

Risk-Constrained Scheduling and Offering Strategies of a Price-Maker Hydro Producer Under Uncertainty

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Abstract—This paper presents a stochastic mixed-integer linear programming (SMILP) approach to maximize total expected profit of one price-maker hydro producer in a pool-based electricity market. Head dependence, commitment decisions, discharge ramping, startup costs and forbidden zones are all effectively handled in our approach. Uncertainty about the competitors' offers is adequately represented by residual demand curves (RDCs) scenarios. The management of risk is suitably addressed by conditional value-at-risk (CVaR) to provide the efficient frontier, i.e., the solutions set for which the expected profit may not be augmented without enlarging the variance of profit. Appropriate offering strategies to the pool are also developed, consisting of supply functions built for different risk levels. A representative cascaded hydro system with 7 reservoirs is considered to analyze and compare risk-neutral versus risk-averse results.

Index Terms—Hydro scheduling, offering strategies, price-maker, risk management, uncertainty.

NOTATION

The notation employed in this paper is shown below:

A. Indexes, Parameters, and Sets

j, J	Reservoir index and set.
k, K	Period index and set.
ω, Ω	Residual demand curve (RDC) scenario index and set.
s, S	RDC step index and set.
r, R	Volume interval index and set.
i, I	Unit performance curve breakpoint index and set.
M_j	Reservoirs upstream set relative to reservoir j .

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δ	Confidence level.
ρ_ω	Probability of scenario ω .
α	Weighting parameter.
$\lambda_{\omega,k}$	Market price in period k in scenario ω .
SU_j	Start-up cost of plant j .
p_j^{\max}, p_j^{\min}	Power output bounds of plant j .
v_j^{\max}, v_j^{\min}	Water volume bounds of reservoir j .
t_j^{\max}, t_j^{\min}	Water discharge bounds of plant j .
R_j	Discharge ramping limit of plant j .

B. Continuous Variables

ζ	Value-at-risk.
η_ω	Auxiliary variable in scenario ω .
$\pi_{\omega,j,k,i}$	Weight of breakpoint i for plant j in period k in scenario ω .
$p_{\omega,j,k}$	Power output of plant j in period k in scenario ω .
$v_{\omega,j,k}$	Water volume in reservoir j at the culmination of period k in scenario ω .
$a_{j,k}$	Water inflow to reservoir j throughout period k .
$t_{\omega,j,k}$	Water discharged by plant j in period k in scenario ω .
$s_{\omega,j,k}$	Water spilled by reservoir j in period k in scenario ω .
$T_{j,i}$	Water discharged by plant j at breakpoint i .
$P_{j,i,r}$	Power output of plant j at breakpoint i for interval r .
$H_{j,r}$	Water volume in reservoir j for interval r .
$q_{\omega,k}$	Hydro producer's quota in period k in scenario ω .
$\lambda_{\omega,k,s}$	Market price matching step s of the RDC in period k in scenario ω .
$f_{\omega,k,s}$	Fraction of the quota of the producer for the selected RDC step s in period k in scenario ω .
$q_{\omega,k,s}^{\min}$	Power blocks summation from step 1 to step $s - 1$ of RDC in period k in scenario ω .

C. Binary Variables

$y_{\omega,j,k}$	Decision to start-up plant j in period k in scenario ω .
$z_{\omega,j,k}$	Decision to shut-down plant j in period k in scenario ω .
$w_{\omega,j,k}$	Unit commitment of plant j in period k in scenario ω .
$d_{\omega,j,k,r}$	Binary variable equal to 1 if $H_{j,r-1} \leq v_{\omega,j,k} < H_{j,r}$.
$m_{\omega,j,k,i}$	Binary variable equal to 1 if $T_{j,i-1} \leq t_{\omega,j,k} < T_{j,i}$ or $T_{j,i} \leq t_{\omega,j,k} < T_{j,i+1}$.
$u_{\omega,k,s}$	Binary variable equal to 1 if step s is the final step to achieve quota $q_{\omega,k}$ in period k in scenario ω .
$g_{\omega,\omega',k}$	Binary variable linking the offers in period k for scenarios ω and ω' .

D. Functions

$\lambda_{\omega,k}(q_{\omega,k})$	RDC in period k in scenario ω .
$\varphi _{\tilde{v}^r}(t_{\omega,j,k})$	Linear piecewise estimation of power output function aimed at a parametric value of the water volume, \tilde{v}^r .

I. INTRODUCTION

DEREGULATION of the electricity industry has induced a formation of mechanisms to encourage competition [1]. In this framework, the aim of deregulation is to assure a clear separation between generation, transmission and distribution activities.

Hydro energy is currently one of the most significant renewable energy sources in the Portuguese system [2] and other parts of the world. Hydro units are fast in terms of operation, compared to coal-fired and natural gas units, ensuring a rapid response to load changes. This feature is important to meet peak demands and ensure network stability. Also, hydro units produce less pollution than competing technologies, being favorable alternatives for electricity generation at intermediate, peak and base loads [3].

Concerning future operations, the optimized management of the available water provides self-scheduling and embodies an important gain for hydro generating companies. Taking this into account, a hydro generating company submits optimal offers to the market, thus hydro scheduling represents a necessary tool that enables optimal bidding decisions [4].

Three time horizons are typically considered in hydro scheduling problems: short- (one day to one week), mid- (one week to one month) and long-terms (few months or even years) [5], [6]. The boundary conditions established by the mid-term scheduling are the reservoir levels that should be met at the day end [7]. Only the short-term hydro scheduling problem is considered here.

Water inflow can be taken as deterministic in the short-term, which is an acceptable assumption, especially when the time

horizon is only one day. Indeed, on a daily basis, water inflows may be forecasted with rather good precision [8]. However, in the mid- and long-terms, water inflow must be treated as stochastic [9], [10].

The market environment is typically composed of a variety of submarkets to facilitate trade between consumers and power producers, such as the pool and the bilateral contracts markets. The pool market is particularly relevant to our problem, namely the day-ahead market, in which most of the energy is negotiated. As a consequence of the market power of some producers, two types of generating companies can be listed: price-takers [11]–[13] and price-makers [14]–[16].

Price-takers accept market prices without being able to affect them. Instead, price-makers have market power, thus being able to influence market prices to increase profit [17].

Price volatility has become a major risk factor for hydro producers. Risk management allows hedging against market uncertainty [18], which in turn can be modeled via scenarios. Uncertainty of the input variables stochastic programming is required, while risk measures can be used to avoid solutions that imply small profits or major costs [19]. Inflow uncertainty in the stochastic hydro-thermal scheduling problem was considered in [20] for the mid-term within a dynamic programming framework.

