$\lambda_{tw}^+/\lambda_{tw}^-$	Positive/negative imbalance market prices in period t and scenario w [euros/MWh].
ρ_{wi}	Probability of occurrence of scenario w of wind farm i .
$\rho_{\Omega_w f_i}$	Set of probabilities of wind farm i .

C. Variables

b_{wti}	Power offer to the day-ahead market associated to wind farm i in period t [MW].
$CVaR_i$	Conditional value at risk of wind farm i [euros].
Δw_{twi}	Imbalance between the actual wind production and the power offer associated to wind farm i in period t and scenario w [MW].
Δw_{twi}^-	Negative imbalance between the actual wind production and the power offer associated to wind farm i in period t and scenario w [MW].
Δw_{twi}^+	Positive imbalance between the actual wind production and the power offer associated to wind farm i in period t and scenario w [MW].
PF_i	Total expected profit of wind farm i [euros].
PF_{twi}	Expected profit of wind farm i in period t and scenario w [euros].
\overline{PF}_{ti}	Mean expected profit of wind farm i in period t [euros].

RV_i	Total expected revenue of wind farm i from selling energy in the day-ahead market [euros].
RV_{twi}	Expected revenue of wind farm i from selling energy in the day-ahead market in period t and scenario w [euros].
η_{wi}	Auxiliary variable of wind farm i in scenario w used to compute CVaR [euros].
ζ_i	Value at risk of wind farm i [euros].
σ_t	Profit standard deviation in period t [euros].

D. Binary Variables

u_{twi}	0/1 variable that is equal to 1 if the imbalance in period t , wind farm i and scenario w is negative, and 0 otherwise.
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In the rest of the document, those parameters and variables that do not depend on the wind farm do not have the subscript i . This is the case of the joint offer, which will be explained in Section II-B. Additionally, when referring to the sum of the variables of all wind farms, the subscript does not appear.

I. INTRODUCTION

WIND power penetration in electric power systems has experienced a large increase in the last years [1]. In the case of the Spanish electricity system, at the end of 2013 the installed wind power accounted for more than 20% of the total installed capacity. Also, more than 20% of the total annual electric energy was produced by wind farms [2].

In most European deregulated electricity markets, producers have to send their energy offers to the day-ahead market for the next day. Those energy quantities that have been selected are paid at the marginal market price obtained for each hour. The majority of day-ahead electricity markets are adapted to conventional generation, such as thermal units, and the time span between the market closure and the delivery hour can be up to 38 h (Spain). In the case of wind producers, due to the wind uncertainty and the long time span, the generation is different from the schedule, incurring in an imbalance.

Energy trading firms are becoming important participants in the energy markets of many countries. The rules of the liberalized Iberian electricity market allow non-dispatchable or renewable generators (special regime) to compensate their imbalances by making a joint offer to the day-ahead market through an external agent [3]. Some wind farms in different network locations can compensate their imbalances and, thus, increase their profits. However, in our paper, several models based on different strategies for trading energy for a set of wind farms to the day-ahead market are presented. The main purpose of this paper is not the minimization of imbalances, but the maximization of the wind power producers profits.

In previous research, the optimal coordination between pumped-storage units and wind farms in electricity markets has been studied [4]–[6], with the result that, when maximizing the

coordinated profit, the total imbalance decreases. Stochastic models for optimal offering strategies for a wind power producer to a short-term electricity market have also been studied [7]–[10].

In [7], a statistical method is used for modeling wind behavior. The paper is focused on determining the amount of energy sold in a short-term electricity market. Several policies for wind energy are described and evaluated under different imbalance price assumptions. It is assumed that the energy will be sold in a short-term electricity market at a fixed price with fixed volume blocks. The energy output of the wind farm is normalized by its rated energy output and divided into energy bands. Each band has a different Markov probability. Risk aversion is considered for the cases where a very unfavorable event is likely to occur.

Authors in [8] address the same problem as in [7] through a stochastic optimization model using mixed-integer programming. With this method, it is possible to compute a higher amount of wind power and price scenarios than in previous works. The method uses statistical data about the forecast error. In [9], historical uncertainty information is taken into account to improve the value of wind power forecasts. Mathematical improvements to the model in [8] are incorporated in [10]. Risk management is taken into account and an adjustment market is proposed in order to minimize the imbalances considering certainty gain. A realistic case study, based on a wind farm in Portugal is carried out in [11] considering risk management. An adjustment market for minimizing imbalances is also proposed in [12]. Apart from introducing an adjustment market, there are other ways of minimizing imbalances, such as combining wind power with energy storage [13]–[15] or combining wind and hydro power generation, as in [5], [16], [17].

The contributions of this paper are stated below:

- 1) A stochastic linear programming model to trade energy to day-ahead electricity markets with a balancing mechanism for wind power producers is presented;
- 2) The model in 1) is available for electricity markets with energy prices equal to zero, which is the case of the systems with high renewable power penetration;
- 3) Consideration of risk through the CVaR methodology to obtain the maximum profit by limiting the scenarios with the worst profits;
- 4) A mechanism for making an optimal joint offer to the day-ahead market for a group of wind farms through an external agent is presented;
- 5) A detailed analysis of a case study in Spain comparing different mechanism for trading energy in the day-ahead market clustering several wind farms.

The remainder of this paper is organized as follows. In Section II, the mathematical formulation of the problem is presented. This section also provides a brief overview of the balancing market framework and the description of the different mechanisms for day-ahead market trading. Scenario generation and scenario reduction are explained in Section II. In Section III, a case study is described and the results are discussed. In Section IV, some relevant conclusions are provided. The model used for the wind speed forecast is explained in the Appendix.

II. PROBLEM DESCRIPTION

A. Balancing Market Framework

In European pool-based electricity markets, there are different types of short-term markets, depending on the time frame: the day-ahead market, the intra-day or adjustment market and the balancing market. In other electrical systems, there can be different short-term electricity markets. Hereafter, the assumptions made in this document are related to electricity markets whose electrical systems are integrated in the ENTSO-E (European Network of Transmission System Operators for Electricity) [18]. After day-ahead market clearing, it is possible to make corrective actions in the adjustment market to increase the producers' revenues. For wind power producers, the adjustment market is very important because it can update the latest information and forecasts considering certainty gain. If the wind power producers are not price makers, their bidding strategies do not affect market prices.

In order to maintain the balance between generation and consumption, there is a balancing market (real-time) which allows the system to operate under normal conditions. In this market, negative imbalance prices and positive imbalance prices are obtained for negative and positive imbalances, respectively. If the imbalance is negative (less generation than the one offered), the producer incurs in a penalty, and has to pay the energy difference at the negative imbalance price. If the imbalance is positive (more generation than the one offered), the producer is paid the energy difference at the positive imbalance price.

In the present document, a single-node model is considered, ignoring transmission constraints. Hence, all wind farms are assumed to be connected to the same node, so that market prices are the same for all the producers. In [19], it is proved that a uniform marginal price can be used without affecting the total economic surplus. However, this is only a simplification of the model. According to [20], locational marginal pricing is the best option for a market design considering congestion management. Some countries or states split up the electric systems in small zones with radial configuration and consider zonal prices, although the definition of zones would be difficult in many European areas, due to the complex network topology. In [21], the effects of congestion management are explained, formulating a bilevel stochastic optimization model to obtain the optimal bidding strategy for a wind power producer in the short-term electricity market.

