A Hybrid Probabilistic Algorithm for Computationally Efficient Estimation of Power Generation in AC Optimal Power Flow

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***Abstract*— Decentralization of power systems is creating a need for tools which can provide fast and accurate optimal power flow (OPF) solutions, without being dependent on the availability of all system information and/or uncertain variables. In this study, a hybrid probabilistic algorithm is proposed to accurately and efficiently predict ideal generation levels of individual generators to minimize the total system cost (as per AC-OPF), while having no information on the grid structure and with limited information on system variables. The proposed hybrid algorithm combines the use of correlation analysis, k-means clusters, and kernel density estimation (KDE), to predict ideal generation levels of each generator based only on historical datasets of local information (i.e. adjacent load centers). By simulating the AC-OPF problem on the IEEE 9-bus test system, a historical dataset of 1000 samples is synthetically generated and randomized local information is given as input for each agent. Quasi-deterministic Monte-Carlo simulations with 100000 samples were used for validation. In the most uncertain operating conditions, the proposed algorithm was capable of predicting the ideal generation level of the most expensive generator with a 1.65% error, while being three times faster than a Neural Network (NN), taking only 0.39 seconds to run on a standard laptop computer.**

Keywords—probabilistic analysis, optimal power flow, power system operation, k-means clustering, kernel density estimation.

I. INTRODUCTION

A. Motivation and Literature Review

While power systems have profoundly changed in recent years both in terms of planning and operation, their primary function remains: feed all loads as economically as possible, in a continuous manner and with high quality service [1]. To ensure system safety and reliability, system performance and conditions are continuously reassessed and associated costs need to be reduced to the minimal. This often requires the continuous employment of complex optimization algorithms which take into account the available generation costs and limitations, structure of the power grid, and the associated physical constraints such as transmission line power and bus voltage angle constraints [2].

As such, in the optimal power flow (OPF) problem, the optimal generation level of each generator is determined such that the total load demand in the system is met. Taking into account the aforementioned constraints, the OPF objective function is to minimize the total cost of the network without jeopardizing its security.

Different variations of the OPF problem can be performed based on additional objectives, the most common being, improvement of voltage angle, minimization of energy losses in the grid, through the setting of the ideal control variables, to meet the system operational constraints [3]–[5].

By accurately modeling all AC load flow equations of power systems, the AC OPF problem becomes a highly complex and highly non-linear one to solve. Therefore, there has been, and continues to be, an interest in developing computationally efficient methods to solve the problem and obtain an exact solution.

Traditionally, deterministic methods based on linear programming were employed to solve the OPF problem, which became popular due to their reliability to obtain an exact solution. However, such approaches can only be applied if the objective functions are differentiable and continuous, and do not consider parameters of uncertainty [6], [7].

With the proliferation of renewable energy sources (RES) and their inherently intermittent nature, accounting for uncertainties in OPF problems became inevitable. Quasi-deterministic approaches such as Monte-Carlo (MC) simulations became popular in finding an "exact" solution in the presence of uncertainties.

The main premise of such methods is to run a very large number of simulations to cover the full possible range of values of uncertain variables. Probability distribution functions (PDFs) could then be constructed by mapping the output variables to the input ones.

As such, MC simulations became the standard quasi-deterministic approach to solve AC OPF problems in the presence of uncertainties, despite being tremendously computationally expensive.

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A variety of approximate methods were developed for faster analysis, provided that prior knowledge of the uncertain variables PDFs was available and the network load flow equations are accurately modeled [8], [9].

Meanwhile, power systems are becoming increasingly decentralized due to the combined effect of different factors ranging from electricity market reforms, to the proliferation of distributed energy resources (DER) and prosumer-based smart grid (SG) structures. This poses an additional challenge to deterministic OPF solvers, since they require complete information about the system constraints and structure. In traditional, centralized, operation structures the system operator would have unlimited access to all information about the current state of the system and thereby be able to solve the deterministic OPF problem. However, in decentralized systems local agents must determine their optimal operation strategy based on locally available information, seldom having access to all system information [10], [11].

One solution effort to solve this issue has been the employment of decomposition techniques to OPF, such as the alternative direction method of multipliers (ADMM) and others [12], [13]. However, such decomposition-based methods often suffer from convergence problems and are difficult to generalize for use with any generic power system configuration (i.e., they must be tuned/configured for each system to achieve reliable performance) [14].