In the technical literature the hydro scheduling problem has been traditionally solved using dynamic programming, posing computational difficulties due to the curse of dimensionality. It is still applied nowadays in the case of a single hydro plant, as in [21], but not for cascaded hydro systems [22].

Soft computing methods have also been used for the hydro scheduling problem, such as differential evolution [23], cultural algorithm [24], immune algorithm [25] and particle swarm optimization variants [26]. However, convergence and numerical problems may occur with the use of heuristic procedures.

Mixed-integer linear programming (MILP) has proven to be a reliable option for solving the hydro scheduling problem [27], since the nonlinearities of the problem (such as the relation between power output of a plant and the corresponding water discharged), can be precisely incorporated with a linear piecewise estimation. Also, the solution can be found within an acceptable computation time with high accuracy [28]. A stochastic model can be utilized to enable profit maximization while minimizing financial risks [29].

Hence, this paper offers a stochastic MILP approach to maximize total expected profit of one price-maker hydro producer operating in a pool-based electricity market, hedging against risk and uncertainty. The head effect is integrated with an improved linearization technique, where binary variables allow us to model performance curves of hydro units, start-up costs, commitment decisions and forbidden zones.

The cascaded configuration, stochasticity and price-maker objective function were not considered in [27].

A price-maker formulation was considered in [14] and [15], but the power output was assumed to be a linear function of the water discharged, thus disregarding the head change.

Also, in [16] the residual demand curve (RDC) was deterministic, whereas, it is stochastic in our paper, considering risk aversion.

Hence, the contributions of the paper are four-fold:

- 1) A price-maker hydro producer owning a cascaded hydro system with 7 reservoirs is modeled using an SMILP approach, considering head dependence, commitment decisions, discharge ramping, start-up costs and forbidden zones;
- 2) Uncertainty about the competitors' offers is adequately represented with RDCs scenarios;
- 3) Risk management is suitably incorporated through conditional value-at-risk (CVaR), providing the efficient frontier used by decision-makers;
- 4) Appropriate offering strategies to a pool-based electricity market are developed, consisting of supply functions built for different risk levels.

The paper is structured as follows. Section II addresses the issues related with the price-maker hydro producer considered. Section III provides the nonlinear and equivalent SMILP problem formulations, objective function and constraints. Section IV illustrates the proposed formulation with a real case study, and the conclusion is provided in Section V.

II. PRICE-MAKER HYDRO PRODUCER

In competitive electricity markets, the profitability of the generating companies depends not only on their own decisions, but also on the decisions of the other companies.

Under perfect competition, the market share of every generating company is small and no company can affect the market price. In this case, every company takes market prices for granted when devising its offering strategy, acting as a price-taker.

However, some generating companies may have a relatively high market share and are capable of exercising their market power, influencing market prices for their own benefit, meaning that a perfect competition model cannot be used, since the companies act as price-makers. When devising its offering strategy, a price-maker hydro producer takes into account the fact that it can affect market prices with its offers.

Our focus is to obtain the short-term operating strategy of a single price-maker hydro producer. Modeling the interaction among several price-makers' strategies using game theory (Nash-Cournot model) is available in [30]. The interaction of the hydro producer with the other players is modeled by RDCs in this paper. The RDC considers the market price like a function (monotonically non-increasing) of the quota of the producer [31], which in turn is the offered quantity that is accepted by the market. Thus, the market price is obtained like a function of the quantity that the price-maker company offers to the market.

A. Residual Demand Curve (RDC)

The problem may be expressed as a nonlinear one with linear constraints, since profit results of the multiplication of price by the quota.

To overcome the difficulty of having a nonlinear optimization problem, several linearizing methods can be adopted to define the RDCs, namely using approximations of the following types: 1) polynomial; 2) piecewise linear; and 3) stepwise [31].

The method adopted in this work is characterized by a stepwise approximation of the RDC, consistent with the way bids

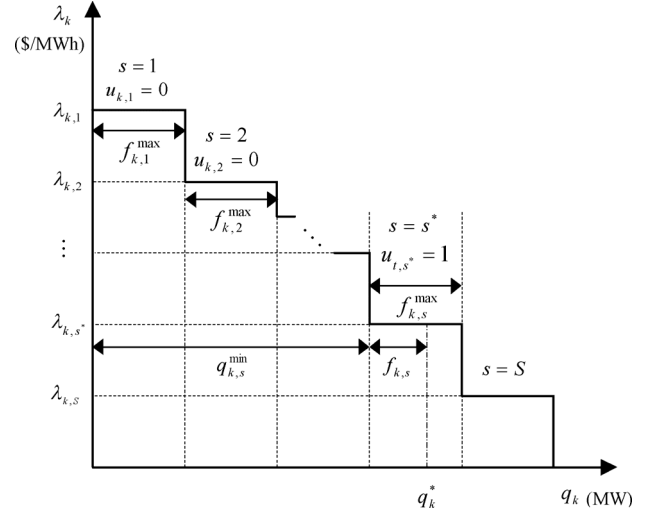


Fig. 1. Characteristic stepwise residual demand curve [33].

are made in most pool-based electricity markets. The stepwise approximation provides a closer agreement between expected and resulting prices, as stated in [31].

According to [32], the number of steps to describe a RDC is small for fairly small changes in the quota, e.g., a variation of 20% in the quota commonly results in less than 10 steps. This provides a convenient framework for the construction of the mentioned curves. Indeed, considering a high number of steps may incur in high computation times or even intractability, due to the overwhelming need of having binary variables for its modeling.

Fig. 1 presents a characteristic stepwise RDC. This curve is represented as a sequence of price-quota pairs [33] that may be given linear constraints set with the use of binary variables that define the active step.

The RDCs of a price-maker producer can be determined by using market simulation or by employing forecasting methods. The RDCs are considered in this paper as known data, as in [33]. A correlations study of RDC patterns can be seen in [34].

As mentioned before, one of our contributions is to model uncertainty by a set of RDC scenarios.

The hydro producer must choose the hourly offer curves which are submitted to the market, aiming for profit maximization. The single offer curve is made up of the points of intersection amongst the RDC scenarios and the optimal price-quantity bids; the curve should be increasing simultaneously in quotas and prices. All scenarios of a particular hour are connected by increasing (monotonically non-decreasing) constraints.

Fig. 2 shows three RDC scenarios and the corresponding offer curve built through $(q_{\omega,k}, \lambda_{\omega,k,s})$ pairs. Each RDC scenario should have just one pair $(q_{\omega,k}, \lambda_{\omega,k,s})$ that should be situated in the RDC curve.