When an imbalance occurs, it has to be compensated. In Spanish electricity markets, as well as in the rest of European electricity markets, the prices for compensating imbalances come from the balancing market. In this market, upward prices (λ_{tw}^{UP}) and downward prices (λ_{tw}^{DW}) are obtained. These prices depend on the imbalance of the global electrical system. If the imbalance of the producer goes in the opposite direction to the imbalance of the global system (the producer imbalance helps to compensate the global imbalance of the system), the imbalance price is the same as the day-ahead market price. If both, the producer and the system imbalances go in the same direction, the imbalance price may be different from the day-ahead market price. In any case, ($\lambda_{tw}^- \geq \lambda_{tw}$) and ($\lambda_{tw}^+ \leq \lambda_{tw}$). Hence, the imbalances from the submitted plan

PRODUCERS IMBALANCE	SYSTEM NEEDS	
	< 0 (lower production / higher consumption) ↓	> 0 (higher production / lower consumption) ↑
Positive Imbalance (overproduction) ↑	$\lambda_{tw}^+ \leq \lambda_{tw}$	$\lambda_{tw}^+ = \lambda_{tw}$
Negative Imbalance (underproduction) ↓	$\lambda_{tw}^- = \lambda_{tw}$	$\lambda_{tw}^- \geq \lambda_{tw}$

Fig. 1. Imbalance prices depending on the system needs.

are penalized, the positive imbalance price being lower than or equal to the day-ahead market price, and the negative imbalance price greater than or equal to the day-ahead market price (see Fig. 1 for further explanation). The mechanism for imbalance prices in European electricity markets is fully explained in [10].

If the system imbalance is positive (more generation than consumption), then:

$$\lambda_{tw}^- = \lambda_{tw} \quad (1)$$

$$\lambda_{tw}^+ = \min(\lambda_{tw}, \lambda_{tw}^{DW}). \quad (2)$$

If the system imbalance is negative (more consumption than generation), then:

$$\lambda_{tw}^+ = \lambda_{tw} \quad (3)$$

$$\lambda_{tw}^- = \max(\lambda_{tw}, \lambda_{tw}^{UP}). \quad (4)$$

B. Day-Ahead Market Offer Cases

This paper is focused on comparing the expected profit of the daily operation of wind power producers with different offer strategies.

- i) Separate wind farm offers (SO).
- ii) Separate wind farm offers with imbalance compensation (IC).
- iii) A coordinated single wind farm offer (JO).

The model proposed does not take into account the adjustment market mentioned in Section II-A. Note that, as aforementioned, the main purpose of this paper is not imbalance minimization. This paper studies the differences between trading energy in the day-ahead market with separate wind farm offers or with a joint offer through an external agent. If an adjustment market were included, the profit would probably be higher and the imbalance lower [10]. The purpose of this paper is not to determine the set of wind farms that make an optimal coalition or to determine sharing mechanisms to allocate the profits to each wind power producer. These topics have been studied in [22] using coalitional game theory. In this paper, the cluster of wind farms is provided beforehand, so the aim is to know the best offer strategy in terms of profit for a given group of wind farms, depending on the risk aversion of the wind producers.

1) *Separate Wind Farm Offers, SO*: In the SO model, each wind farm performs its own optimization. Thus, there are as many optimization problems as wind farms, N_I . Each wind farm has its own wind power scenarios. Once all the optimizations are carried out, offers, revenues, and profits are added to obtain the total offer to the day-ahead market, as well as the total

TABLE I
OPTIMIZATION INPUTS AND OUTPUTS

Separate Offer (SO) & Imbalance Compensation (IC)		
WF	Input data	Output data
1	$\lambda_{tw}, \lambda_{tw}^+, \lambda_{tw}^-, gw_{tw1}$ $\alpha, \beta, \rho_{\omega 1}$	$bw_{t1}, CVaR_1, \Delta w_{tw1}, \Delta w_{tw1}^+,$ $\Delta w_{tw1}^-, PF_{tw1}, RV_{tw1}$
\vdots	\vdots	\vdots
N_I	$\lambda_{tw}, \lambda_{tw}^+, \lambda_{tw}^-, gw_{twN_I}$ $\alpha, \beta, \rho_{\omega N_I}$	$bw_{tN_I}, CVaR_{N_I}, \Delta w_{twN_I}, \Delta w_{twN_I}^+,$ $\Delta w_{twN_I}^-, PF_{twN_I}, RV_{twN_I}$
Joint Offer (JO)		
	Input data	Output data
	$\lambda_{tw}, \lambda_{tw}^+, \lambda_{tw}^-, \Sigma_i gw_{twi}$ $\alpha, \beta, \rho_{\omega}$	$bw_t, CVaR, \Delta w_{tw}, \Delta w_{tw}^+,$ $\Delta w_{tw}^-, PF_{tw}, RV_{tw}$

TABLE II
EXPECTED PROFIT AND VARIATION OF EXPECTED PROFIT

	SO PF [€]	IC PF [€]	JO PF [€]	$\Delta PF_{SO,IC}$ [%]	$\Delta PF_{SO,JO}$ [%]
$\beta = 0.05$	60584	61763	61948	1.95	2.25
$\beta = 0.25$	60392	61428	61899	1.71	2.50
$\beta = 0.50$	59853	60660	61711	1.33	3.10
$\beta = 0.75$	59223	59890	61476	1.10	3.80
$\beta = 0.95$	58779	59359	61325	0.96	4.33

revenue and profit for the set of wind farms (5)–(7). The total CVaR is also obtained as the sum of the CVaR of all wind power producers (8). As each wind farm has to pay for its underproduction and it is paid for its overproduction, negative (9) and positive (10) imbalances are added separately in order to obtain the total amount of each one. In Table I, it can be observed that the total imbalance (Δw_{tw}) as well as the negative (Δw_{tw}^-) and positive (Δw_{tw}^+) imbalances in each period t and scenario ω are available for each wind farm as output data of the optimization models. The total standard deviation (TSD) is obtained as the sum of the standard deviations of each wind farm (11).

$$bw_t^{SO} = \sum_i bw_{ti}, \quad \forall t \quad (5)$$

$$RV_{wt}^{SO} = \sum_i \lambda_{tw} bw_{ti}, \quad \forall t \quad (6)$$

$$PF_{tw}^{SO} = \sum_i PF_{twi}, \quad \forall t \quad (7)$$

$$CVaR^{SO} = \sum_i CVaR_i \quad (8)$$

$$\Delta w_{tw}^{-SO} = \sum_i \Delta w_{twi}^- \quad (9)$$

$$\Delta w_{tw}^{+SO} = \sum_i \Delta w_{twi}^+ \quad (10)$$

$$\sigma_t^{SO} = \sum_i \sqrt{\sum_{\omega} \rho_{\omega i} (PF_{twi} - \overline{PF}_{ti})^2}. \quad (11)$$

2) *Separate Wind Farm Offers With Imbalance Compensation, IC*: In the IC model, there is an optimization for each wind farm with the same price and wind power scenarios as in the SO model. The optimization outputs are also the same as in the SO

model, as can be seen in Table I. The difference between both models is that, once the optimizations for all wind farms are done, some corrective actions are carried out.