Another solution approach is the use of probabilistic methods. While having been around for a long time, they have recently drawn attention in scientific literature as a suitable approach to obtain fast and reliable solutions for OPF problems, particularly in the presence of high levels of uncertainties. In addition, probabilistic methods only rely on the statistical relationships between input and output variables without the need of any deterministic model of the system. Therefore, lack of certain information about the system or different grid configurations are irrelevant for the operation of these methods, making them attractive to deal with the aforementioned challenges [8], [15].

B. Contributions

In this study, a hybrid probabilistic algorithm is proposed to accurately and efficiently predict ideal generation levels of individual generators to minimize the total system cost (as per an AC-OPF study), whilst having no information on the grid structure and limited information on system variables. This makes the proposed approach ideal for decentralized operation of power systems, being capable of giving highly accurate estimates with a very fast processing speed, relying only on local historical data.

C. Paper Organization

This manuscript is organized as follows: Section I provided the motivation behind this work and a review or recent literature pertaining to different solution approaches for the OPF problem. Section II presents the mathematical formulation of the proposed hybrid probabilistic algorithm. Section III presents the 9-bus test system, used as a case study to test and validate the proposed approach, also showing how randomized historical datasets are synthetically generated to provide local historical data for each agent. In Section IV, the proposed algorithm is tested using the synthesized historical dataset and is validated by comparison with a Neural Network (NN) and MC simulations. Finally, the conclusions and prospects for future work are presented in Section V.

II. Proposed Algorithm

A flowchart of the proposed algorithm and the analysis performed in this study is provided in Fig. 1. The figure describes the pre-processing stage, which in this study is used to synthetically generate a random set of historical variables to be used as input for the proposed hybrid probabilistic algorithm. The algorithm consists of three main components: correlation analysis, k-means clustering, and kernel density estimation (KDE). All implementation is performed using MATLAB R2019b, on a standard laptop computer with the following specifications: Intel Core i7-8550U CPU @ 1.80 GHz, 16.0 GB RAM, Windows 10 64-bit operating system. The mathematical formulation of each component of the proposed algorithm is subsequently presented in this section.

A. Correlation Analysis

Given a historical set of load demand values *PD*, and the corresponding ideal generation values *PG*, the linear dependence between them (i.e., correlation coefficient) can be calculated by means of the Pearson coefficient [16], which is defined in (1):

$$\rho(PG, PD) = \frac{1}{N_s - 1} \sum_{i=1}^{N_s} \left(\frac{PG_i - \mu_{PG}}{\sigma_{PG}} \right) \left(\frac{PD_i - \mu_{PD}}{\sigma_{PD}} \right) \quad (1)$$

where, $\rho(PG, PD)$ is the correlation coefficient between *PG* and *PD*. *Ns* is the number of samples in the historical dataset. $\mu_{PG}, \mu_{PD}, \sigma_{PG}$, and σ_{PD} are the mean and standard deviation values of *PG* and *PD*, respectively.

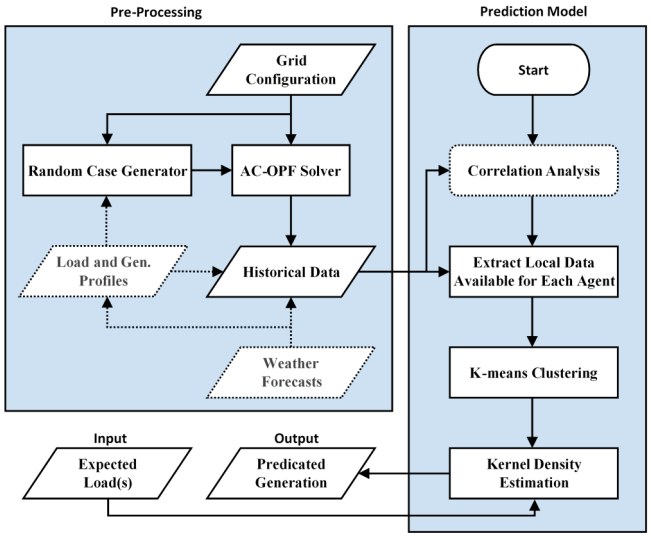

Fig. 1. Flowchart of the proposed hybrid probabilistic algorithm (right), in addition to the process used to randomly generate a synthetic historical dataset (left). Dotted elements are optional (i.e., not critical for the functionality of the algorithm), while solid elements are mandatory.