B. Risk Management

Value-at-Risk, ζ , is defined as

$$\text{VaR} = \text{Max}(x | p\{B \leq x\} \leq 1 - \delta). \quad (1)$$

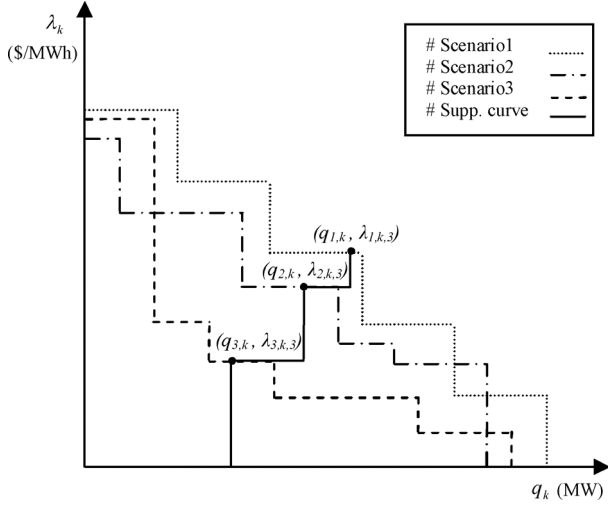


Fig. 2. Supply curve constructed over $(q_{\omega,k}, \lambda_{\omega,k,s})$ pairs.

CVaR is expected profit not surpassing ζ :

$$\text{CVaR} = E(B | B < \zeta). \quad (2)$$

The discrete formulation of CVaR is the following:

$$\text{CVaR} = \frac{1}{1-\delta} \sum_{\omega \in \Omega | B_{\omega} \leq \zeta} \rho_{\omega} B_{\omega}. \quad (3)$$

A corresponding representation of CVaR is given by

$$\text{CVaR} = \frac{1}{1-\delta} \sum_{\omega=1}^{\Omega} \rho_{\omega} \eta_{\omega} \quad (4)$$

$$\eta_{\omega} = \begin{cases} B_{\omega} & \text{if } B_{\omega} \leq \zeta \\ 0 & \text{if } B_{\omega} > \zeta. \end{cases} \quad (5)$$

The values of δ are usually between 0.90 and 0.99 [35], thus in this paper it is regarded identical to 0.95. Hence, CVaR can be formulated as

$$\text{Max} \quad \zeta - \frac{1}{1-\delta} \sum_{\omega=1}^{\Omega} \rho_{\omega} \eta_{\omega} \quad (6)$$

subject to

$$-B_{\omega} + \zeta - \eta_{\omega} \leq 0 \quad (7)$$

$$\eta_{\omega} \geq 0. \quad (8)$$

III. STOCHASTIC PROBLEM FORMULATION

The problem consists in determining the optimal price-quota combination that maximizes the total profits of a price-maker hydro producer in a day-ahead pool-based electricity market.

A. Nonlinear Problem Formulation

The operation of a hydro producer performing as a price-maker can be mathematically modeled as

$$\text{Max} \quad \sum_{\omega=1}^{\Omega} \rho_{\omega} \sum_{k=1}^K \left[\lambda_{\omega,k}(q_{\omega,k}) q_{\omega,k} - \sum_{j=1}^J \text{SU}_j y_{\omega,j,k} \right] \quad (9)$$

subject to

$$p_{\omega,j,k} \in \Pi_j \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (10)$$

$$q_{\omega,k} = \sum_{j=1}^J p_{\omega,j,k} \quad \forall \omega \in \Omega, \quad \forall k \in K. \quad (11)$$

The first term in (9) provides the revenue of the hydro producer coming from the market, while the second term corresponds to the start-up costs. The first term is nonlinear, since the producer's revenue is resulting from the multiplication of price and quota.

The set of constraints (10) allows us to model the technical features of the hydro units, such as the start-up and shutdown procedures, discrete hydro unit commitment constraints, ramp rates, minimum and maximum power output constraints, and the head effects using an improved linearization method as in [11] and [27]. The set of constraints (11) sets the quota of the price-maker as the total power output summation of its units for each period.

B. Equivalent SMILP Problem Formulation

The previous optimization problem cannot be directly solved using standard software due to its discontinuous, nonlinear, and large-scale nature. To overcome this difficulty it is assumed that the RDCs can be expressed as stepwise curves. An equivalent formulation of the problem based on SMILP is presented hereafter, using continuous and binary variables.

1) *Objective Function*: The problem can be defined as

$$\text{Max} \sum_{\omega=1}^{\Omega} \rho_{\omega} \sum_{k=1}^K \left[\sum_{s=1}^S \lambda_{\omega,k,s} (f_{\omega,k,s} + u_{\omega,k,s} q_{\omega,k,s}^{\min}) - \sum_{j=1}^J \text{SU}_j y_{\omega,k,j} \right] + \alpha \left[\zeta - \frac{1}{1-\delta} \sum_{\omega=1}^{\Omega} \rho_{\omega} \eta_{\omega} \right]. \quad (12)$$

The objective function (12) to be maximized incorporates the revenue, the start-up costs and CVaR term multiplied by a weighting positive parameter, $\alpha \in [0, \infty[$, required to attain a proper trade-off between risk and profit. A risk-neutral hydro producer corresponds to $\alpha = 0$, while a risk-averse hydro producer corresponds to large values of α . In either case the solution obtained always has an associated profit standard deviation (STD). The producer's revenues are estimated by a stepwise function employing binary variables, as shown in Fig. 1.

2) *Risk Constraints*: These constraints are defined as follows:

$$- \left[\sum_{k=1}^K \sum_{s=1}^S \lambda_{\omega,k,s} (b_{\omega,k,s} + u_{\omega,k,s} q_{\omega,k,s}) - \sum_{j=1}^J \text{SU}_j y_{\omega,k,j} \right] + \zeta - \eta_{\omega} \leq 0 \quad (13)$$

$$\eta_{\omega} \geq 0. \quad (14)$$

In (13), η_{ω} is an auxiliary variable that is identical to zero if profit is greater than the ζ ; otherwise, η_{ω} is given by the difference among them.

3) *Price-Maker Constraints*: These constraints are defined as follows:

$$q_{\omega,k} = \sum_{j=1}^J p_{\omega,j,k} \quad \forall \omega \in \Omega, \quad \forall k \in K \quad (15)$$

$$q_{\omega,k} = \sum_{s=1}^S (f_{\omega,k,s} + u_{\omega,k,s} q_{\omega,k,s}^{\min}) \quad \forall \omega \in \Omega, \quad \forall k \in K \quad (16)$$

$$0 \leq f_{\omega,k,s} \leq u_{\omega,k,s} f_{\omega,k,s}^{\max} \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (17)$$

$$\sum_{s=1}^S u_{\omega,k,s} = 1 \quad \forall \omega \in \Omega, \quad \forall k \in K, \quad \forall s \in S. \quad (18)$$

The set of constraints (15) is identical to (11). The set of constraints (16) defines the value of producer's quota, $q_{\omega,k}$, in all hours, depending on the variables $f_{\omega,k,s}$ and $u_{\omega,k,s}$. The quota's minimum value for step s is established as $q_{\omega,k,s}^{\min}$ (note that $q_{\omega,k,1}^{\min} = 0, \forall k$), while nonnegative continuous variables $f_{\omega,k,s}$ provide the added portion of step s that is occupied.