The total offer, which is the sum of the individual offers (12), remains the same as in the SO model and, due to this fact, the revenue coming from selling energy in the day-ahead market is also the same (13). Since the CVaR depends on the auxiliary variable $\eta_{\omega i}$, it is the same for the IC and SO models for each value of the weighting parameter β (14). In the SO model, negative and positive imbalances are added separately. In the IC model, a single global imbalance is obtained by adding all wind farms imbalances, both positive and negative (15). The purpose of this is that, if there are wind farms with positive and negative imbalances in a particular period t and scenario w , then, they can compensate their imbalances and increase their profits, although the global offer is the same as in the SO model. The goal of this mechanism is to optimize its own energy offer to the day-ahead market for each wind power producer, so that each producer knows what its revenue is from selling the energy, although the imbalance cost is shared among all the producers. The global imbalance per period and scenario is equal to the summation, for all wind farms, of the difference between the positive and the negative imbalances resulting from the optimization model (15). If the global imbalance is higher than or equal to zero, then it is considered to be a positive imbalance ($\Delta w_{tw}^{+IC} = \Delta w_{tw}^{+IC}$). On the other hand, if the global imbalance is lower than zero, it is considered as a negative imbalance ($\Delta w_{tw}^{-IC} = \Delta w_{tw}^{-IC}$). This is done through (16) and (17). The expected profit is recalculated in (18), with all the terms known. The TSD is computed in (19) considering the values of the expected profit coming from (18).

$$bw_t^{IC} = bw_t^{SO} = \sum_i bw_{ti} \quad (12)$$

$$RV_{wt}^{IC} = RV_{wt}^{SO} = \sum_i \lambda_{tw} bw_{ti} \quad (13)$$

$$CVaR^{IC} = CVaR^{SO} = \sum_i CVaR_i \quad (14)$$

$$\Delta w_{tw}^{IC} = \sum_i [\Delta w_{twi}^+ - \Delta w_{twi}^-] \quad (15)$$

$$\Delta w_{tw}^{+IC} = \Delta w_{tw}^{IC} | \Delta w_{tw}^{IC} \geq 0 \quad (16)$$

$$\Delta w_{tw}^{-IC} = \Delta w_{tw}^{IC} | \Delta w_{tw}^{IC} < 0 \quad (17)$$

$$PF_{tw}^{IC} = \sum_i [\lambda_{tw} bw_{ti} - c^g gw_{twi}]$$

$$+ \lambda_{tw}^+ \Delta w_{tw}^{+IC} - \lambda_{tw}^- \Delta w_{tw}^{-IC} \quad (18)$$

$$\sigma_t^{IC} = \sqrt{\sum_{\omega} \rho_{\omega} (PF_{tw}^{IC} - \overline{PF}_t^{IC})^2}. \quad (19)$$

3) *Joint Offer, JO*: In the JO model, an optimal joint offer of the group of wind farms for the day-ahead market is proposed. There is a single optimization problem involving all wind farms. Wind power scenarios are introduced as described in Section II-C. All output data are unique, as can be observed in Table I. The aim of this model is to maximize the total expected

profit of all wind farms as if they were owned by a single producer. All the results are obtained directly from one optimization, except the TSD of the profit

$$\sigma_t^{JO} = \sqrt{\sum_{\omega} \rho_{\omega} \left(PF_{t\omega}^{JO} - \overline{PF}_t^{JO} \right)^2}. \quad (20)$$

Looking at equations (5)–(20), it is observable that the main difference between the aforementioned offer strategies is the way the imbalances are treated. This difference is specially important when comparing SO and IC models.

C. Scenario Generation

Hourly wind speed scenarios of each wind farm have been forecasted considering one year of historical data, since March 20, 2013 to March 19, 2014. The wind speed data of the meteorological stations is obtained from [23], and it is adapted from the altitudes of the meteorological stations to the altitudes of the wind turbines by

$$v(h) = v_s \cdot \left(\frac{h}{h_s} \right)^{\gamma} \quad (21)$$

where $v(h)$ is the wind speed adapted to the wind farm, v_s is the wind speed measured at the meteorological station, h is the altitude of the wind turbine, h_s is the altitude of the meteorological station and γ is a parameter which value depends on weather. The ARIMA model used for the wind forecast is explained in the Appendix.

Once wind speed scenarios are generated, they are converted into wind power scenarios through the turbine power curves. In order to obtain price scenarios, historical price time series are obtained from the Spanish transmission system operator, REE [24]. The day-ahead market price scenarios as well as the positive and negative imbalance price scenarios correspond to the same days.

D. Scenario Reduction

If there are m price scenarios and s wind power scenarios, each wind farm has a total of $m \cdot s$ scenarios using a common scenario tree. In the case of the joint offer, which involves several wind farms, if the same scenario tree were used, there would be $m \cdot s^{N_I}$ scenarios, where N_I is the number of wind farms. If the number of wind farms is greater than 2, any number raised to the N_I -th power is very high. In order to solve the problem mathematically, a scenario reduction approach is used to reduce the wind power scenarios [25]–[27]. The heuristic algorithm used is the so-called *backward reduction algorithm*, which determines the scenarios that have to be deleted.

Due to computational limitations, scenario reduction is done in several steps. Firstly, the scenario reduction algorithm explained in [25]–[27] is applied, where s^2 scenarios are reduced to s_{red} scenarios for each pair of wind farms. The pairs reduced whose original set of scenarios and the reduced set of scenarios are at the minimum distance (Kantorovich Distance) are selected for the next step.

Let Ω_{wf_i} be the set of power scenarios of wind farm i , $\Omega_{(wf_i, wf_j)}$ the set of power scenarios of the pair of wind farms i and j , and $\Omega_{red(wf_i, wf_j)}$ the set of reduced power scenarios of the pair of wind farms i and j . According to this, the cardinalities or sizes of each single (one wind farm) and combined (two wind farms) set of scenarios are:

$$\begin{aligned} |\Omega_{wf_i}| &= s, \quad \forall i = 1 \cdots N_I \\ |\Omega_{(wf_i, wf_j)}| &= s^2, \quad \forall i = 1 \cdots N_I, \quad \forall j = i + 1 \cdots N_I \\ |\Omega_{red(wf_i, wf_j)}| &= s_{red}, \quad \forall i = 1 \cdots N_I, \quad \forall j = i + 1 \cdots N_I. \end{aligned}$$

Stepwise Reduction Algorithm

The iterative reduction algorithm used for the reduction of several sets of scenarios is explained next. Note that the aforementioned algorithm only explains how to select the set of reduced scenarios to be used in each step. The reduction algorithm itself is the one studied in [25], [26].

Step 1

The number of pairs of wind farms is: $n_{pairs} = \sum_{i=1}^{N_I-1} i$

Then, the number of reduced set of scenarios is the integer part of k :

$$k = \frac{n_{pairs}}{(N_I - 1)} = \frac{N_I}{2}$$

The selected sets of reduced scenarios are chosen based on the minimum distance between the original sets of scenarios and the reduced ones.

$$\begin{aligned} \Omega_1^{(1)} &:= \Omega_{red(wf_{x_1}, wf_{x_2})} | d(wf_{x_1}, wf_{x_2}) = \min d_{ij}, \\ &\quad \forall i = 1 \cdots N_I, \quad \forall j = i + 1 \cdots N_I \\ \Omega_2^{(1)} &:= \Omega_{red(wf_{x_3}, wf_{x_4})} | d(wf_{x_3}, wf_{x_4}) = \min d_{ij}, \\ &\quad \forall i = 1 \cdots N_I | i \neq x_1, x_2, \quad \forall j = i + 1 \cdots N_I | j \neq x_1, x_2 \\ &\quad \vdots \end{aligned}$$

where x_1, x_2 refer to wind farms i and j whose distance between the original set of scenarios of the pair of wind farms and the reduced set is the minimum of the reduced set of all pairs of wind farms. They are called as $wf_{x_1}, wf_{x_2}, \dots, wf_{x_{N_I}}$ instead of $wf_1, wf_2, \dots, wf_{N_I}$ in order to avoid confusing them with wind farm number 1, wind farm number 2, \dots , wind farm number N_I .