As indicated in the flowchart in Fig.1, this is optional and not critical to run the algorithm. The objective of correlation analysis is to determine the most relevant inputs in case more than one is provided. For instance, if historical data of multiple load centers is provided, those upon which the generator's optimal value is least relevant (i.e., with the lowest values of ρ) can be discarded from the analysis.

In the cases for which the proposed algorithm is intended for (decentralized operation with limited access to information), this step is seldom influential, as abundance of redundant information is rarely a problem. However, it is important to maintain this component in the algorithm as it has an insignificant computational burden while always ensuring rapid processing by preventing any redundant data from reaching the subsequent steps, which make up the majority of the processing time. This ensures that statistically irrelevant data is discarded from the beginning enhancing the computational speed of the algorithm as much as possible.

B. K-Means Clustering

The next step is to employ k-means clustering to group correlated historical data pairs into clusters, or partitions. Consider *Ns* samples of historical values of *PD* and the corresponding *PD*. Those can be represented as a dataset in the form of a vector X, with each element x_i containing a pair of values $\{PD_i, PG_i\}$ as shown in (2) and (3).

$$PG = f(PD) \tag{2}$$

$$\mathrm{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_S} \end{bmatrix} = \begin{bmatrix} \{PD_1, PG_1\} \\ \{PD_2, PG_2\} \\ \vdots \\ \{PD_{N_S}, PG_{N_S}\} \end{bmatrix} \tag{3}$$

In k-means clustering, the objective is to partition the dataset into a number of *Nc* clusters, each with a centroid *c*, such that the sum of all distances between the data points and the centroids (ϕ) is minimized:

$$\arg\min \phi \coloneqq \sum_{x \in X} \min_{c \in C} d(x, c) \tag{4}$$

The distance between a point *x* and a cluster *c* is denoted by *d(x,c)*. C is the vector of cluster centroids, each having a number of dimensions equal to the number of variables in each historical data point. In this case, each data point *x* has two variables, and so each centroid has two dimensions:

$$\mathrm{C} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{N_C} \end{bmatrix} = \begin{bmatrix} \{c_1(1), c_1(2)\} \\ \{c_1(1), c_2(2)\} \\ \vdots \\ \{c_{N_C}(1), c_{N_C}(2)\} \end{bmatrix} \tag{5}$$

As such, *d(x,c)* can be calculated with different metrics. In this study, the square Euclidean distance is used:

$$d(x, c) = \sqrt{\big(x(1) - c(1)\big)^2 - \big(x(2) - c(2)\big)^2} \tag{6}$$

To determine the location of the centroids satisfying (4), the k-means++ algorithm is employed due to its high computationally efficiency [17]:

Step 1: Choose the first centroid c_1 as a random (with uniform probability distribution) selection from the dataset.

$$c_1 \overset{R}{\leftarrow} \mathrm{X} \tag{7}$$

Step 2: Calculate the distances between the first centroid and all data points.

Step 3: Select the next centroid c_i from the dataset:

$$c_i = x' \overset{R}{\leftarrow} \mathrm{X} \tag{8}$$

In this case the random selection probability is as follows:

$$f_R = \frac{d^2(x', c_{i-1})}{\sum_{x \in \mathrm{X}} d^2(x, c_{i-1})} \tag{9}$$

Step 4: Repeat step 3 iteratively until the centroids of all N_c clusters are determined.

Step 5: Calculate the distances from each data point to each centroid, assigning each data point to the cluster with the nearest centroid.

C. Kernel Density Estimation

To estimate the PDF of a given dataset, two classes of methods exist: parametric and nonparametric. Parametric methods are employed when the data can be fitted to a typical (e.g. Normal, Gaussian, etc.) parametric distribution, i.e., when some knowledge of the PDF is known beforehand. Non-parametric methods are employed when no prior knowledge is available. The simplest and most well-known non-parametric method is histogram analysis, in which data points are grouped in bins and the frequency distribution is obtained. While being simple, it is unsuitable for large sets of data due to the large computational burden [18].