In (17) the lower bound takes nonnegative values for $f_{\omega,k,s}$, while the upper bound doesn't surpass the quota's maximum value for the selected step s of the RDC in period k ; if step s is not chosen, $u_{\omega,k,s} = 0$, then the upper bound of $f_{\omega,k,s}$ is null and, thus, $f_{\omega,k,s} = 0$.

In (18) the binary variables summation that characterizes every step s of RDC in period k is equal to one. This means that just one variable, $u_{\omega,k,s}$, is active in each period, selecting the optimal step that aggregates the producer's quota. If $u_{\omega,k,s} = 1$ then only the step s is selected for period k .

4) *Market Offer Constraints*: These constraints are defined as follows:

$$q_{\omega,k} - q_{\omega',k} \geq -M^q g_{\omega,\omega',k} \quad \forall \omega, \omega' \in \Omega, \quad \forall k \in K \quad (19)$$

$$q_{\omega',k} - q_{\omega,k} \geq -(1 - g_{\omega,\omega',k}) M^q \quad \forall \omega, \omega' \in \Omega, \quad \forall k \in K \quad (20)$$

$$\sum_{s=1}^S \lambda_{\omega,k,s} u_{\omega,k,s} - \sum_{s=1}^S \lambda_{\omega',k,s} u_{\omega',k,s} \geq -M^p g_{\omega,\omega',k} \quad \forall \omega, \omega' \in \Omega, \quad \forall k \in K, \quad \forall s \in S \quad (21)$$

$$\sum_{s=1}^S \lambda_{\omega',k,s} u_{\omega',k,s} - \sum_{s=1}^S \lambda_{\omega,k,s} u_{\omega,k,s} \geq -(1 - g_{\omega,\omega',k}) M^p \quad \forall \omega, \omega' \in \Omega, \quad \forall k \in K, \quad \forall s \in S. \quad (22)$$

These conditions are imposed for each pair of offers (ω, ω') to ensure that only monotonically non-decreasing offers are submitted to the market, as seen in Fig. 2.

Accordingly to Fig. 2, each RDC scenario should have just one pair $(q_{\omega,k}, \lambda_{\omega,k,s})$ which should be situated in the RDC. M^q and M^p represent a large quota and a large price, respectively. If $g_{\omega,\omega',k} = 0$, constraints (19) and (21) are active, whereas, if $g_{\omega,\omega',k} = 1$, constraints (20) and (22) are active.

5) *Hydro Constraints*: These constraints are defined as follows:

$$p_{\omega,j,k} - \varphi |v^r(t_{\omega,j,k}) = 0 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (23)$$

$$v_{\omega,j,k} = v_{\omega,j,k-1} + a_{j,k} + \sum_{m \in M_j} (t_{\omega,m,k-\tau_{m_j}} + s_{\omega,m,k-\tau_{m_j}}) - t_{\omega,j,k} - s_{\omega,j,k} \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (24)$$

$$v_j^{\min} \leq v_{\omega,j,k} \leq v_j^{\max} \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (25)$$

$$w_{\omega,j,k} t_j^{\min} \leq t_{\omega,j,k} \leq w_{\omega,j,k} t_j^{\max} \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (26)$$

$$t_{\omega,j,k} - R_j \leq t_{\omega,j,k+1} \leq t_{\omega,j,k} + R_j \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (27)$$

$$p_j^{\min} \leq p_{\omega,j,k} \leq p_j^{\max} \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (28)$$

$$s_{\omega,j,k} \geq 0 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (29)$$

$$y_{\omega,j,k} - z_{\omega,j,k} = w_{\omega,j,k} - w_{\omega,j,k-1} \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (30)$$

$$y_{\omega,j,k} + z_{\omega,j,k} \leq 1 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (31)$$

$$t_{\omega,j,k} - \sum_{i \in Z} T_{j,i} \pi_{\omega,j,k,i} = 0 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (32)$$

$$\sum_{i \in Z} \pi_{\omega,j,k,i} - w_{\omega,j,k} = 0 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (33)$$

$$\pi_{\omega,j,k,i} - m_{\omega,j,k,i} \leq 0 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (34)$$

$$m_{\omega,j,k,i} + m_{\omega,j,k,l} \leq 1 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K, \quad \forall l \in Z : i < l - 1 \quad (35)$$

$$\sum_{r \in R} d_{\omega,j,k,r} = 1 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (36)$$

$$p_{\omega,j,k} - \sum_{i \in Z} P_{j,i,r} \pi_{\omega,j,k,i} - \Delta P_{j,r} (1 - d_{\omega,j,k,r}) \leq 0 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (37)$$

$$v_{\omega,j,k} - \sum_{r \in R} H_{j,r-1} d_{\omega,j,k,r} \geq 0 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (38)$$

$$v_{\omega,j,k} - \sum_{r \in R} H_{j,r} d_{\omega,j,k,r} \leq 0 \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (39)$$

$$v_{\omega,j,0} = v_j^{\text{Initial}} \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K \quad (40)$$

$$v_{\omega,j,K} = v_j^{\text{Final}} \quad \forall \omega \in \Omega, \quad \forall j \in J, \quad \forall k \in K. \quad (41)$$

In (23) the power output, $p_{\omega,j,k}$, is connected to water discharged and characteristics of the reservoir.

An improved method of linearization is implemented considering the head change effect. This method corresponds to: an improvement of [11] for a parametric value of water volumes, and a more accurate assessment of the power output upper bound using a convex mixture methodology taking into account both discharges and volumes [27]. Three preset water volume quantities $H_{j,r}$ are implemented. The $p_{\omega,j,k} - q_{\omega,j,k}$ relation is denoted by a linear piecewise estimation with 4 breakpoints for each water volume.

Equation (24) represents the continuity equation (water balance) for the reservoirs. In (24) it is assumed that the water travel

time between consecutive reservoirs is less than 1 hour, due to the distances involved.

In (25), the upper and lower water volume bounds are defined. In (26), the same happens for the water discharged. The binary variable $w_{\omega,j,k}$ is identical to 1 if plant j is working in period k ; else, it is identical to zero. Constraints on discharge ramp rates are implemented in (27). In (28), the upper/lower bounds on power output are defined. Equation (29) defines water spillage as a null or non-negative value. Equations (30) and (31) model the start-up and shutdown of hydro plants.