If N_I is even, k is an integer, and the last set of reduced scenarios is given by:

$$\Omega_k^{(1)} := \Omega_{red(wf_{x_{N_I-1}}, wf_{x_{N_I}})}$$

If N_I is odd, k has a decimal part, which means that there is one original set of scenarios that has not been reduced. This set of scenarios is considered for the next reduction step.

$$\begin{aligned}\Omega_k^{(1)} &:= \Omega_{red} \left({}^{wf x_{N_I-2}, wf x_{N_I-1}} \right) \\ &\quad \left| d({}^{wf x_{N_I-2}, wf x_{N_I-1}}) = \min d_{ij}, \right. \\ &\quad \forall i = 1 \cdots N_I | i \neq x_1 \cdots x_{N_I-3}, \\ &\quad \forall j = i + 1 \cdots N_I | j \neq x_1 \cdots x_{N_I-3} \\ \Omega_{k+1}^{(1)} &:= \Omega_{wf | i \neq x_1, \dots, x_{N_I-1}}\end{aligned}$$

Step 2

$$N_I^{(1)} := \text{nearest integer to } k$$

The number of sets to be reduced is: $n_{pairs}^{(1)} = \sum_{i=1}^{N_I^{(1)}-1} i$, and the number of reduced sets is the integer part of $k^{(1)}$, where $k^{(1)} = N_I^{(1)}/2$

$$\begin{aligned}\Omega_1^{(2)} &:= \Omega_{red(x_1, x_2)}^{(1)} \left| d_{x_1 x_2} = \min d_{ij}, \right. \\ &\quad \forall i = 1 \cdots N_I^{(1)}, \forall j = i + 1 \cdots N_I^{(1)} \\ \Omega_2^{(2)} &:= \Omega_{red(x_3, x_4)}^{(1)} \left| d_{x_3 x_4} = \min d_{ij}, \right. \\ &\quad \forall i = 1 \cdots N_I^{(1)} | i \neq x_1, x_2, \forall j = i + 1 \cdots N_I^{(1)} | j \neq x_1, x_2 \\ &\quad \vdots\end{aligned}$$

If $N_I^{(1)}$ is even:

$$\Omega_{k^{(1)}}^{(2)} := \Omega_{red}^{(1)} \left({}^{x_{N_I^{(1)}-1}, x_{N_I^{(1)}}} \right)$$

If $n^{(1)}$ is odd:

$$\begin{aligned}\Omega_{k^{(1)}}^{(2)} &:= \Omega_{red}^{(1)} \left({}^{x_{N_I^{(1)}-2}, x_{N_I^{(1)}-1}} \right) \left| d_{N_I^{(1)}-2, N_I^{(1)}-1} = \min d_{ij}, \right. \\ &\quad \forall i = 1 \cdots N_I^{(1)} | i \neq x_1 \cdots x_{N_I^{(1)}-3}, \\ &\quad \forall j = i + 1 \cdots N_I^{(1)} | j \neq x_1 \cdots x_{N_I^{(1)}-3}\end{aligned}$$

$$\Omega_{k^{(1)}+1}^{(2)} := \Omega_{wf | i \neq x_1, \dots, x_{N_I^{(1)}-1}}^{(1)}$$

\vdots

Step T

$$N_I^{(T-1)} := \text{nearest integer to } k^{(T-2)}$$

$$k^{(T-1)} := \frac{N_I^{(T-1)}}{2}$$

\vdots

Step n_{steps}

The same procedure is carried out until only one set of scenarios remains. This happens when the number of steps is

$$n_{steps} = \text{rounding up to the next integer of } \left(\frac{\log N_I}{\log 2} \right)$$

At this step:

$$\begin{aligned}N_I^{(n_{steps}-1)} &:= 2 \\ k^{(n_{steps}-1)} &:= 1\end{aligned}$$

E. CVaR as a Risk Measure

Value at Risk (VaR) has been used throughout the last few years to measure and quantify the level of financial risk within an investment portfolio. However, VaR is difficult to optimize when it is calculated using the scenarios approach and for distributions that are not Normal. An alternative measure with better properties is the CVaR. CVaR is strongly related to VaR but, in the case of profits, it is lower than VaR because CVaR quantifies the tail risk. From a mathematical point of view, when maximizing profits, CVaR is the expected value of $(1 - \beta) \cdot 100\%$ of the lowest profits at a given confidence interval, α [28]. CVaR can be defined as:

$$CVaR_i = \zeta_i - \frac{1}{1 - \alpha} \sum_w \rho_{wi} \eta_{wi}, \quad \forall i \quad (22)$$

subject to the following constraints needed for a linear formulation of the CVaR:

$$- \sum_t PF_{twi} + \zeta_i - \eta_{wi} \leq 0, \quad \forall w, i \quad (23)$$

$$\eta_{wi} \geq 0, \quad \forall w, i \quad (24)$$

where PF_{twi} is the profit of the wind farm i in period t and scenario w , ζ_i is the VaR, β is the weighting factor of CVaR, α is the confidence interval and η_{wi} is an auxiliary variable used to compute CVaR. The variable η_{wi} is equal to zero if the profit of wind farm i in scenario w is greater than VaR. If the profit in scenario w is not greater than VaR, η_{wi} value is the difference between the profit of scenario w and VaR.

F. Objective Function

The risk-constrained formulation of the problem is composed of two blocks, profit and CVaR. Both blocks depend on the weighting factor, β , which models the tradeoff between the expected profit and the CVaR. As β increases, the producer becomes more risk averse. In this formulation, $\beta \in [0, 1]$. The objective function maximizes the sum of the expected profit and the CVaR. For all day-ahead market offer cases, the price scenarios, the confidence interval, α , and the weighting parameter of the CVaR, β , are the same in such a way that all cases can be compared with the same price conditions. In Table I, the inputs and outputs of the optimization problems are shown for each case. Note that the mathematical model is the same for all cases. The difference resides in the input data in each case. For the joint offer case, only one optimization is done, while for SO and IC cases one optimization is necessary for each wind farm.

$$\max(1 - \beta) \cdot PF_i + \beta \cdot CVaR_i \quad (25)$$

$$PF_i = \sum_w \rho_{wi} \sum_t (\lambda_{tw} b w_{ti} + \lambda_{tw}^+ \Delta w_{twi}^+ - \lambda_{tw}^- \Delta w_{twi}^- - c^g g w_{twi}). \quad (26)$$

The expected profit of the wind power producer is calculated as the difference between the revenues and the operation costs plus the penalties. The revenues are the energy sold in the day-ahead market, paid at the day-ahead market price, plus the positive imbalance, paid at the positive imbalance price. The negative imbalance, paid at the negative imbalance price, is considered as a penalty. Note that the expected energy sold in the day-ahead market, bw_{ti} , does not depend on the scenarios. The operational costs of the wind farm are taken from [29]. Thus, (23) becomes:

$$-\sum_t (\lambda_{tw} bw_{ti} + \lambda_{tw}^+ \Delta w_{twi}^+ - \lambda_{tw}^- \Delta w_{twi}^- - c^g gw_{twi}) + \zeta_i - \eta_{wi} \leq 0, \quad \forall w, i. \quad (27)$$

G. Constraints

The following constraints are needed for the problem resolution:

$$0 \leq bw_{ti} \leq \bar{P}_i, \quad \forall t, i \quad (28)$$

$$\Delta w_{twi} = gw_{twi} - bw_{ti}, \quad \forall t, w, i \quad (29)$$

$$\Delta w_{twi} = \Delta w_{twi}^+ - \Delta w_{twi}^-, \quad \forall t, w, i \quad (30)$$

$$0 \leq \Delta w_{twi}^+ \leq \bar{P}_i (1 - u_{twi}), \quad \forall t, w, i \quad (31)$$

$$0 \leq \Delta w_{twi}^- \leq \bar{P}_i \cdot u_{twi}, \quad \forall t, w, i. \quad (32)$$

The maximum power offer to the day-ahead market has to be lower than the capacity of the wind farm (28). The imbalance is defined as the difference between the final generation of the wind farm and the energy scheduled (29), and is decomposed into the sum of a positive and a negative imbalance (30). In order to force that positive and negative imbalances cannot exist simultaneously, a binary variable is used, u_{twi} , as in [8]. The imbalances, both positive and negative, have to be lower than the maximum capacity of the wind farm, (31) and (32). In several previous formulations, the use of u_{twi} was not necessary [10]–[12] because the optimal solution was guaranteed if one of the variables, λ_{tw}^+ or λ_{tw}^- , was equal to zero. For this purpose, the imbalance was multiplied by a ratio: the market imbalance price divided by the day-ahead market price. As is currently common in many electricity markets, such as the Iberian one, the day-ahead market price can be equal to zero, then, it is not possible to get a variable in which the denominator is equal to zero.