KDE is a reliable and computationally efficient non-parametric method to estimate PDFs of random variable datasets. The mathematical formulation of KDE is provided subsequently.

1) 1D Univariate KDE

Consider that for a fixed value of PD_j, a corresponding subset of historical PG_{PDj} values are available. The size of this subset can be denoted as N_{sub}. The probability of each PG_i, $\hat{f}(PG_i)$, can then be estimated using KDE as follows, to obtain the PDF for the entire range of *PG* values in this subset [19]:

$$\hat{f}\left(PG_{PD_j,i}\right) = \frac{1}{h\, N_{sub}} \sum_{k=1}^{N_{sub}} K\left(\frac{PG_{PD_j,i} - PG_{PD_j,k}}{h}\right) \tag{10}$$

$$\forall\, i = 1, 2, \ldots N_{sub}$$

K and *h* correspond to the kernel smoothing function (KSF) and its bandwidth, respectively. In this study, a normal distribution is used as the KSF. Silverman's rule is used to determine the optimal bandwidth for each data subset [20]:

$$h = \sigma \cdot \left(\frac{4}{(N_d + 2) \cdot N_s}\right)^{\frac{1}{(N_d+4)}} \tag{11}$$

where σ is the standard deviation and of the subset, and N_d is the number of dimensions.

2) 2D Bivariate KDE

In the current study the objective is to predict values of generation based on demand, meaning that the historical dataset is two-dimensional as expressed in (2)-(3). Thereby, the univariate definition of KDE must be extended to the 2D bivariate one.

In this case, take the vector of historical variables in Eq. (3) such that the number of dimensions is 2. For each variable in the dataset there exists *Ns* historical measurements (10) can then be extended to the two-dimensional case as follows [20]:

$$\hat{f}(\mathrm{X}) = \frac{1}{h_1 h_2 N_s} \sum_{i=1}^{N_{sub}} \prod_{j=1}^{2} K\left(\frac{x(j) - x_i(j)}{h_j}\right) \tag{12}$$

The bandwidth used for each dimension (h_1, h_2) is calculated by adapting Silverman's rule in (11) for a bivariate case as follows:

$$h_j = \left(\frac{\sigma_{xj}}{N_s}\right)^{\frac{1}{6}} \quad \forall \quad j = 1,2 \tag{13}$$

3) Superposition of PDFs

In the proposed algorithm, bivariate KDE is applied for each cluster of historical data. The superposition of the resulting PDFs of each cluster is then performed.

$$\hat{f}(\mathrm{X}) \approx \frac{1}{N_c} \cdot \left(\sum_{c=1}^{N_C} \hat{f}(\mathrm{X_c})\right) \tag{14}$$

where $\mathrm{X_c}$ denotes the subset of the historical data belonging to each cluster *c*, obtained as a result of the k-means clustering in the previous step. At this stage, a 2D PDF can be obtained linking historical load values to ideal generator output levels.

III. Case Study

The proposed algorithm is tested by considering the standard IEEE 9-bus test system [21], shown in Fig. 2. The system is comprised of three conventional generators. The quadratic cost function coefficients, in addition to the generator limits are listed in Table I. The network contains three load centers, two adjacent to each of the generators. The initial/default loading conditions are presented in Table II.

A. Synthetic Generation of Historical Data

In order to test the proposed method, a historical dataset is synthetically generated by running an AC-OPF analysis for 1000 ($i=1,2,\ldots,N_H=1000$) randomly generated operating scenarios on the three load buses (PD_5, PD_7, and PD_9).

$$\mathbb{H}_{PD_b} = \{PD_{1,b}, PD_{2,b}, \ldots PD_{N_H,b}\} \ \forall \ b \in \{5,7,9\} \tag{15}$$

To do this, the first step is to determine the maximum loadability of the network according to the method described in [22]. Using this method, the total load of the system is increased while maintaining the power angle (or power factor) on each load bus. By running this test it was found that the maximum loadability of this network was around 21% of the initial loading (Table II).