Constraints (32)–(39) complete the model (12)–(31) approaching the power output function (23) by a parametric value of water volumes.

Equations (32)–(33) express the water to be discharged by plant j in period k . According to (34), $\pi_{\omega,j,k,i}$ may be nonzero only when the related binary variable $m_{\omega,j,k,i}$ is equal to one. Note that $0 \leq \pi_{\omega,j,k,i} \leq 1$.

Equation (36) establishes the logical value of the d variables responsible for determining the volume intervals. Equation (37) expresses the power output, $p_{\omega,j,t}$, for interval r .

Equations (38) and (39) define, for every period k , the two farthest water volumes within the interval in which the water volume, v_k , is calculated. In (40) and (41), the initial and final reservoir volumes are set.

The algorithm used to solve the entire problem is represented with a flowchart in Fig. 3. The major steps involved in the algorithm implementation are described as follows.

Step 1: Obtaining the information about the RDC scenarios of the price-maker hydro producer for each hour of the bidding period. In this problem, RDC scenarios are assumed to be known.

Step 2: Initializing the weighting parameter ($\alpha = 0$).

Step 3: Solving the SMILP hydro producer problem (12)–(41) described previously. Once the solution is obtained, the hydro producer derives its optimal hourly supply function, i.e., the optimal price and energy amount that the hydro producer needs to offer in every period.

Step 4: After obtaining the optimal offers, update the weighting parameter, α . If the chosen value of α has been obtained, the simulation is concluded; else, the simulation will continue in the previous step.

Step 5: Finally, taking into account the portfolio of the optimal solutions for all weighting parameters, it is possible to build the efficient frontier.

IV. CASE STUDY

The SMILP problem formulation has been tested on a representative cascaded hydro system with 7 reservoirs, as the one in the Douro River in Portugal. The hydro data and topology can be seen in [12].

The modeling is carried out in MATLAB environment [36] and solved using CPLEX 12.1, considering a 3.47-GHz dual processor with 48 GB RAM.

A pre-defined final water storage is included equal to 80% of the maximum storage. Storage targets can be derived from medium-term models [37].

The start-up costs of the hydro units are assumed to be given by $SU_j = p_j^{\max} \times 2.5$ [8], and forbidden zones are taken into account using (26). The time horizon is one day, since a day-ahead

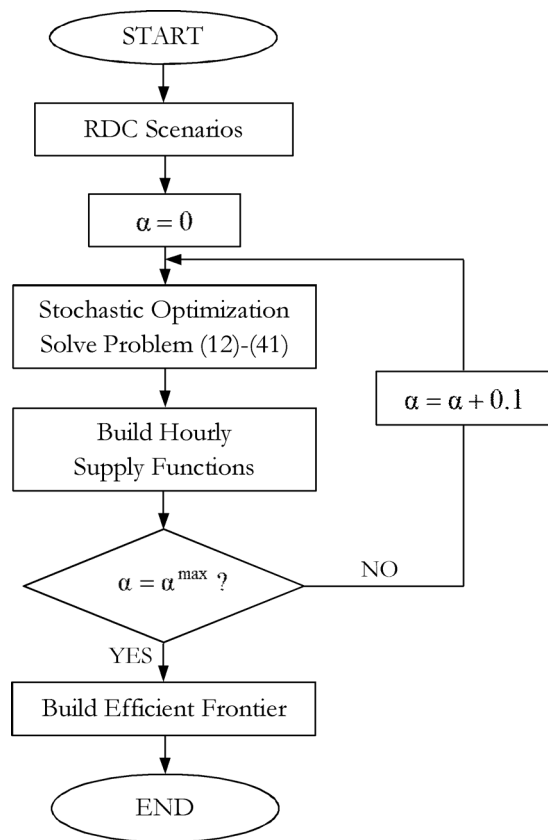


Fig. 3. Flowchart of the algorithm.

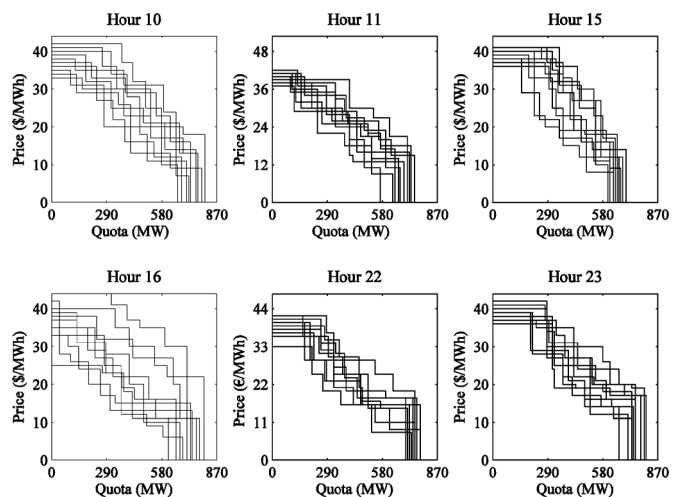


Fig. 4. RDCs scenarios using 5 steps at hours 10, 11, 15, 16, 22, and 23.

market is considered. The RDCs are given as stepwise functions with 5 steps. The total number of RDCs scenarios is 10.

Fig. 4 presents the RDCs scenarios at hours 10, 11, 15, 16, 22, and 23. The optimal solution corresponds to the specific points that define the offering strategies for each RDC. Table I provides the dimension of the optimization problem.

The efficient frontier curve, built in terms of the expected profit versus STD of profit, is given in Fig. 5 for six values of α . Analyzing Fig. 5 it can be seen that a risk-neutral hydro producer ($\alpha = 0$) aims to achieve a \$266 060 profit with a \$25 522 STD.

TABLE I
CHARACTERISTICS OF THE CASE STUDY

#	SMILP Problem Formulation
Constraints	47770
Continuous variables	14891
Binary variables	19080

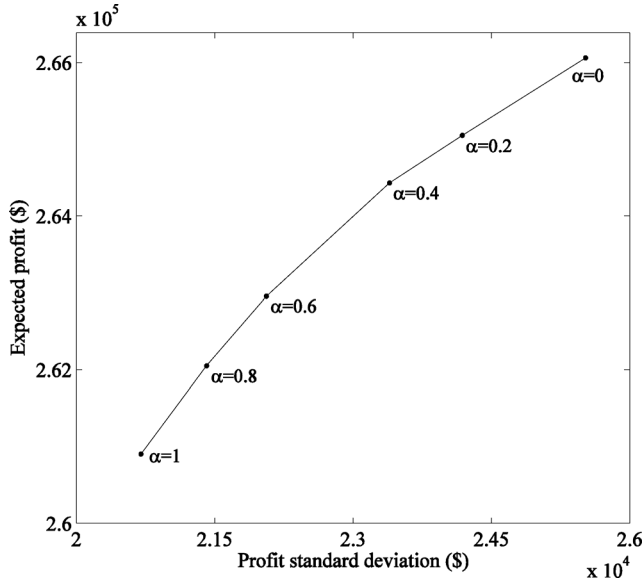


Fig. 5. Expected profit versus profit STD.