III. CASE STUDY

The case study is based on five wind farms. Each wind farm has 25 turbines of 2 MW. The installed wind power capacity is 50 MW per wind farm, with a total capacity for all wind farms of 250 MW. The wind farms are situated in the north of Spain. All wind turbines are assumed to be operating in every period. The number of scenarios for the day-ahead and balancing markets is 30 for both, taken from [24], [30]. The number of wind power scenarios of each wind farm is 50, 1500 being the final number of scenarios of the scenario tree for each wind farm, considering wind power and prices. For the case of the joint offer, 200 wind power scenarios are considered, with a final number of scenarios of 6000. The scenario tree used is explained in Sections II-C and

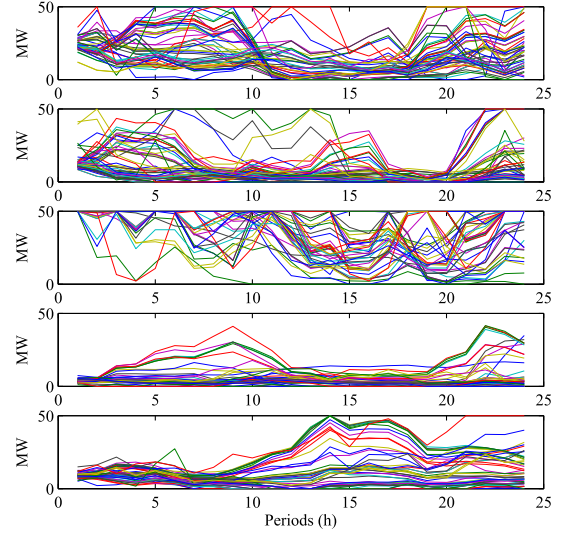


Fig. 2. Wind power scenarios of each wind farm.

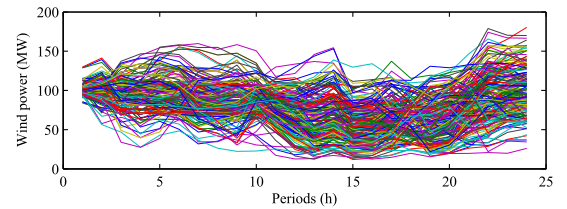


Fig. 3. Reduced wind power scenarios involving all wind farms.

II-D. The time frame of the case study is 24 hours divided into hourly periods. A confidence level $\alpha = 0.9$ is used to compute CVaR in all periods. In Fig. 2 and 3 the initial scenarios of each wind farm and the final reduced scenarios are presented, respectively.

A. Daily Expected Offer, Profit and Imbalances

The three offer mechanisms proposed in the problem description are compared in detail. As β increases, the expected offer decreases, the expected negative imbalance being lower and the expected positive imbalance greater. This means that, as producers become more risk averse, they prefer to trade lower quantities of energy in the day-ahead market to get lower negative imbalances and sell the rest of the generation in the balancing market at the positive imbalance price, λ_{tw}^+ . The variation of the expected offer and the expected imbalances with risk aversion is more pronounced in the case of separate offers, SO, and in separate offers with imbalance compensation, IC (Fig. 4). The slope of the lines representing JO variables vs. β is lower, which means that SO and IC are more influenced by risk aversion.

The expected profit is maximum in the JO for all β values. Also, the IC expected profit is higher than the SO expected profit. Since the energy offers to the day-ahead market are the same in the SO and IC cases, the producers' revenues coming from the energy sold are also equal. Thus, the differences in the expected profits come from the payments and incomes resulting from the clearing of the balancing market.

By looking at Fig. 4 and 5, it can be said that a negative imbalance represents a high loss of the expected profit. As can be

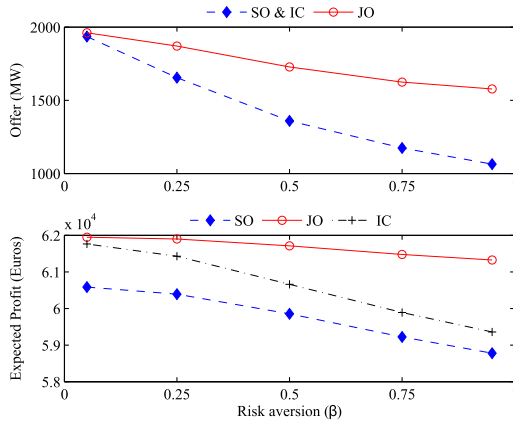


Fig. 4. Evolution of the optimal offers and expected profits with risk aversion.

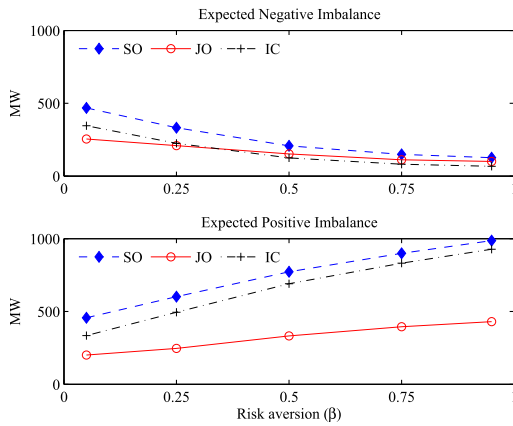


Fig. 5. Evolution of the expected imbalances with risk aversion.

 TABLE III
 IMBALANCE PERCENTAGES FOR DIFFERENT β VALUES

$\beta = 0.05$					
	Offer [MW]	Δ^- [MW]	Δ^+ [MW]	$\% \Delta^-$	$\% \Delta^+$
SO	1935.6	468.0	456.9	24.2	23.6
IC	1935.6	345.5	334.4	17.8	17.3
JO	1961.2	254.9	201.2	13.0	10.3
$\beta = 0.5$					
	Offer [MW]	Δ^- [MW]	Δ^+ [MW]	$\% \Delta^-$	$\% \Delta^+$
SO	1359.1	207.4	772.8	15.3	56.9
IC	1359.1	125.1	690.4	9.2	50.8
JO	1727.8	152.3	331.9	8.8	19.2
$\beta = 0.95$					
	Offer [MW]	Δ^- [MW]	Δ^+ [MW]	$\% \Delta^-$	$\% \Delta^+$
SO	1064.1	126.3	986.7	11.9	92.7
IC	1064.1	67.5	927.9	6.3	87.2
JO	1557.3	100.4	430.5	6.4	27.6

observed in Fig. 4 and Table III-A, as β increases, the expected profit decreases for all offer mechanisms. However, the increase of the expected profit when comparing the JO with the other cases, is greater for high β values.