For each generated scenario, a loadability coefficient $\alpha_{b,i}$ is chosen at random (uniform distribution) for each load bus, such that the loadability of each bus is between 0.9 and 1.2 (maximum limit) of the initial/default value. This coefficient is then used to determine the active and reactive powers at each load bus for each randomly generated scenario, as shown in (16)-(20). Finally, each randomly generated scenario is solved as an AC-OPF problem using the MATPOWER 7.0 solver [23].

$$\alpha_{b,i} \overset{R}{\leftarrow} \mathbb{R} \mid 0.9 \leq \alpha_{b,i} \leq 1.2 \tag{16}$$

$$PD_{i,b} = PD_b^{initial} \cdot \alpha_{b,i} \tag{17}$$

$$\left|SD^{initial}\right| = \sqrt{(PD^{initial})^2 + (QD^{initial})^2} \tag{18}$$

$$pf_b = \cos(\theta_b) = \frac{PD^{initial}}{|S^{initial}|} \tag{19}$$

$$QD_{i,b} = PD_{i,b} \cdot \tan(pf_b) \tag{20}$$

As such, the required historical dataset indicated earlier in Eq. (2)-(3) is synthetically generated in order to test the proposed method on this test system. The results of the preliminary step (correlation analysis) are shown Table III between the generation levels of each generator and different combinations of load centers.

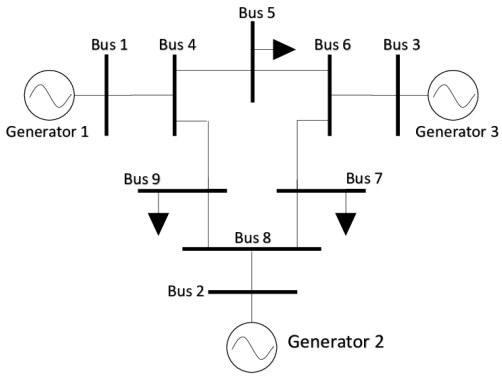

Fig. 2. Single line diagram of the IEEE 9-bus test system [21].

TABLE I. Generator specifications in the 9-bus test system: quadratic cost function coefficients and generation limits.

Generator	a	b	c	PG^{max}	PG^{min}
G1	150	5.0	0.1100	250	10
G2	600	1.2	0.0850	300	0
G3	335	1.0	0.1225	270	0

TABLE II. Initial (default) active and reactive power loading conditions at the three load centers of the 9-bus test system.

Bus	$PD^{initial}$	$QD^{initial}$	$\lvert SD^{initial}\rvert$	θ	pf
5	90	30	94.868	0.322	0.949
7	100	35	105.948	0.337	0.944
9	125	50	134.629	0.381	0.928

TABLE III. Calculated correlation indices between each agent/generator and each load center or combination thereof.

	PD_5	PD_7	PD_9	PD_{57}	PD_{59}	PD_{79}	PD_{579}
G1	0.5113	0.5071	0.7004	0.7135	0.8653	0.8649	0.9988
G2	0.4962	0.5492	0.6793	0.7344	0.8394	0.8747	1.0000
G3	0.5041	0.5492	0.6736	0.7397	0.8394	0.8702	1.0000

B. *Locally Available Data for Each Agent*

From Table III, it can be seen that each agent (generator) can accurately determine the ideal generation level if they are given information about the total load of the system (due to exact linear dependency as shown in the last column). It was previously mentioned that one of the main goals of the proposed method is the capability of accurately estimating generation values by relying only on locally available information for each agent in the network. To test this aspect, the following conditions are assumed:

- Each generator only has access to the total (aggregated) load of the two adjacent load centers (G1 to PD_{59} . G2 to PD_{79}, and G3 to PD_{57}) as shown in Fig. 3.
- The agents have no access to any other information about the network, including values of individual loads, levels of other generators, and the network configuration.

In other words, each agent/generator employs the proposed method to predict their ideal generation levels based solely on the information provided to them: the aggregated load of the most adjacent load centers. This becomes the input to the algorithm for each agent, as described in (2)-(3).

IV. Results

The results of the proposed method for each of the three agents/generators is shown in Fig. 4 and Fig 5, for the K-means clustering and the final KDE stages, respectively. The proposed method is capable of effectively producing a probabilistic estimate for each generator, satisfying the condition of minimizing overall system costs, given only local historical data.