TABLE II
PROFIT VARIATION FOR SOME RISK LEVELS

Risk Level	Profit St. Dev. (\$)	CVaR (\$)	Expect. Profit (\$)	Decrease (%)	Computation Time (s)
0.0	25,522	238,576	266,060	-	1405
0.2	24,185	241,488	265,050	0.38	1458
0.4	23,396	245,207	264,430	0.61	1500
0.6	22,061	247,200	262,953	1.17	1541
0.8	21,413	248,249	262,050	1.51	1573
1.0	20,701	249,199	260,901	1.94	1592

On the contrary, a risk-averse hydro producer ($\alpha = 1$) expects a lower profit of \$260 901, with a lower STD, \$20 701.

Table II establishes the profit variation for different levels of risk, ranging from a risk-averse to a risk-neutral hydro producer. The expected profit increases about 2% from $\alpha = 1$ to $\alpha = 0$. Nevertheless, the STD of the profit may also increase significantly. Also, it can be seen in Table II that a CVaR increase of about 4.5% diminishes expected profit almost 2%. Hence, diverse hydro producers can have different attitudes towards risk, considering Fig. 5 and Table II.

Figs. 6 and 7 provide the power output of each plant for risk levels $\alpha = 0$ and $\alpha = 1$, respectively. Hydro generation is adequately distributed along the day to obtain the optimum intersections with the RDCs, i.e., the best combination (quota, price)

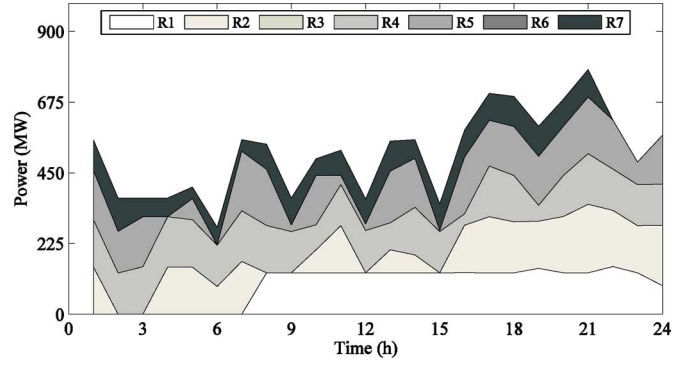


Fig. 6. Hydro units' generation in scenario 3 and risk level $\alpha = 0$.

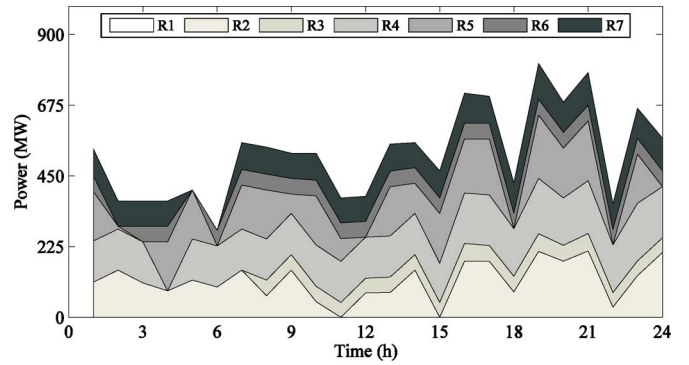


Fig. 7. Hydro units' generation in scenario 3 and risk level $\alpha = 1$.

TABLE III
COMPUTATIONAL RESULTS OBTAINED FOR SCENARIO 3

Risk Level	Av. Market Price (\$/MWh)	Av. Quota (MW)
0.0	21.92	537.65
1.0	19.96	541.21

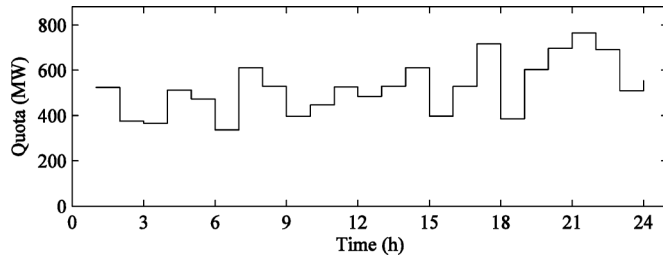
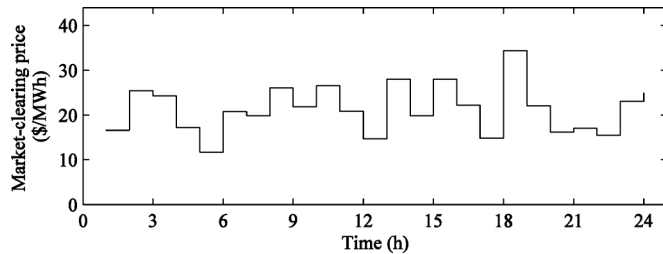
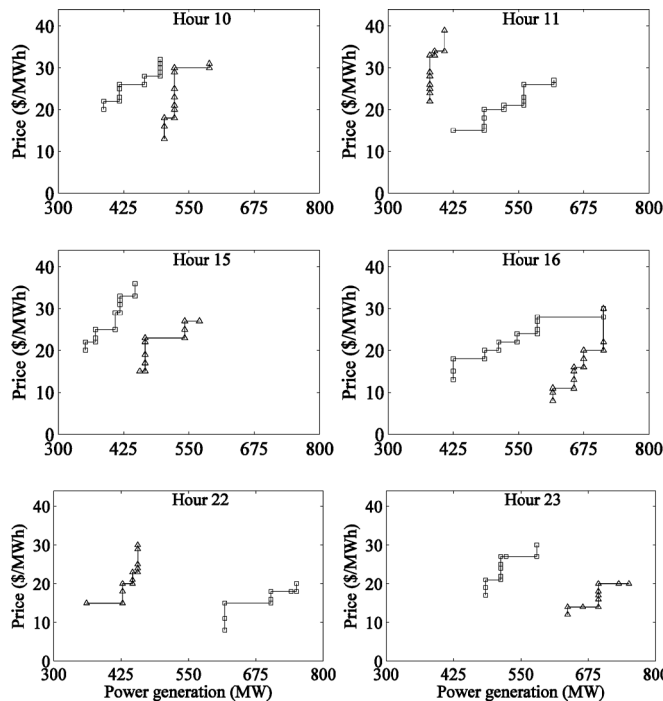
in each hour for expected profit maximization. Note that the risk-averse hydro producer shown in Fig. 7 produces a relatively high amount of energy due to lower electricity prices.