Table III presents the percentages of the expected imbalances with respect to the expected offers. It is observed that the percentages of the expected negative imbalances have a much lower variation than the percentages of the expected positive

 TABLE IV
 CVaR AND TSD FOR DIFFERENT β VALUES

	CVaR [€]			TSD [€]		
	SO	IC	JO	SO	IC	JO
$\beta = 0.05$	19795	19795	38040	29134	20035	14365
$\beta = 0.25$	20925	20925	38334	28514	19561	14197
$\beta = 0.50$	21819	21819	38647	27904	19111	13987
$\beta = 0.75$	22220	22220	38793	27602	18965	13818
$\beta = 0.95$	22301	22301	38823	27487	18948	13748

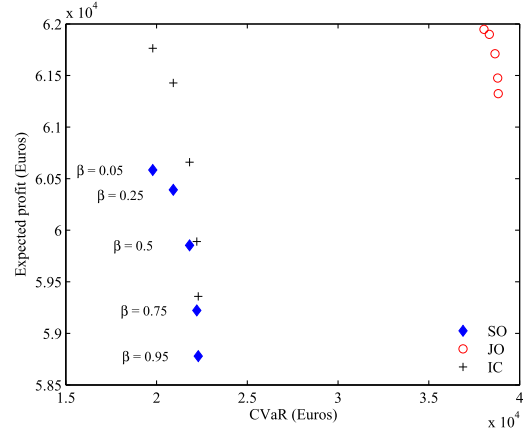


Fig. 6. CVaR efficient frontier.

imbalances, which have a significantly greater increase as producers become more risk averse. Equations (33) and (34) are used to calculate the parameters which represent the imbalance percentages, $\% \Delta^-$ and $\% \Delta^+$, respectively:

$$\% \Delta^- = \left[\frac{\sum_w \rho_w \sum_t \Delta w_{tw}^-}{\sum_t bw_t} \right] \cdot 100 \quad (33)$$

$$\% \Delta^+ = \left[\frac{\sum_w \rho_w \sum_t \Delta w_{tw}^+}{\sum_t bw_t} \right] \cdot 100. \quad (34)$$

B. CVaR and TSD Versus Expected Profit

The curve relating expected profits with CVaRs is usually known as the *efficient frontier*. In (25) the objective function is divided into two terms, the expected profit and the CVaR, and both of them are multiplied by the weighting factor. CVaR is directly multiplied by β and the expected profit by $(1 - \beta)$. Considering this equation, the producers have to select the value of β in order to rise the expected profit or the CVaR. Obviously when one of them increases, the other one decreases. In Fig. 6, the optimal efficient frontier is shown for the three offer cases.

The efficient frontier corresponding to the JO model is displaced to the upper right hand corner, with the highest expected profits and the highest CVaRs. Comparing the two models in which each wind farm makes its own offer to the day-ahead market, SO and IC, it can be observed that, for the same value of CVaR, the expected profit is greater for the IC case. It can also be observed that the variation of both, expected profit and CVaR, with risk aversion is less pronounced for the JO model, the different points of the efficient frontier being closer. This

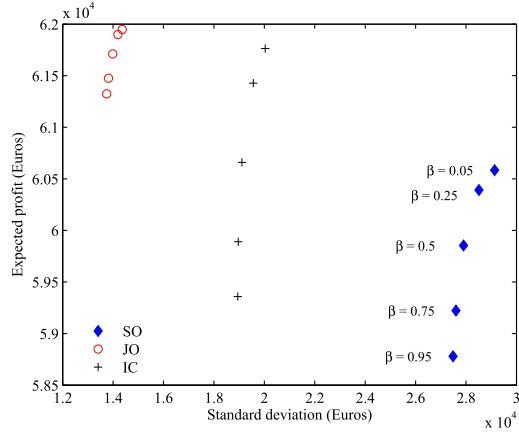

 Fig. 7. Standard deviation and expected profit for several β values.

 TABLE V
 VARIATION OF CVaR, STANDARD DEVIATION AND EXPECTED PROFIT

	$\beta = 0.05 \rightarrow \beta = 0.25$			$\beta = 0.5 \rightarrow \beta = 0.95$		
	SO	IC	JO	SO	IC	JO
Δ CVaR [%]	10.22	10.22	1.60	2.21	2.21	0.46
Δ TSD [%]	-4.22	-4.61	-2.63	-1.49	-0.85	-1.71
Δ PF [%]	-1.21	-1.79	-0.38	-1.79	-2.15	-0.63

confirms that, when trading energy with a joint offer, the producers suffer less risk impact. This is due to the risk evaluation of the profit distributions. For the SO and IC models, one optimization is done for each wind farm, which leads to one different profit distribution (depending on scenarios) for each wind farm. In the JO model, only a profit distribution is obtained and the risk evaluation is done for this distribution, which results in higher CVaR values.

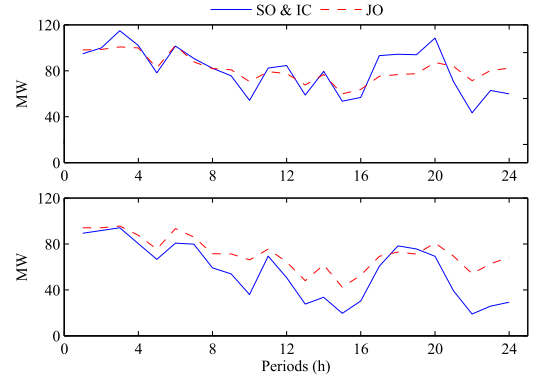
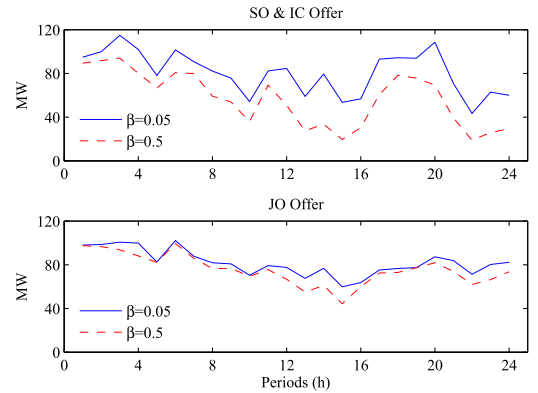
The expected profits and their corresponding standard deviations are represented in Fig. 7. As it happens with the CVaR efficient frontier, the different points relating the expected profit and its associated TSD are closer in the JO case. In all cases, as β increases, the standard deviation decreases because of the lower imbalance costs associated.

The values of CVaR and TSD are presented in Table III-B and the changes of CVaR, TSD and the expected profit for different β values are presented in Table V. It is remarkable that, from $\beta = 0.5$ onwards, the increase of CVaR is very low compared with the decrease of the expected profit.

It is expected that the maximum value of β that the producers are willing to consider is that value for which the CVaR efficient frontier begins to be almost vertical. This β value depends on the case study and the price and wind speed scenarios and may not even exist if the CVaR efficient frontier has an approximate horizontal shape.

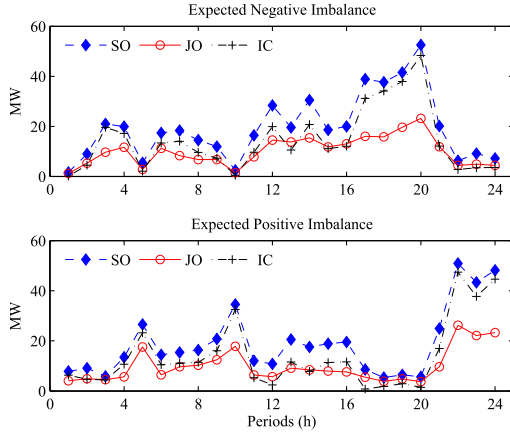
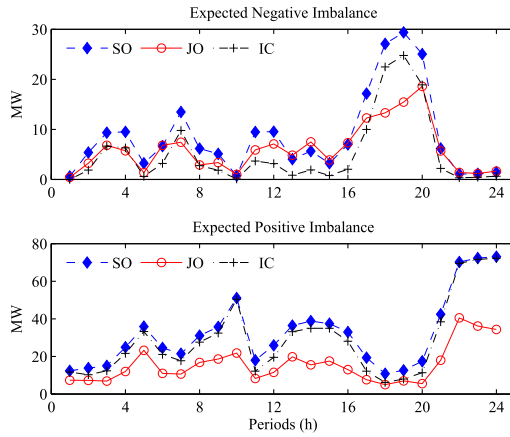
C. Hourly Analysis

In order to analyze the differences between considering high risk aversion and low risk aversion, the hourly results are compared for $\beta = 0.05$ and $\beta = 0.5$. As mentioned in Section III-A, for low values of β , in which producers are less risk averse, the expected energy sold in the day-ahead market tends to be greater than for higher β values. The optimal offers depending


 Fig. 8. Optimal offers for $\beta = 0.05$ and $\beta = 0.5$.