In Fig. 5, the most likely (highest probability) values seem to exhibit steep jumps or fluctuations. This is expected due to the high non-linearity of the problem with the unknown variables creating a multi-model probability distribution for the generation at each value of adjacent load demand.

To investigate this in more detail, the results for the most expensive generator (i.e., G2) are highlighted in Fig. 6. In figure, the most probable scenario is shown, along with the confidence intervals shown corresponding to the 20% most probable scenarios. To evaluate the performance of the algorithm and validate the obtained results, an operating point which exhibits a high value of uncertainty is chosen (PD_{79}=*218.1* MW). The predicted value based on the proposed algorithm is compared with a Neural Network (1 hidden layer with 10 neurons), and quasi deterministic MC1000, 10000, and 1000000. The results listed in Table IV show that the proposed method was capable of predicting the most likely scenario with an error of 1.65%. The Neural Network managed to achieve an error of 0.86%, however was three times slower than the proposed method. The results demonstrate the exceptional computational efficiency of the proposed model.

TABLE IV. Performance Comparison between the proposed method, Neural Networks, and Monte-Carlos Simulations.

	Predicted Generation (MW)	Prediction Error (vs. MC100000)	Running Time (seconds)
Proposed Method	**131.62**	**1.65 %**	**0.39**
Neural Network	132.68	0.86 %	1.29
Monte-Carlo 1000	133.73	0.07 %	32.07
Monte-Carlo 10000	133.81	0.01 %	288.62
Monte-Carlo 100000	133.83	--	2822.72

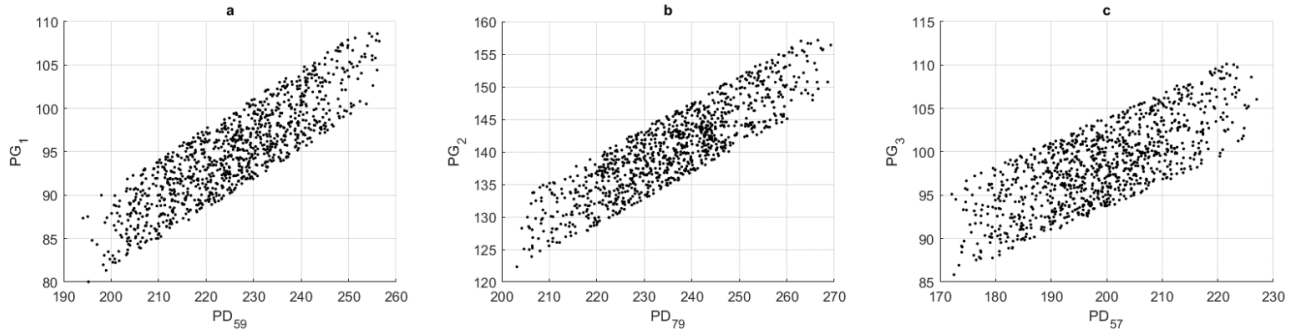

Fig. 3. Locally accessible data: (a) G1 has access to PD_{59} (aggregated load from buses 5 and 9), (b) G2 has access to PD_{79}, and (c) G3 has access to PD_{79}

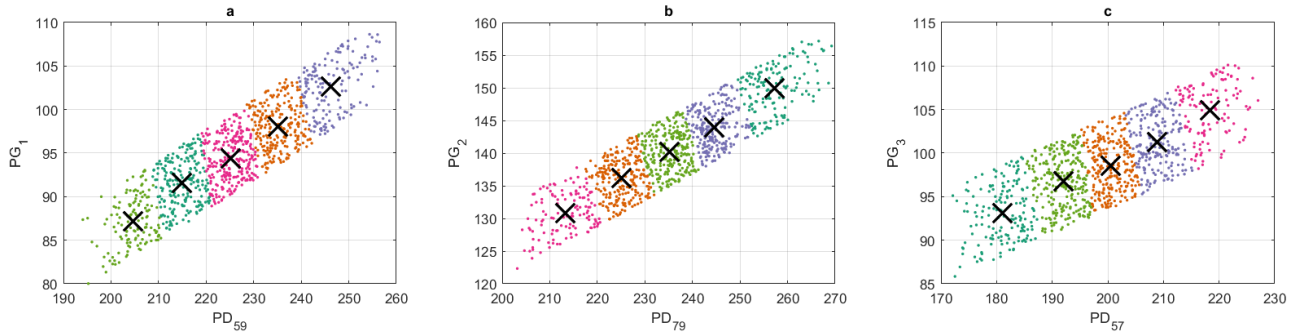

Fig. 4. Results of the k-means clustering for each input dataset for each agent/generator: (a) G1, (b) G2, and (c) G3. Colors denote data points belonging to the same cluster, and crosses indicate cluster centroids.