Table III shows that the average market price for a risk-averse hydro producer ($\alpha = 1$) is lower than the price of a risk-neutral hydro producer ($\alpha = 0$).

As a consequence of the lower average market price obtained by the risk-averse hydro producer, its average quota is slightly higher. Nevertheless, the price-quota combination achieved by the risk-neutral hydro producer is more profitable but at the expense of having higher values of the STDs, as shown in Fig. 5 and Table II.

The mean hourly production for the risk-neutral producer is illustrated in Fig. 8, while the respective mean market prices are illustrated in Fig. 9. Fig. 8 is equivalent to the quotas summation for all scenarios in every period k . According to Figs. 8 and 9, market prices do not commonly follow the quota variation of the hydro producer during the day, meaning that market power is enforced. This variation pattern may be employed to monitor market power.

Fig. 10 provides the offer curves to the market for the whole system as a group, considering both risk-neutral ($\alpha = 0$) and risk-averse ($\alpha = 1$) hydro producers. It can be realized that, usually, offers for $\alpha = 0$ present higher prices for smaller

Fig. 8. Mean hourly production for risk level $\alpha = 0$.Fig. 9. Mean market prices for risk level $\alpha = 0$.Fig. 10. Hourly supply functions for levels of risk $\alpha = 0$ (\square) and $\alpha = 1$ (\triangle).

amounts of energy offered, in order to increase the expected profit. Nevertheless, this is not always the case, as can be seen for hours 11 and 22 in Fig. 10.

Taking into account the mean value of the production and the market prices in each period, shown in Figs. 8 and 9, respectively, a risk-neutral hydro producer can vary the amount of market power exerted depending on the energy block chosen through those supply curves depicted in Fig. 10.

To assess the effectiveness of the stochastic approach over a deterministic approach, the value of stochastic solution (VSS) has been determined. The VSS is given by

$$VSS = Z^{S*} - Z^{D*} \quad (42)$$

as Z^{S*} and Z^{D*} represent optimal objective function values of stochastic and deterministic problems, respectively.

To have a fair and unambiguous comparison, constraints (19)–(22) were not included in this comparison. VSS is given by $VSS = \$295\,026 - \$271\,198 = \$23\,828$, i.e., $VSS(\%) = 8.79\%$.

Note that \$271 198 is the average profit achieved for the deterministic problem. The solution obtained using the stochastic approach is noticeably better than using a deterministic approach.

Hence, the SMLP approach is simultaneously accurate and computationally satisfactory.

V. CONCLUSION

RDCs scenarios and risk-aversion using CVaR are simultaneously considered in the optimal scheduling and strategies' development of a hydro producer performing as a price-maker. All major hydro generation technical characteristics are modeled through a stochastic mixed-integer linear programming formulation. The good performance of the decision making procedure is tested using a case study based on a real hydro chain in Portugal where head dependence plays a major role. The trade-off between maximizing profit and minimizing risk is provided by the efficient frontier curve, which represents an important added-value to the decision makers. The optimal combination of price-quantity bids allows us maximizing the hydro producer's expected profits in the market. This is reflected in greater expected profits attained by the risk-neutral hydro producer at the expense of higher STDs. The value of the stochastic solution shows a 9% improvement over the deterministic solution. Furthermore, the computation time is acceptable, according to the large number of continuous and binary variables, showing the value of the approach.

REFERENCES

- [1] J. P. S. Catalão, *Electric Power Systems: Advanced Forecasting Techniques and Optimal Generation Scheduling*. Boca Raton, FL: CRC, 2012.
- [2] J. P. S. Catalão, H. M. I. Pousinho, and J. Contreras, "Optimal hydro scheduling and offering strategies considering price uncertainty and risk management," *Energy*, vol. 37, no. 1, pp. 237–244, Jan. 2012.
- [3] G. K. Toh and H. B. Gooi, "Starting up rapid-start units for reserve contributions in a system with interruptible loads," *IET Gener. Transm. Distrib.*, vol. 5, no. 11, pp. 1081–1090, Nov. 2011.
- [4] S. Fleten and T. Kristoffersen, "Stochastic programming for optimizing bidding strategies of a Nordic hydropower producer," *Eur. J. Oper. Res.*, vol. 181, no. 2, pp. 916–928, Sep. 2007.
- [5] M. Pereira, S. Granville, M. Fampa, R. Dix, and L. Barroso, "Strategic bidding under uncertainty: A binary expansion approach," *IEEE Trans. Power Syst.*, vol. 20, no. 1, pp. 180–188, Feb. 2005.
- [6] M. Liu and F. F. Wu, "Risk management in a competitive electricity market," *Elect. Power Energ. Syst.*, vol. 29, no. 9, pp. 690–697, Nov. 2007.
- [7] J. Cabero, A. Baillo, S. Cerisola, M. Ventosa, A. García-Alcalde, F. Perán, and G. Relajo, "Medium-term integrated risk management model for a hydrothermal generation company," *IEEE Trans. Power Syst.*, vol. 20, no. 3, pp. 1379–1388, Aug. 2005.
- [8] J. García-González, E. Parrilla, and A. Mateo, "Risk-averse profit-based optimal scheduling of a hydro-chain in the day-ahead electricity market," *Eur. J. Oper. Res.*, vol. 181, no. 3, pp. 1354–1369, Sept. 2007.
- [9] L. Hongling, J. Chuanwen, and Z. Yan, "A review on risk-constrained hydropower scheduling in deregulated power market," *Renew. Sustain. Energy Rev.*, vol. 12, no. 5, pp. 1465–1475, June 2008.
- [10] G. B. Shrestha, B. K. Pokharel, T. T. Lie, and S.-E. Fleten, "Medium term power planning with bilateral contracts," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 627–633, May 2005.