 Fig. 9. Day-ahead market offer comparison for $\beta = 0.05$ and $\beta = 0.5$.

on the risk aversion are presented in Fig. 8. When considering a low value of β , the expected offers for all mechanisms are very similar. However, considering a higher value of the parameter, the offer to the day-ahead market is considerably higher for the JO mechanism. This is due to the fact that, when the producers are very risk averse, they try to withhold the energy sold in the day-ahead market. Since in the SO and IC cases each producer withholds energy, the sum of the non-sold energy is more pronounced. The differences in the optimal offers depending on the risk aversion of each mechanism are shown in Fig. 9. It can be observed that the optimal offer has a lower variation in the JO case.

In Figs. 10 and 11, the expected hourly imbalances are presented for $\beta = 0.05$ and $\beta = 0.5$, respectively. When β is close to zero, the predominant imbalance is negative, and positive when it takes a value of 0.5. This happens because it is preferable to trade less energy in the day-ahead market and sell the remaining generation in the balancing market. This trend is more accused in the SO and IC cases. For both values of the weighting parameter, the lowest expected hourly negative imbalance corresponds to the IC case in some periods, and to the JO in others, with the result that the value of the mean expected daily negative imbalances are alike for the aforementioned cases. However, the expected daily positive imbalance is lower if the JO strategy is considered, although for a low β value, a lower hourly positive imbalance is associated to the IC strategy in the same periods (Fig. 10). As can be observed in Fig. 11, the expected positive


 Fig. 10. Expected imbalances for $\beta = 0.05$.

 Fig. 11. Expected imbalances for $\beta = 0.5$.

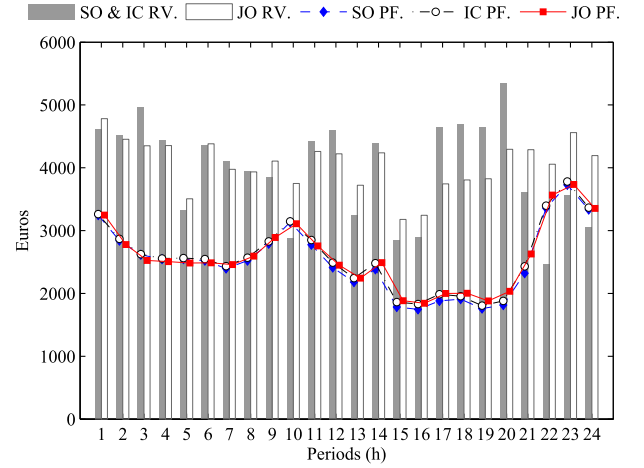
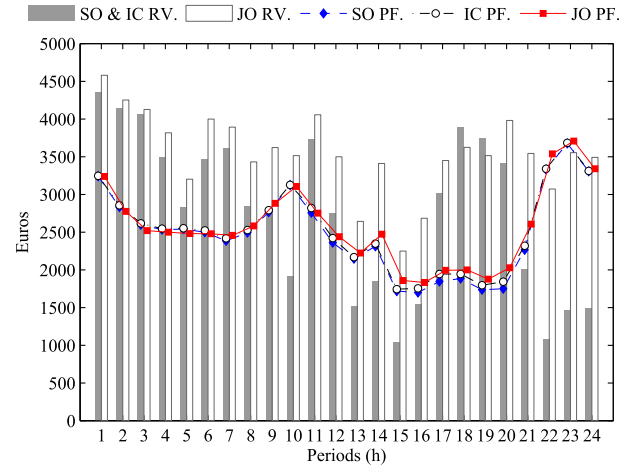
imbalance when $\beta = 0.5$ is always lower when the producers make a joint offer.

In any case, the biggest imbalances, both negative and positive, correspond to the SO model.

In fact, looking at Figs. 8–11, it is clearly observed that, when the expected offer to the day-ahead market is high (low risk aversion), the expected negative imbalances are also high and the expected positive imbalances tend to be lower, and just the opposite occurs when the producers become more conservative and trade less amount of energy to the day-ahead market, considering the same wind speed and market price scenarios.

In Fig. 12 and Fig. 13 the revenues from selling the energy in the day-ahead market are compared with the final profits. The bars represent the revenues and the profits are plotted with lines. Since SO and IC cases have the same revenue coming from the energy traded, only one bar is displayed for both.

The hourly revenues are greater for $\beta = 0.05$, being almost always higher than the expected profits, because of the amount of negative imbalances (Fig. 12). Nevertheless, when the value of β is equal to 0.5, the expected profit is higher than the revenue from selling the energy in the day-ahead market, due to the amount of positive imbalances (Fig. 13). As explained in


 Fig. 12. Expected revenue and profit for $\beta = 0.05$.

 Fig. 13. Expected revenue and profit for $\beta = 0.5$.

(26), the imbalances make the revenue different from the final expected profit. In addition, the differences among the JO expected profits and the SO and IC expected profits increase as β rises.

As an example, the periods with the highest differences between revenue and profit ($t = 19$ and $t = 22$) are compared in Table VI for $\beta = 0.5$. In period $t = 22$, the expected offer to the day-ahead market is very low for the SO and IC cases, but their positive imbalances are very large. Because of that, the expected profits increase considerably. In the IC case, the income coming from the energy sold in the balancing market accounts for 67.87% of the total expected profit. Moreover, in period $t = 19$, the revenues from selling the energy in the day-ahead market are high for both SO and IC cases and for the JO case. Notwithstanding, the negative imbalances are the highest of all periods. Thus, the expected profits suffer an important reduction since the producers have to pay for the non-generated energy at a price greater than or equal to the day-ahead market price. In the IC case this means a loss of profit of about 52% and about 51.41% in the JO case.

TABLE VI
EXPECTED REVENUES AND EXPECTED PROFITS FOR $\beta = 0.5$

t = 19						
	Offer [MW]	Δ_t^- [MW]	Δ_t^+ [MW]	RV [€]	PF [€]	PF-RV [€]
SO	75.67	29.37	12.55	3742	1736	-2006
IC	75.67	24.76	7.94	3742	1796	-1946
JO	72.94	16.76	5.44	3516	1877	-1639
t = 22						
	Offer [MW]	Δ_t^- [MW]	Δ_t^+ [MW]	RV [€]	PF [€]	PF-RV [€]
SO	18.90	1.08	70.25	1074	3334	2260
IC	18.90	0.33	69.51	1074	3342	2268
JO	54.61	1.14	37.60	3073	3541	468

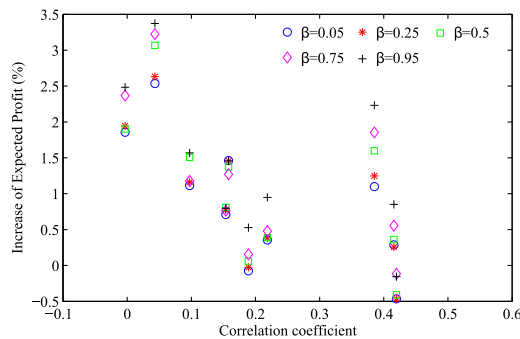


Fig. 14. Comparison of the increase of profit and the correlation coefficient for each pair of wind farms.

D. Spatial Correlation

In order to know how the spatial correlation between wind farms improves the profitability of a joint offer, the correlation coefficients between each pair of wind farms are computed as:

$$r_{ij} = \frac{S_{ij}}{\sqrt{S_{ii}S_{jj}}} \quad (35)$$

where S_{ij} denotes the sample covariance between the wind power productions of wind farm i and wind farm j . In Fig. 14 the increase in the expected profit between the SO and JO cases has been compared with the correlation coefficient of each pair of wind farms for different β values. It can be observed that the lower the correlation between wind farms, the higher the increase of the expected profit, this increase being greater when considering high β values, as seen in Table III-A. In Fig. 15 the correlation coefficients for $t = 19$ and $t = 22$ are compared with the purpose of knowing if the differences obtained when these two periods are analyzed are due to significant variations in the correlation coefficients. The degree of correlation is presented as a continuous gradient between black ($r = 0$) and white ($r = 1$). Although the correlations are a little bit higher in period $t = 22$, the variation is not meaningful enough to ensure that this is the reason for the differences between periods.