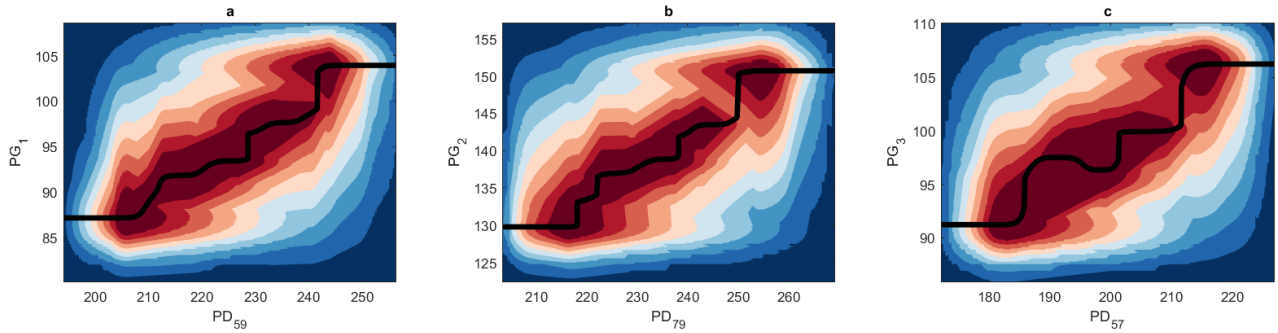

Fig. 5. Results of the 2D Bivariate KDE stage after superposing PDFs of individual clusters. Red and blue indicate higher probability and lower probabilities, respectively. The solid black line shows the estimated value (highest probability) for ideal generation corresponding to each load value.

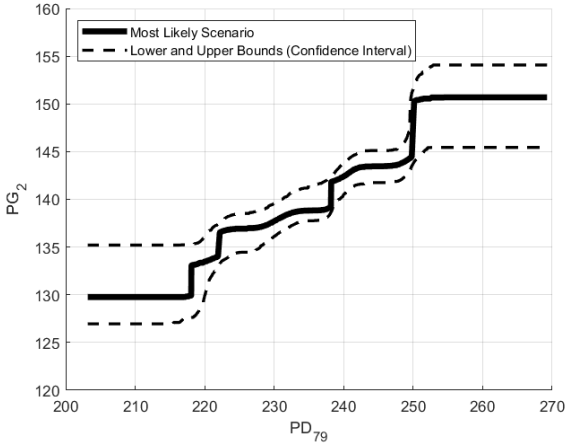

Fig. 6. Results for the most expensive generator (G2), showing most likely generation value and confidence intervals (top 20% probable scenarios).

V. Conclusions

In this study, a hybrid probabilistic algorithm was proposed to accurately and efficiently predict ideal generation levels of individual generators which minimize the total system cost (as in an AC-OPF study), while having no information on the grid structure and with limited information on system variables. The proposed algorithm combines correlation analysis, k-means clustering, and KDE to predict ideal generation levels of each generator based only on local historical information (i.e. aggregated adjacent load centers). By simulating the AC-OPF problem on the IEEE 9-bus test system, a historical dataset of 1000 samples was synthetically generated and local information was provided as input for each agent. Quasi-deterministic Monte-Carlo simulations with 100000 samples were used for validation. In the most uncertain operating conditions, the proposed algorithm was capable of predicting the ideal generation level of the most expensive generator with a 1.65% error, while being three times faster than a Neural Network (NN), taking only 0.39 seconds to run on a standard laptop computer. This method exhibits great potential for use in decentralized operation of power systems, especially when lack of system information is an issue. For future work, a study incorporating uncertainties of renewable generation in addition to the scalability of the method for larger networks is recommended.

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