- [11] A. J. Conejo, J. M. Arroyo, J. Contreras, and F. A. Villamor, "Self-scheduling of a hydro producer in a pool-based electricity market," *IEEE Trans. Power Syst.*, vol. 17, no. 4, pp. 1265–1272, Nov. 2002.
- [12] J. P. S. Catalão, S. J. P. S. Mariano, V. M. F. Mendes, and L. A. F. M. Ferreira, "Scheduling of head-sensitive cascaded hydro systems: A nonlinear approach," *IEEE Trans. Power Syst.*, vol. 24, no. 1, pp. 337–346, Feb. 2009.
- [13] F. J. Díaz, J. Contreras, J. I. Muñoz, and D. Pozo, "Optimal scheduling of a price-taker cascaded reservoir system in a pool-based electricity market," *IEEE Trans. Power Syst.*, vol. 26, no. 2, pp. 604–615, May 2011.
- [14] C. G. Baslis and A. G. Bakirtzis, "Optimal yearly scheduling of generation and pumping for a price-maker hydro producer," in *Proc. 7th Int. Conf. European Energy Market*, Madrid, Spain, Jun. 2010.
- [15] C. G. Baslis and A. G. Bakirtzis, "Mid-term stochastic scheduling of a price-maker hydro producer with pumped storage," *IEEE Trans. Power Syst.*, vol. 26, no. 4, pp. 1856–1865, Nov. 2011.
- [16] B. Flach, L. Barroso, and M. Pereira, "Long-term optimal allocation of hydro generation for a price-maker company in a competitive market: Latest developments and a stochastic dual dynamic programming approach," *IET Gener. Transm. Distrib.*, vol. 4, no. 2, pp. 299–314, Feb. 2010.
- [17] S. de la Torre, A. J. Conejo, and J. Contreras, "Simulating oligopolistic pool-based electricity markets: A multiperiod approach," *IEEE Trans. Power Syst.*, vol. 18, no. 4, pp. 1547–1555, Nov. 2003.
- [18] H. M. I. Pousinho, V. M. F. Mendes, and J. P. S. Catalão, "Scheduling of a hydro producer considering head-dependency, price scenarios and risk-aversion," *Energy Conv. Manag.*, vol. 56, pp. 96–103, Apr. 2012.
- [19] V. Cuiques and C. Sagastizábal, "The value of rolling-horizon policies for risk-averse hydro-thermal planning," *Eur. J. Oper. Res.*, vol. 217, no. 1, pp. 129–140, Feb. 2012.
- [20] S. Rebennack, B. Flach, M. Pereira, and P. Pardalos, "Stochastic hydro-thermal scheduling under CO₂ emissions constraints," *IEEE Trans. Power Syst.*, vol. 27, no. 1, pp. 58–68, Feb. 2012.
- [21] J. Pérez-Díaz, J. Wilhelmi, and L. Arévalo, "Optimal short-term operation schedule of a hydropower plant in a competitive electricity market," *Energy Convers. Manag.*, vol. 51, no. 12, pp. 2955–2966, Dec. 2010.
- [22] A. F. Ribeiro, M. C. M. Guedes, G. V. Smirnov, and S. Vilela, "On the optimal control of a cascade of hydro-electric power stations," *Elect. Power Syst. Res.*, vol. 88, pp. 121–129, Jul. 2012.
- [23] X. H. Yuan, Y. C. Zhang, L. Wang, and Y. B. Yuan, "An enhanced differential evolution algorithm for daily optimal hydro generation scheduling," *Comput. Math. Appl.*, vol. 55, no. 11, pp. 2458–2468, Jun. 2008.
- [24] X. H. Yuan, Y. Y. Wang, J. Xie, X. W. Qi, H. Nie, and A. J. Su, "Optimal self-scheduling of hydro producer in the electricity market," *Energy Convers. Manag.*, vol. 51, no. 12, pp. 2523–2530, Dec. 2010.
- [25] X. Fu, A. Q. Li, L. P. Wang, and C. M. Ji, "Short-term scheduling of cascade reservoirs using an immune algorithm-based particle swarm optimization," *Comput. Math. Appl.*, vol. 62, no. 6, pp. 2463–2471, Sep. 2011.
- [26] A. Mahor and S. Rangnekar, "Short term generation scheduling of cascaded hydro electric system using novel self adaptive inertia weight PSO," *Int. J. Elect. Power Energy Syst.*, vol. 34, no. 1, pp. 1–9, Jan. 2012.
- [27] A. Borghetti, C. D'Ambrosio, A. Lodi, and S. Martello, "An MILP approach for short-term hydro scheduling and unit commitment with head-dependent reservoir," *IEEE Trans. Power Syst.*, vol. 23, no. 3, pp. 1115–1124, Aug. 2008.
- [28] G. W. Chang, M. Aganagic, J. G. Waight, J. Medina, T. Burton, S. Reeves, and M. Christoforidis, "Experiences with mixed integer linear programming based approaches on short-term hydro scheduling," *IEEE Trans. Power Syst.*, vol. 16, no. 4, pp. 743–749, Nov. 2001.
- [29] L. V. L. Abreu, M. E. Khodayar, M. Shahidehpour, and L. Wu, "Risk-constrained coordination of cascaded hydro units with variable wind power generation," *IEEE Trans. Sustain. Energy*, vol. 3, no. 3, pp. 359–368, Jul. 2012.
- [30] L. Barroso, R. Carneiro, S. Granville, M. Pereira, and M. Fampa, "Nash equilibrium in strategic bidding: A binary expansion approach," *IEEE Trans. Power Syst.*, vol. 21, no. 2, pp. 629–638, May 2006.
- [31] A. R. M. Guerra, J. L. M. Ramos, and A. G. Expósito, "A comparison of residual demand models for oligopolistic markets," in *Proc. Power Syst. Comput. Conf.*, 2005.
- [32] A. Conejo, J. Contreras, J. Arroyo, and S. Torre, "Optimal response of an oligopolistic generating company to a competitive pool-based electric power market," *IEEE Trans. Power Syst.*, vol. 17, no. 2, pp. 424–430, May 2002.
- [33] S. de la Torre, J. M. Arroyo, A. J. Conejo, and J. Contreras, "Price maker self-scheduling in a pool-based electricity market a mixed-integer LP approach," *IEEE Trans. Power Syst.*, vol. 17, no. 4, pp. 1037–1042, Nov. 2002.
- [34] E. Lobato, A. Franco, L. Rouco, J. Fernandez-Caro, J. de-Benito, J. Chofre, and J. De-la-Hoz, "Stochastic model of residual demand curves with decision trees," in *Proc. IEEE Power Engineering Society General Meeting*, Toronto, ON, Canada, 2003.
- [35] A. J. Conejo, R. García-Bertrand, M. Carrión, A. Caballero, and A. Andrés, "Optimal involvement in futures markets of a power producer," *IEEE Trans. Power Syst.*, vol. 23, no. 2, pp. 701–711, May 2008.
- [36] The Mathwork Inc., MATLAB. [Online]. Available: <http://www.mathworks.com/>.
- [37] W. Uturbey and A. S. Costa, "Dynamic optimal power flow approach to account for consumer response in short term hydrothermal coordination studies," *IET Gener. Transm. Distrib.*, vol. 1, pp. 414–421, May 2007.



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