E. Computational Characterization of the Models

All the problems have been solved using MATLAB R2012a [31] and CPLEX under GAMS 24.0 [32] on a Windows 8-based Dell Server R920 with four processors Intel Xeon E7-4820 clocking at 2 GHz and 128 GB of RAM. The total CPU time

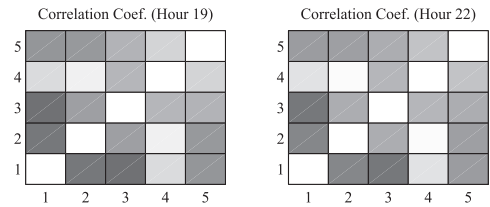


Fig. 15. Correlation matrices for all wind farms during hours 19 and 22.

TABLE VII
ORDER OF COMPLEXITY

	JO	SO & IC
Binary variables	$N_W N_T$	$N_I N_W N_T$
Continuous variables	$7N_W N_T + 2N_W + N_T + 4$	$N_I (7N_W N_T + 2N_W + N_T + 4)$
Constraints	$8N_W N_T + 2(N_W + N_T) + 3$	$N_I (8N_W N_T + 2(N_W + N_T) + 3)$

required to solve the stochastic model is 7410 s, analyzing 5 different β values. The running times of the optimization problems are 128 s for each 1500-scenario simulation and 842 s for each 6000-scenario simulation (joint offer). Table VII illustrates the comparison between the order of complexity of the proposed mechanisms. Since the optimization model is the same for all cases, the difference between them requires to run the model N_I times for the SO and IC cases. The number of binary variables and constraints depends on the number of scenarios (N_W) and the number of periods (N_T). However, the number of scenarios is more relevant for the number of continuous variables.

IV. CONCLUSION

In this paper, an optimal joint offer to the day-ahead market for a group of wind farms is compared with other offering strategies using a stochastic programming approach. Wind power producers cope with uncertainty in wind speed and market prices. The risk aversion of the wind power producers has been incorporated using the CVaR. A complete study has been carried out with extended results to compare different strategies for offering energy to the day-ahead market considering different risk aversions. The analysis allows wind power producers to obtain greater profits while ensuring a high CVaR. The main conclusion of this paper is that an optimal joint offer for trading energy through an external agent is more profitable for the wind power producers. The system operation also improves due to the imbalance reduction. It has been found that the risk evaluation is different depending on the offer strategy. Also, there is a clear trend to withhold the energy offered to the day-ahead market in order to sell it in the balancing market and reduce the negative imbalance. This trend is more pronounced as producers become more risk averse and when they offer their energies separately.

APPENDIX WIND FORECAST

Hourly wind speed scenarios of each wind farm have been forecasted with time series models [33] using ECOTOOL [34], a MATLAB toolbox [31], considering one year of historical data.

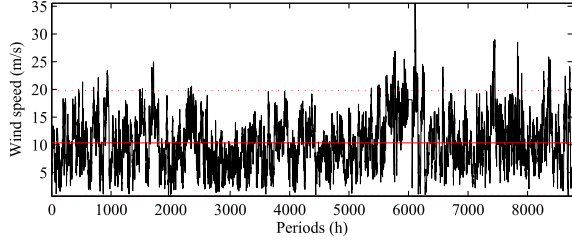


Fig. 16. Wind speed adapted to the wind farm altitude.

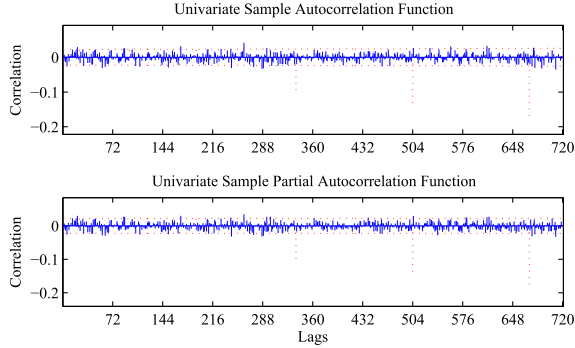


Fig. 17. ACF and PACF of the residuals.

The procedure carried out for the wind speed forecast used to compute the scenarios for one of the wind farms is explained. The rest of wind farms follow the same procedure. The time series presented in Fig. 16, composed of 8760 hours of historical wind speed data (one year), adapted to the altitude of the wind turbines (21), is transformed through a logarithmic transformation to make the dispersion constant when the mean rises. Then, the mathematical transformation of the time series is applied to the ARIMA model.

The proposed general ARIMA formulation is the following:

$$y_t = c + \frac{1}{(1-B)^{d_0}(1-B^{s_1})^{d_1} \dots (1-B^{s_k})^{d_k}} \times \frac{\theta_{q_0}(B)}{\phi_{p_0}(B)} \frac{\theta_{q_1}(B^{s_1})}{\phi_{p_1}(B^{s_1})} \dots \frac{\theta_{q_k}(B^{s_k})}{\phi_{p_k}(B^{s_k})} \epsilon_t \quad (36)$$

where y_t is the observed time series, ϵ_t is the residual term, s_j , $j = 0, \dots, k$ are a set of seasonal periods, $s_0 = 1$, $(1-B_j^s)$, $j = 0, 1, \dots, k$ are the $k+1$ differencing operators necessary to reduce the time series to achieve mean stationary, $\phi_{p_j}(B^{s_j})$ and $\theta_{q_j}(B^{s_j})$, $j = 0, 1, \dots, k$ are the AR and MA polynomials of the backshift operator B : $B^l y_t = y_{t-l}$ of $\theta_{q_j}(B^{s_j}) = (1 + \theta_1 B^{s_j} + \theta_2 B^{2s_j} + \dots + \theta_j B^{q_j s_j})$ and c is a constant.

The particular ARIMA model used is as follows:

$$\log y_t = c + \frac{1}{(1-B)(1-B^{24})(1-B^{168})} \times \frac{(1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3)}{(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3)} \times \frac{(1 - \theta_{24} B^{24})}{(1 - \phi_{24} B^{24} - \phi_{48} B^{48} - \phi_{96} B^{96})} \times \frac{1}{(1 - \phi_{168} B^{168} - \phi_{336} B^{336} - \phi_{504} B^{504})} \epsilon_t. \quad (37)$$

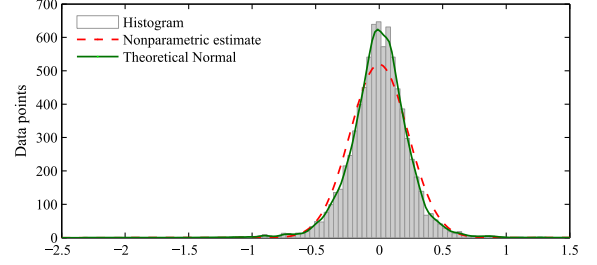


Fig. 18. Histogram of the residuals.

The particular ARIMA model presents three differencing operators necessary to achieve mean stationary and selects the seasonal periods: 1, 24, and 168, reducing the trend. The $AR(\phi)$ polynomial parameters used are: 1, 2, 3, 24, 48, 96, 168, 336, and 504. In the case of the $MA(\theta)$ polynomial, the parameters selected are 1, 2, 3, and 24.

The residuals of the particular ARIMA model follow a white noise: zero mean, constant variance, uncorrelated process and Normal distribution. The autocorrelation function (ACF) and the partial autocorrelation function (PACF) are portrayed in Fig. 17 and the histogram of the residuals in Fig. 18.

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